UNIVERSITI PUTRA MALAYSIA

IMPROVING EMBEDDED LINEAR ELASTICITY IN DEFORMABLE MODELS USING STIFFNESS MATRIX

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FSKTM 2015 9

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By

NOR AZURA BINTI MD ALI

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Master of Science

April 2015

DEDICATION

To Fakhrou Razi,

Thank you very much for my beloved husband, for his unfailing support and contribution as an enormous and important portion of the fulfilment of this study.

To Puan Siti Jariah,

Thank you very much for my beloved mother for her support, patient and willing to take care of my lovely son and daughter throughout the duration of my study.

To Umar Uqail and Nur Sofea Khadeeja,

My lovely children, you are my inspiration and motivation to finish my study.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Master of Science

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By

NOR AZURA BINTI MD ALI

April 2015

Chair: **Lili Nurliyana Abdullah, PhD**
Faculty: **Computer Science and Information Technology**

Physical animation deformation is an important part of computer animation. Most geometric models commonly used in graphics have hundreds of thousands of vertices. Embedding is also a good approach because of its simplicity and ability to preserve geometric features but a standard embedding technique does not correctly model geometry with complex branching. Complex models may have a lot of parts with different properties of different materials. In such cases, it is more likely that a coarse element will contain a mix of materials, soft and hard, and not just one material. Therefore, it is difficult to select an appropriate material in the element, whether stiff or soft, that will deform in the same manner. Thus, many GPU-based collision detection algorithms have been limited to examining the circumstances of the collision in discrete time.

In this research, embedding of a linear elastic deformable model is presented. This research has resulted in a significant improvement in efficient animation based on physical objects that are very detailed. To perform embedding, topology information should be taken into account. This means that parts of disconnected elements that fall into the same coarse element can be animated freely. Thus, the properties of different materials are accounted for by calculating the interpolation function together with appropriate stiffness for the coarse elements that are similar to the embedded material. Finally, coarse embedding space is also included to provide a better animation of the border. The result is a simple approach to a complex deformation simulation model with ease and speed associated with coarse regular embedding, with quality and detail that can be made at a finer resolution. Finally, in order to obtain better GPU processing time compared to the computer, an anisotropic visco-hyperelastic constitutive formulation is presented for implementation in a graphical processor unit (GPU).

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk Ijazah Master Sains

MENINGKATKAN PEMBENAMAN KEANJALAN SELARI DI DALAM MODEL YANG BERUBAH BENTUK MENGGUNAKAN KEKAKUAN MATRIKS

Oleh

NOR AZURA BINTI MD ALI

April 2015

Pengerusi: **Lili Nurliyana Abdullah, PhD**
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Mengubah bentuk animasi fizikal adalah satu bahagian penting dalam animasi komputer. Kebanyakan model geometri yang biasa digunakan dalam grafik mempunyai beratus-ratus ribu mercu. Pebenaman juga adalah satu pendekatan yang baik kerana kesederhanaan dan keupayaan untuk mengekalkan ciri-ciri geometri tetapi teknik pembenaman yang biasa adalah tidak sesuai untuk geometri model yang mempunyai cawangan yang kompleks. Model kompleks mungkin mempunyai banyak bahagian dengan sifat yang berbza daripada bahan-bahan yang berbeza. Dalam kes sedemikian, ia lebih berkemungkinan bahawa unsur kasar akan mengandungi campuran bahan-bahan seperti lembut dan keras, bukan hanya satu bahan sahaja. Oleh itu, adalah sukar untuk memilih bahan yang sesuai dalam unsur, sama ada keras atau lembut akan berubah mengikut cara yang sama. Manakala kebanyakan GPU-algoritma pengesanan perlanggaran adalah terhad untuk memeriksa keadaan pelanggaran dalam masa yang diskret.

Dalam kajian ini, satu pembenaman model ubah bentuk anjal linear dibentangkan. Keputusan dalam peningkatan yang ketara dalam animasi yang berkesan berdasarkan objek fizikal yang sangat terperinci. Untuk melakukan pembenaman, maklumat topologi perlu diambil kira. Ini bermakna bahagian elemen terputus yang jatuh ke dalam unsur kasar yang sama adalah bebas. Oleh itu, sfat-sifat bahan yang berbeza menyumbang dengan mengira fungsi interpolas dan dengan kekakuan yang sesuai untuk unsur-unsur kasar yang serupa dengan bahan yang terbenam. Akhirnya, ruang pembenaman kasar juga termasuk untuk menyediakan animasi yang lebih baik daripada sempadan. Hasilnya adalah satu pendekatan yang mudah untuk model simulasi perubahan bentuk kompleks dengan mudah dan kelajuan yang berkaitan dengan pembenaman tetap kasar, dengan kualiti yang teperinci akan dibuat ada resolusi yang lebih halus. Akhir sekali, untuk mempunyai masa pemprosesan GPU yang lebih baik berbanding komputer, anisotropik Visco-hyperelastic formula dikemukakan untuk dilaksanakan pada unit pemprosesan grapfik (GPU).

ACKNOWLEDGEMENTS

Thanks to God for everything during my voyage of knowledge exploration. First and foremost, I would like to express my innermost to my supervisor Associate Professor Dr. Lili Nurliyana Abdullah to who I am indebted to for the whole my life. She is my mentor and I wish I could repay her. Next, I would like to forward my warmest appreciation to the supervisory committee member Dr. Fatimah Khalid for her guides, valuable suggestion and advice throughout this work a success.

My deepest thanks go to all the multimedia department and staff who are so kind to me and assisted me during my study. I would also like to express my highest appreciation to my course mates and friends who assisted me in this study.

I certify that a Thesis Examination Committee has met on 20 April 2015 to conduct the final examination of Nor Azura Binti Md Ali on her thesis entitled "Improving Embedded Linear Elasticity in Deformable Models using Stiffness Matrix" in accordance with the Universities and University College Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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LIST OF ABBREVIATIONS

DOF	Degree of Freedom
GPU	Graphic Processing Unit
CPU	Central Processing Unit
TLED	Total Langrangian Explicit Dynamic
CD	Collision Detection
VR	Virtual Reality
3D	Three Dimensions
2D	Two Dimensions
1D	One Dimensions
FEM	Finite Element Method
XFEM	Extended Finite Element Method
GHz	Giga Hertz
GB	Giga Byte
RAM	Random Access Memory
CUDA	Compute Unified Device Architecture
API	Application Programming Interface

CHAPTER 1

INTRODUCTION

1.1 Introduction

Embedding is mostly a generally rough calculation that animates an exceedingly complete geometric model turning it into a lower difficulty rough grid of mechanical elements (for example the use of finite element method for coarse grid animation). Embedding is also a good practice because of its effortlessness and its capability to maintain its geometry. Building a nested grid is very easy compared to a hexahedral grid because it is as effortless as identifying the vertices that drop into voxels. When a block undergoes elastic deformation, the material is deformed easily using interpolation.

The most important part of computer animation is physical animation deformation. Creating interactive animation is one of the biggest obstacles. Most geometric models that are commonly applied in graphics have hundreds of thousands of vertices. The combination of simplification, estimates, and pre-computation is important to decrease this level of difficulty to a point where a change in the physical-based interactive forms is possible.

Animation involves numerically solving partial differential equations that manage the deformation of objects that grow and move through space. Recently, the rotation model, Felippa, proposed by CA et al. (2006) has introduced geometric non-linearity to address a number of boundaries of earlier solutions, in which the constitutive law is to remain linear for all models. Miller et al. (2007) introduced the Total Langrangian algorithm explicit Dynamics (TLED) that describes a non-linear finite element algorithm. Then, a Graphics Processing Unit (GPU) based on Tylor et al. (2008) was implemented. A CPU of more than 16 times computing performance can be achieved with profits and great meshes, thus allowing the implementation of real-time animation of deformable models.

1.2 Problem Statement

Complex models may have a lot of parts with different properties of different materials. Coarse elements do not only have one material but they also contain joint materials such as hard and soft ones. Hence, it is quite complex to choose a suitable material in the element, whether soft or stiff, that will disfigure in a similar manner. For example, take a balloon with an exposed centre point seen in Figure 1. The centre is harder than the entire balloon and therefore it needs to maintain its structure when deformed.

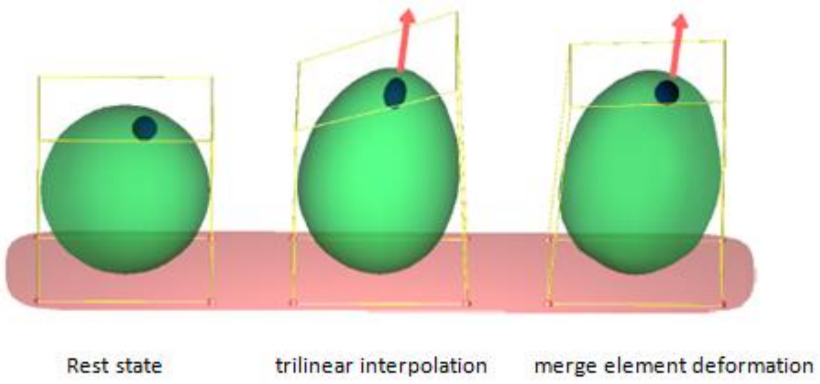

Figure 1-1 Merge element deformation, a soft balloon with a point of stiffness

The standard embedding techniques is an important problem in that it does not properly model the complex geometry of branches. For instance, if materials drop into two sides, then the natural embedding including two mechanical elements installed in the four corners of their faces will share vertices and the adjacent elements need not always be mechanically connected. There are also cases where the collection of fines in respect to all the branches fall into a similar voxel. Elements that are applied in the animation of these structures should have more power than full uniform elements with the same material (Nesme et al., 2006).

Much of GPU-based collision detection (CD) algorithms have been limited to examining the circumstances of collision in discrete time. Recently, several GPU-based methods were used for CD as well. Redon et al. (2004) used a CULIDE algorithm to perform continuous collision detection for a specified model and avatar. Algorithms model the movement of each link using a swept-sphere line volume and may not include a general deformation model. (Govindaraju et al., 2005) However, this algorithm can only use collisions at 1-2 frames per second on a cloth model complex and may not be fast enough for Virtual Reality (VR) applications.

1.3 Research Objectives

The main objectives are:

i. To introduce merge element techniques to improve the behaviour of coarse elements that contains a mixture of materials with diverse properties.

ii. To propose a technique that optimize the model for complex geometry of branches.

iii. To optimise GPU processing time for presenting results on the models performance.

1.4 Research Scope

i. This research aims to introduce a merged element formulation, which is effective in different materials and conditions.

ii. Second, to introduce a technique that builds a proper embedding model in which our method does not need to create a skeleton of restraint.

1.5 Research Contribution

The main contributions of this research is the introduction of a merged element formulation, that enables the construction of coarse embeddings in which the elements properly describe the linear elastic behaviour and both deformation and stiffness interpolations of the materials contained therein. Various materials from soft to hard can be entered in any given element. We consider the entry of an unauthorised implementing essential to make a thin geometry, which has important implications on the strength, deformation interpolation, and mechanical connection of topology models. Our coarse embeddings correctly preserve the mechanical topology of the model, which is essential for the simulation of complex systems with branches, holes or mechanically distant parts, which live in close proximity. A technique for setting a correct surface that interacts with and transfers power from an exceedingly full geometry to the mechanical degrees of a freedom model combining these elements will be presented.

1.6 Summary of Chapter

The rest of this thesis is organised as follows: Chapter 2 provides a comprehensive literature review. This chapter is divided into several sub-sections. Each sub-section contains a discussion concerning a kind of embedding linear elasticity method and its recent related works. Chapter 3 presents the methodology of this research. The experimental design, research framework and research plan, and Method Overview and Ideology are explained in this chapter. In Chapter 4, the proposed merge elements in 3D are presented. Chapter 5 presents the family of topology, mesh topology, and topology branching that is related with this research. Chapter 6 presents the number of experiments that are run for the purpose of evaluating each part of the proposed methods separately. Chapter 7 presents a summary of the work, which has been done in this study and outlines the conclusions that can be drawn. In addition, this chapter also includes suggestions for future work.

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