UNIVERSITI PUTRA MALAYSIA

CLASSIFICATION OF SECOND ORDER PARTIAL DIFFERENTIAL EQUATION USING MAPLE AND COMPARISON FOR THE SOLUTIONS

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By

GHADEER OMAR S ALGABISHI

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

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DEDICATIONS

First and foremost I will like to dedicate this research work to our national "Baba" King Abdullah Bin Abdul Aziz (Rahimahulla); to my Father; to my Mother "Mona Alshowair"; to my grand mother "Salmi Alshowair" and finally to my auntie "Nora Alshowair".

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

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Chair: Prof. Adem Kılıçman, PhD.
Faculty: Science

The study of partial differential equations plays a significant role in many fields including mathematics, physics, and engineering. A partial differential equation (PDE) relates the partial derivatives of a function of two or more independent variables together. The general linear second order partial differential equation with constant coefficient has the form

$$aZ_{xx} + bZ_{xy} + cZ_{yy} + dZ_x + eZ_y + fZ = g(x, y).$$

There are many methods of solving this type of PDE's such as finite elements, finite different and crank Nicolson depends on its classifications based on $\Delta = b^2 - 4ac$. It is also well known that PDE is hyperbolic when $\Delta > 0$, parabolic when $\Delta = 0$ and elliptic when $\Delta < 0$.

In this research study, classification of the partial differential equation with constant coefficient is achieved by using Maple program. The classifications of variable coefficients of partial differential equations by Maple program are also given.

The PDE's after the convolution has the form

$$AZ_{xx} + BZ_{xy} + CZ_{yy} + DZ_x + EZ_y + FZ = G,$$

where A, B and C are coefficients of the PDE's after the convolution. Further more the classification PDE's with convolution are addressed by using $\Delta_1 = B^2 - 4AC$. Similarities in the classification of PDE's before and after convolution were found.

The solution of some important problems such as the wave equation is highly need of and occurs as one of three fundamental equations in mathematical physics that occurs in many branches of physics, applied mathematics, and engineering. In this research work, some problems of PDE's with constant coefficient are solved by double Laplace transforms method. The same problems of the PDE's are modified by some convolution function. The solutions of this new PDE's are obtained by double Laplace transform. Graphical comparisons indicated that the methods are the same. In the same way, the PDE's with variable coefficient are solved by double Laplace transforms methods. Then the same problem of the PDE's is modified by some convolution functions. The new PDE's are solved after convolution. However, graphical comparison made revealed that this two PDE's before and after convolution are the same.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

KLASIFIKASI DAN PERUBAHAN LAPLACE BERGANDA DUA TERHADAP PERSAMAAN PEMBEZAAN SEPARA PADA TERTIB KEDUA MENGGUNAKAN MAPLE

Oleh

GHADEER OMAR S ALGABISHI

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Kajian terhadap persamaan pembezaan separa memainkan peranan penting dalam bidang matematik, fizik dan kejuruteraan. Persamaan pembezaan separa (PPS) mengaitkan terbitan separa pada fungsi yang mempunyai dua pembolehubah yang bebas atau lebih secara bersama. Persamaan pembezaan separa pada tertib kedua dengan pekali tetap mempunyai persamaan umum seperti berikut

$$aZ_{xx}+bZ_{xy}+cZ_{yy}+dZ_x+eZ_y+fZ=g(x,y).$$

Banyak kaedah yang digunakan untuk menyelesaikan PPS seperti elemen tak terhingga, pembezaan tak terhingga dan crank Nicolson bergantung pada klasifikasi asas iaitu $\Delta=b^2-4ac$. Ianya hiperbolik apabila $\Delta>0$, parabolik apabila $\Delta=0$ dan eliptik apabila $\Delta<0$.

Dalam penyelidikan ini, klasifikasi persamaan pembezaan separa yang mempunyai pekali tetap dapat dicapai melalui penggunaan program Maple. Maple juga telah menyediakan klasifikasi pekali pembolehubah pada persamaan pembezaan separa.

Selepas PPS melalui proses perlingkaran, ia mempunyai bentuk persamaan

$$AZ_{xx}+BZ_{xy}+CZ_{yy}+DZ_x+EZ_y+FZ=G,$$

dimana A, B dan C adalah pekali PPS selepas perlingkaran. Seterusnya, klasfikasi PPS dengan perlingkaran diperkenalkan oleh $\Delta_1=B^2-4AC$. Persamaan dalam klasifikasi PPS sebelum dan selepas perlingkaran telah ditemui.

Penyelesaian untuk masalah-masalah penting seperti persamaan gelombang adalah amat diperlukan dan terjadi sebagai salah satu daripada tiga persamaan asas dalam fizik matematik dalam bidang fizik, matematik gunaan dan kejuruteraan. Dalam kerja penyelidikan ini, beberapa permasalahan dalam PPS yang mengandungi pekali tetap diselesaikan oleh kaedah perubahan Laplace berganda dua. Masalah PPS yang sama diubah suai melalui penggunaan fungsi perlingkaran. Penyelesaian-penyelesaian yang terhasil adalah dari kaedah perubahan Laplace berganda dua. Perbandingan grafik menggambarkan kedua-dua kaedah ini adalah sama. Melalui cara yang sama, PPS yang mempunyai pekali pembolehubah diselesaikan oleh kaedah perubahan Laplace berganda dua. Kemudian, permasalahan PPS yang sama diubahsuai melalui beberapa funsi pelingkaran. PPS baru ini diselesaikan selepas pelingkaran. Bagaimanapun, perbandingan grafik menunjukkan bawaha dua PPS sebelum dan selepas pelingkaran ini adalah sama.

ACKNOWLEDGEMENTS

This project could not have been made possible without the help and support from many individuals and organizations. Firstly I would like to extend my sincere appreciation to everyone's support. For most I am highly indebted to my creator Allah Subhanahu Wataalah for His guidance, strength and health He rendered to me. My appreciation goes to the Saudi Arabian Embassy for the financial support in completing this project. There is no amount of words that can adequately express the debt I owe to my Advisor and guide, Prof. Dr. Adem Kılıçman, a Professor in Mathematics, Faculty of Science for his none relenting and continuous words of encouragement and educative discussion during the course of the present work. I am very much grateful for introducing me to PDE, appreciating my ideas and giving me the freedom to take on the tasks independently, enriching me with amassed knowledge in the area and helping to explore the things myself. My special appreciation and thanks also goes to Assoc. Prof. Dr. Norazak Senu, in Mathematics department faculty of Science. I would also want to take the opportunity to express my deep appreciation to Sirajo Lawan Bichi for helping me in my entire work and always willing to engage in thoughtful discussion with me. Finally, yet very important, my heartfelt thanks to my beloved father and mother for their blessings, my friends/classmates for their help and success wishes.

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science.

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LIST OF ABBREVIATIONS

ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
BVPs	Boundary Value Problems
HOBVPs	Higher-Order Boundary Value Problems
IVPs	Initial Value Problems
DT	Differential Transform
LT	Laplace Transform
DLT	Double Laplace Transform
DE	Differential Equation
SLT	Single Laplace Transform

CHAPTER 1

INTRODUCTION

1.1 Background

The study of partial differential equations plays a significant role in many fields including mathematics, physics, and engineering. Partial differential equation (PDE) is an equation that provides the relationship between the partial derivatives of a function of two or more independent variables together. The second order PDE in two independent variables can be written in general form given by (Strauss, 2007) as

$$F(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy}) = 0. \tag{1.1}$$

The form of linear second order PDE's with constant or variable coefficients in general is

$$aZ_{xx} + bZ_{xy} + cZ_{yy} + dZ_x + eZ_y + fZ = g(x,y). \tag{1.2}$$

Many methods of solving this PDE's such as finite elements, finite different and crank Nicolson depends on its classifications base on $\Delta = b^2 - 4ac$. It is hyperbolic when $\Delta > 0$, parabolic when $\Delta = 0$, and elliptic when $\Delta < 0$. (Kilicman and Eltayeb, 2008), considered the PDE's with constant coefficients of hyperbolic and elliptic; then by applying double convolutions, equations with polynomial coefficients are obtained and then classified. It was found that classifications of hyperbolic and elliptic equations with variable coefficients are similar to those of the original equations, see (Kilicman and Eltayeb, 2009).

These new equations are then classified by applying the classification method for the second order linear PDE's. It was found that the classification of PDE's having the polynomial coefficients depends on the signs of the coefficients. In particular, by applying continuously differential functions, some boundary value problems having singularity can be solved because the convolution regularizes the singularity. In (Kilicman and Eltayeb, 2012) the authors extend further the classification of PDE's by applying the convolutions products. The solutions of some specified initial boundary value problems are computed by Laplace transform for some problems of wave equation of one-dimensional with variable coefficients which in general has no solution.

1.1.1 Concepts of Partial Differential Equations

The general idea behind defining property of a PDE's is that it has one more independent variable $x, y, \cdots$ and the dependent variable as the unknown function of these variables, $Z(x, y, \cdots)$. The derivatives are usually denoted by subscripts; thus $\frac{\partial Z}{\partial x} = Z_x$. PDE expresses independent variables x together with the dependent variable Z, and or the partial derivatives of Z, see (Walter, 2007). It can be expressed as

$$G(x,y,Z(x,y),Z_x(x,y),Z_y(x,y)) = G(x,y,Z,Z_xZ_y) = 0. \tag{1.3}$$

Eq. (1.3) above gives general form of first order PDE for two independent variables x and y. It's order can be identified from the highest derivative.

A function $Z(x,y,\cdots)$ is a solution of partial differential equation that satisfies the equation identically in some region of variables $x,y,\cdots$. For the variable separable ODE $\frac{dZ}{dx}=Z^3$, the roles of the independent and the dependent variables one may be reversed. However, the role of the dependent and independent variable is always maintained. In the next we provide some examples of PDE's which occur in physical sciences:

$$\begin{aligned} \text{transport equation:} \quad & Z_x+Z_y=0, \\ \text{transport equation:} \quad & Z_x+yZ_y=0, \\ \text{shock wave equation:} \quad & Z_x+ZZ_y=0, \\ \text{Laplace's equation:} \quad & Z_{xx}+Z_{yy}=0, \\ \text{dispersive wave equation:} \quad & Z_t+ZZ_x+Z_{xxx}=0, \\ \text{vibrating bar:} \quad & Z_{tt}+Z_{xxxx}=0. \end{aligned}$$

All of the above PDE's shown in the immediate examples above are also linear, except the shock wave equation and dispersive wave equation, which are non linear.

A PDE is said to be homogeneous type if each term in the equation contains either the dependent variable or one of its derivatives. That is the right hand side of equation is zero. Otherwise, the equation is non-homogeneous

Solution of a PDE in some region R of the space of independent variables, is a function which satisfies all the derivatives that appear on the equation, and also satisfies the equation everywhere in R.

In general there should be as many initial conditions or boundary as the highest order of the corresponding partial derivative.

1.1.2 Initial and Boundary Conditions

Since PDE's are known to posses many solutions, it is possible for the solution to be single out via imposing auxiliary conditions which result in formulating a unique solution. The particular conditions are important in physics appearing in one and or two varieties; initial and boundary conditions. The case of initial condition specifies the physical state at a particular time t_0. In the diffusion equation for instance

$$Z(x,t_0)=\zeta(x), \tag{1.4}$$

where $\zeta(x)=\zeta(x,y,z)$, usually function given and $\zeta(x)$ represents initial concentration. In the case of heat flow problems, $\zeta(x)$ signifies initial temperature. In Schrödinger equation, too, Eq.(1.4) is the usual initial condition. In wave equation there is a pair of initial conditions

$$Z(x,t_0)=\zeta(x) \text{ and } \frac{\partial Z}{\partial t}(x,t_0)=\psi(x), \tag{1.5}$$

where $\zeta(x)$ describes initial position and $\psi(x)$ initial velocity. On physical grounds all must be specified in order to determine the position $z(x,t)$ at later times.

From physical problem, valid domain D should exist for the PDE's. The D is the interval $0<x<\rho$, are usually for the vibrating string problems, as such boundary of D has only of the two points $x=0$ and $x=\rho$. The case of drumhead model has plane region domain and closed curve as its boundary. Model of diffusing chemical substance has D as the container holding the liquid and its boundary as surface $S=D$. The hydrogen atom model has domain as all of space as such it has no boundary.

One-dimensional problems with D an interval $0<x<\rho$ has boundary at the two endpoints only and where these boundary conditions are of simple form

$$\text{(D)} \qquad Z(0,t)=g(t) \text{ and } Z(\rho,t)=h(t)$$

$$\text{(N)} \qquad \frac{\partial Z}{\partial x}(0,t)=g(t) \text{ and } \frac{\partial Z}{\partial x}(\rho,t)=h(t).$$

1.1.3 Basics of Laplace Transform (LT)

This section discusses some concepts of single and double LT that are useful in further discussion. According to (Schiff, 1999) and (Kilicman, 2006). For real or complex valued functions g of variable t such that $t>0$ and s being real or complex parameter, then the LT is defined as

$$G(s)=\mathscr{L}[g(t)]=\int_0^\infty e^{-st}g(t)dt=\lim_{\tau\to\infty}\int_0^\tau e^{-st}g(t)dt, \tag{1.6}$$

provided that the limit exists, the integral of equation Eq.(1.6) converges. Otherwise, the integral Eq.(1.6) diverges and else no LT defined for g. In (Lopez, 2001) it is mentioned that the LT of a product is not product of the transform, since, for example,

$$\mathscr{L}[t]=\frac{1}{s^2} \text{ but } \mathscr{L}[t^2]=\frac{2}{s^3}\neq\left(\frac{1}{s^2}\right)^2.$$

Therefore, the product of two LT does not invert back to the product of the inverses of factor. So in general,

$$\mathscr{L}^{-1}[G(s)H(s)]\neq g(t)h(t).$$

Like wise, the inverse of a product of two LT is not the product of the inverse. Instead, it is the convolution product, that is

$$\mathscr{L}[G(s)H(s)]=g(t)*h(t).$$

Theorem 1.1.1 *(Jefrey, 2002)* Linearity of LT. Let the functions $g_1(t), g_2(t), ..., g_n(t)$ have LT, and let $c_1, c_2, ...c_n$ be any set of arbitrary constants then

$$\mathscr{L}[c_1 g_1(t) + c_2 g_2(t) + ... + c_n g_n(t)] = c_1 \mathscr{L}[g_1(t)] + c_2 \mathscr{L}[g_2(t)] + ... + c_n \mathscr{L}[g_n(t)].$$

This theorem has many applications and it uses is essential when working with LT.

Theorem 1.1.2 *(Jefrey, 2002)* Transform of derivative. Let $g(t)$ be continuous on $0 \leq t < \infty$, and let $g'(t), g''(t), ..., g^{n-1}(t)$ be piecewise continuous on every finite interval contained in $t \geq 0$. Then if $\mathscr{L}[g(t)] = G(s)$,

$$\mathscr{L}[g^n(t)] = s^n G(s) - s^{n-1} g(0) - s^{n-2} g'(0) - ... - s g^{n-2}(0) - g^{n-1}(0).$$

The detail of proving can be obtained in (Jefrey, 2002).

Definition 1.1.1 *(Mei, 1997; James, 2012)* A function $g(t)$ is said to be of exponential order as $t \to \infty$ if there exist real number σ and positive constant m and T such that $|g(t)| < me^{\sigma t}$ for all $t > T$.

Theorem 1.1.3 *(Kreyszig, 1999)* The First transform of shifting property: If $\mathscr{L}[g(t)] = G(s)$ where $s > k$, $\mathscr{L}[e^{at} g(t)] = G(s-a)$, where $s - a > k$. In order to apply LT to PDEs, it is necessary to invoke the inverse transform. If $\mathscr{L}[g(t)] = G(s)$, then inverse LT is denoted by $\mathscr{L}^{-1}[G(s)] = g(t),\ t > 0$.

Theorem 1.1.4 (Mei, 1997) If the LT of $g(t)$ and $h(t)$ are $G(s)$ and $H(s)$, respectively, then the inverse transform of $G(s)H(s)$ is the convolution integral. That is

$$\mathscr{L}^{-1}[G(s)H(s)] = \int_0^\infty g(t-\tau)h(\tau)d\tau = g * h.$$

Theorem 1.1.5 (Dyke, 2000) If Laplace transform of $G(t)$ exists, that is the $G(t)$ is of exponential order and

$$g(s) = \int_0^\infty e^{-st} G(t) dt,$$

then

$$G(t) = \lim_{k \to \infty} \left[\frac{1}{2\pi i} \int_{\sigma - ik}^{\sigma + ik} g(s) e^{-st} ds \right],\ t > 0.$$

The detail of proving can be obtained in (Dyke, 2000).

Theorem 1.1.6 (Inverse LT of function with poles). Let $G(s)$ be analytic in the s-plane except a finite number of poles that lie to the left of some vertical line $Res = a$. Suppose there exist positive constants, m, R_0, and k such that for all s lying in the half plane $Res \leq a$, and satisfying $|s| > R_0$, we have $|G(s)| \leq \frac{m}{|s|^k}$. Then for $t > 0$,

$$\mathscr{L}^{-1}[G(s)] = \sum Res[G(s)e^{st}],$$

at all poles of $G(s)$. For more detail see (Wunsch, 2005).

Example 1.1.1 *(Wunsch, 2005) Find inverse Laplace Transform for the function*

$$\mathscr{L}_s^{-1}\left[\frac{1}{(s+1)^2(s-2)}\right].$$

Solution:

The function has poles at $s=2$ and $s=-1$. Thus

$$f(t)=Re\left[\frac{e^{st}}{(s-1)(s+2)^2},2\right]+Re\left[\frac{e^{st}}{(s-1)(s+2)^2},-1\right].$$

The first residue given by

$$\lim_{s\to 2}\frac{e^{st}}{(s+1)^2}=\frac{1}{9}e^{2t},$$

while the second residue which involves a pole of second order, is

$$\lim_{s\to -1}\frac{d}{ds}\frac{1}{(s-1)}=\frac{1}{3}te^{-t}-\frac{1}{9}e^{-t},$$

then

$$f(t)=\frac{1}{9}e^{2t}-\frac{1}{3}te^{-t}-\frac{1}{9}e^{-t}.$$

The definition of LT of the Heaviside unit function (Iyengar, 2004; Graf, 2004)

$$H(t-a)=\begin{cases}1, & t\geq a,\\ 0, & t<a,\end{cases} \tag{1.7}$$

is given by

$$\mathscr{L}[H(t-a)]=\frac{1}{s}e^{-as}.$$

In particular,

$$\mathscr{L}[H(t)]=\frac{1}{s}.$$

Similarly, LT of unit Impulse function (or Dirac delta function)

$$\delta(t-a)=\begin{cases}\infty, & t=a,\\ 0, & t\neq a,\end{cases} \tag{1.8}$$

is given by

$$\mathscr{L}[\delta(t-a)]=e^{-as}.$$

In particular, (Estrada and Fulling, 2002).

$$\mathscr{L}[\delta(t)]=1$$

two dimensional LT used by (Hillion, 1997)

$$G(p,q)=\mathscr{L}[g(x,y)]=\int_0^\infty\int_0^\infty g(x,y)e^{-px-qy}dxdy, \tag{1.9}$$

with $x>0, y>0$ and also see (Sneddon, 1972a; Moorthy, 2009) are defined DLT by

$$\mathscr{L}_x\mathscr{L}_t[g(x,t)]=G(p,s)=\int_0^\infty e^{-px}\int_0^\infty e^{-st}g(x,t)dtdx,$$

where $x,t>0$ and p,s are complex and DLT is defined for first order partial derivative as

$$\mathscr{L}_x\mathscr{L}_t\left[\frac{\partial g(x,t)}{\partial x}\right]=pG(p,s)-G(0,s) \tag{1.10}$$

for second partial derivative with respect to x, DLT is given by

$$\mathscr{L}_x\mathscr{L}_t\left[\frac{\partial^2 g(x,t)}{\partial x^2}\right]=p^2G(p,s)-pG(0,s)-\frac{\partial G(0,s)}{\partial x} \tag{1.11}$$

and for second partial derivative with respect to t DLT is similarly given as above by

$$\mathscr{L}_x\mathscr{L}_t\left[\frac{\partial^2 g(x,t)}{\partial t^2}\right]=p^2G(p,s)-sG(p,0)-\frac{\partial G(p,0)}{\partial t}. \tag{1.12}$$

More so, DLT of a mixed partial derivative can be deduced from SLT as

$$\mathscr{L}_x\mathscr{L}_t\left[\frac{\partial^2 g(x,t)}{\partial t\partial x}\right]=psG(p,s)-pG(p,0)-sG(0,s)-G(0,0). \tag{1.13}$$

Theorem 1.1.7 (Convolution theorem (Schiff, 1999; Zayed, 1996)) If f and g are piecewise continues on $[0,1)$ and of exponential order α then

$$\mathscr{L}[(g*h)(t)]=\mathscr{L}(g)\mathscr{L}(h),\ (Re(s)>\alpha).$$

Theorem 1.1.8 If, the integrals

$$G_1(p,q)=\int_0^\infty\int_0^\infty e^{-px-qy}g_1(x,y)dxdy,$$

bounded and convergent, and

$$G_2(p,q)=\int_0^\infty\int_0^\infty e^{-px-qy}g_2(x,y)dxdy,$$

absolutely converges, then

$$G(p,q)=G_1(p,q)G_2(p,q)$$

gives LT of function

$$g(x,y) = \int_0^x \int_0^y g_1(x-\zeta, y-\eta) g_2(\zeta,\eta) d\zeta d\eta$$

and

$$G(p,q) = \int_0^\infty \int_0^\infty e^{-px-qy} g(x,y) dx dy,$$

becomes bounded and convergent at the point (p,q). The detail of proving can be obtained in (Hillion, 1997), from this Theorem and DLT of partial derivatives very useful results for solving non constant coefficients linear second order PDE's can be obtained later we give more details. In the next we gives table of double Laplace transform (it is very important in this thesis) as follow.

$f(x,y)$	$F(p,q)$
$\sin(x+y)$	$\frac{p+q}{(p^2+1)(q^2+1)}$
$\cos(x+y)$	$\frac{pq-1}{(p^2+1)(q^2+1)}$
e^{x+y}	$\frac{1}{(p-1)(q-1)}$
$x^n y^m$	$\frac{n!m!}{(p^n+1)(q^m+1)}$
$f(x,y)**g(x,y)$	$F(p,q)G(p,q)$
$\delta(x-a)\delta(y-b)$	e^{-pa-qb}
$\frac{\partial}{\partial x}\delta(x-a)\frac{\partial}{\partial y}\delta(y-b)$	pqe^{-pa-qb}
$\delta(x_1-a_1)\delta(x_2-a_2)\ldots\delta(x_n-a_n)$	$e^{-p_1a_1-p_2a_2\ldots-p_na_n}$
$f(x,y)**(g(x,y)**h(x,y))$	$F(p,q)G(p,q)H(p,q)$
$f(x,y)**\frac{\partial u(x,y)}{\partial x}$	$F(p,q)[pU(p,q)\ldots U(0,q)]$
$f(x,y)**\frac{\partial^2 u(x,y)}{\partial^2 x}$	$F(p,q)\left(p^2U(p,q)\ldots pU(0,q)\ldots\frac{\partial U(0,q)}{\partial x}\right)$
$f(x,y)**\frac{\partial^2 uf(x,y)}{\partial x \partial y}$	$F(p,q)(pqU(p,q)\ldots pU(p,0)\ldots qU(0,q)$ $\ldots U(0,0)$
$f(x,y)**\frac{\partial u(x,y)}{\partial y}$	$F(p,q)[qU(p,q)\ldots U(p,0)]$
$f(x,y)**\frac{\partial^2 u(x,y)}{\partial^2 y}$	$F(p,q)\left(q^2U(p,q)\ldots qU(0,q)\ldots\frac{\partial U(0,p)}{\partial y}\right)$
$f(x,y)**(g(x,y)**h(x,y))$	$F(p,q)G(p,q)H(p,q)$

Table 1.1: Table of double Laplace transforms

1.2 Aims and Objectives

1. To obtain classification of PDE's with constant coefficients before and after convolutions

2. To obtain classification of PDE's with variable coefficients before and after convolutions and their comparison

3. To obtain solution of PDE’s with constant coefficients by double Laplace transforms before and after convolution and their comparison

4. To obtain solution of PDE’s with variable coefficients by double Laplace transforms before and after convolution and their comparison

1.3 Scope of the Study

The research work is limited to

- studying parabolic, hyperbolic and elliptic types of partial differential equations for their classifications both before and after the convolution.
- using the simple polynomial functions as our convolution functions.
- using maple program for classification of PDEs with constants and variable coefficients.
- applying double Laplace transforms to obtain the solution of PDE’s both before and after the convolution.
- using Maple programme to achieve the immediate above solutions.

1.4 Organisation of the thesis

The organization of thesis is as follows:

In chapter 1 under introduction of the thesis, we present background, aims and objectives and the scope of the study.

Chapter 2 under the literature review discusses on classification of PDEs. Linear second order PDEs, Laplace transforms, inverse Laplace transforms, some properties of Laplace transforms, double Laplace transforms, inverse of double Laplace transform, properties of DLT, type of integral transform and convolution.

We present in Chapter 3, methodology and results. Under which we discussed the classification of PDEs with convolution, the maple program of classification PDE’s for linear PDEs, comparison between constant coefficient PDE’s before and after the convolution, comparison between coefficient PDEs before and after the convolution and summary.

Chapter 4 under linear second order PDEs with double Laplace transform, we give the methods of double convolution, results and discussion and summary.

Chapter 5 presents summary, conclusion and recommendations.

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