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NUMERICAL ANALYSIS OF A SYSTEM OF WIRELESS ENERGY TRANSFER VIA RESONANCE OF MAGNETIC INDUCTION

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By

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NUMERICAL ANALYSIS OF A SYSTEM OF WIRELESS ENERGY TRANSFER VIA RESONANCE OF MAGNETIC INDUCTION

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Chairman: Professor Ishak Aris, PhD

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In the early 19th century Nikola Tesla, the inventor well known for his contribution for the development of the present day alternating current system gave rise to the idea of wireless electricity, however it was short lived and sparks insignificant interest to the then society. In 2007, physicist at Massachusetts Institute of Technology (MIT) has demonstrated an efficient scheme of wireless energy transfer via magnetic induction at resonance frequency. The literature has reported several methods to optimize the power delivery and efficiency; as well as theories to explain the behavior of the wireless energy transfer system. However, most of them are based on the Coupled Mode Theory (which was put forward by the MIT physicist), and the Impedance Analysis Model. There are some limitations imposed by these theories, the Coupled Mode Theory despite being widely accepted in the literature are unfamiliar to electrical engineers, in that it introduces variables that does not directly correlate to variables accustomed to electrical engineers, as a result an ample time had to be spent to understand the theory. While the Impedance Analysis is more familiar and simpler, it only solves the system in the frequency domain and does not explain the system relationship to time; thus it only considers the steady state condition and does not provide the transient behavior. This thesis fills the gap by providing a set of numerical equations to solve the currents of a two-coil Wireless Energy Transfer via Resonance of Magnetic Induction in the time domain from an initial condition to a steady state condition by using variables familiar to electrical engineers; with the objective of obtaining the conditions for that maximizes the efficiency and power delivery. The behavior of the system is governed by variables defined by the values of the system's circuit components, and relationship between these variables is investigated by solving the ordinary differential equations of the system's current and providing both the analytical and the numerical solution to the differential equation problem. It was found that, analytical solution of the system's current in the time domain results in a very long algebraic expression, while the numerical solution produces equation much shorter in length. Analysis is done on the numerical solution by simulating the equation in MATLAB programming. The hardware was constructed to test the validity of the numerical solutions to the equations presented. The data presented shows an agreeable result between the hardware and the equation based simulation. Result of analyses performed found that there exist un-

symmetrically between the lower resonant frequency and the upper resonant frequency at high coupling coefficient. It was also found that there exist several load and frequency conditions that give peak power delivery in over-coupled mode. Moreover, a theoretical analysis performed suggest that efficiency at very low coupling coefficient could be maximize by optimizing the ratio self-inductance to the series capacitance in conjunction with a correct terminated load, as such it was shown that efficiency of 55% is obtainable at a very low coupling coefficient of 0.005. Finally, this thesis also provides a set of equations to calculate the mutual inductance and magnetic field of the system's coil; this is done by applying the Biot-Savart equation to perform finite element calculation.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

ANALISIS PENYELESAIAN BERANGKA KEPADA SISTEM PEMINDAHAN TENAGA TANPA WAYAR MELALUI ARUHAN MAGNET PADA RESONAN

Oleh

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Pada awal abad ke-19 Nikola Tesla, pencipta terkenal dengan sumbangan beliau kepada pembangunan arus ulang alik telah membawa idea tentang elektrik tanpa wayar. Walau bagaimanapun, idea beliau tidak bertahan lama dan tidak mendapat minat daripada masyarakat zaman itu. Pada tahun 2007, ahli-ahli fizik di Massachusetts Institute of Technology (MIT) telah menunjukkan skim pemindahan tenaga tanpa wayar yang cekap melalui aruhan magnet pada frekuensi resonan. Sorotan ilmiah telah melaporkan beberapa kaedah untuk mengoptimumkan penghantaran kuasa dan kecekapan; serta teori-teori untuk menerangkan sifat-sifat sistem pemindahan tenaga tanpa wayar. Walau bagaimanapun, kebanyakannya adalah berdasarkan kepada Teori Mod Berpasangan (yang telah dikemukakan oleh ahli fizik MIT), dan Model Analisis Impedans. Terdapat beberapa batasan pada teori-teori ini, Teori Mod Berpasangan walaupun diterima secara meluas dalam sorotan ilmiah adalah asing kepada jurutera elektrik, ini kerana Teori Mod Berpasangan memperkenalkan pembolehubah yang tidak langsung ada kaitan dengan pembolehubah yang biasa digunakan jurutera elektrik, akibatnya masa yang banyak diperlukan untuk memahami teori ini. Walaupun Analisis Impedans adalah tidak asing dan lebih mudah, ia hanya menyelesaikan sistem dalam domain frekuensi dan tidak menjelaskan hubungan sistem di dalam domain masa; oleh itu ia hanya menunjukan kelakuan akhir sistem dan tidak memberikan kelakuan antara permulaan dan pengakhiran. Tesis ini memenuhi jurang dengan menyediakan satu set persamaan berangka untuk menyelesaikan arus dalam sistem dua gegelung pemindahan tenaga tanpa wayar oleh induksi magnet pada frekuensi resonan dalam domain masa dari keadaan awal kepada keadaan akhir dengan menggunakan pembolehubah yang biasa digunakan jurutera elektrik; dengan objektif untuk mendapatkan kondisi yang memaksimumkan kecekapan dan penghantaran kuasa. Kelakuan sistem ini dikawal oleh pembolehubah yang ditakrifkan oleh nilai-nilai komponen litar sistem, dan hubungan antara pemboleh ubah ini disiasat dengan menyelesaikan persamaan pembezaan kepada arus sistem dan menyediakan kedua-dua penyelesaian analitikal dan penyelesaian berangka kepada masalah persamaan pembezaan. Didapati bahawa penyelesaian analitikal kepada arus sistem dalam di dalam domain masa dalam menghasilkan ungkapan algebra yang sangat panjang, manakala penyelesaian berangka menghasilkan ungkapan algebra yang lebih pendek.

Analisis penyelesaian berangka dilakukan oleh simulasi persamaan dalam pengaturcaraan MATLAB . Perkakasan telah dibina untuk menguji kesahihan penyelesaian berangka yang dibentangkan. Data menunjukkan persetujuan antara perkakasan dan simulasi persamaan berangka. Keputusan analisis mendapati bahawa terdapat ketidak-simetri antara frequency resonan yang lebih rendah dan frekuensi resonan yang lebih tinggi pada pekali gandingan yang tinggi. Ia juga mendapati bahawa wujud beberapa kondisi beban dan frekuensi yang memberikan penghantaran kuasa yang maksimum dalam mod 'over-coupled'. Selain itu, keputusan analisis mencadangkan bahawa kecekapan pada pekali gandingan yang sangat rendah boleh dimaksimumkan dengan mengoptimumkan nisbah kearuhan sendiri kepada pemuat siri, beserta beban ditamatkan yang betul. Keputusan telah menunjukkan bahawa kecekapan sebanyak 55% boleh diperolehi pada pekali gandingan yang sangat rendah iaitu 0.005. Akhir sekali , tesis ini juga menyediakan satu set persamaan untuk mengira kearuhan bersama dan medan magnet gegelung sistem; ini dilakukan dengan menggunakan persamaan Biot - Savart untuk melaksanakan pengiraan unsur terhingga.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfillment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

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DECLARATION

I declare that the thesis is my original work except for quotation and citations which have been duly acknowledged. I also declare that it has not been previously and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or other institutions.

RADIN ZA'IM RADIN UMAR

Date: 25 November 2013

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

A	Area of Overlapping Parallel Capacitor Plate
a	Index of Mesh Element in X-Axis
a_1	Energy Field Amplitude of Transmitting Coil
a_2	Energy Field Amplitude of Receiving Coil
AC	Alternating Current
a_m	Energy Field in Object m
B	Magnetic field Intensity
b	Index of Mesh Element in Y-Axis
C	Capacitance
C_1	Transmitter Series Capacitors
C_2	Receiver Series Capacitor
C_{gs}	MOSFET Gate-Source Capacitance
D	Separation Distance of Coil to Z-Plane
d	Separation of plate
dB	Partial Magnetic Field Intensity
DC	Direct Current
ds	Derivative of Current Source
dx	Finite Mesh Increment in X-Axis
dy	Finite Mesh Increment in Y-Axis
emf	Electromotive Force
EPCOS	Film Capacitor Manufacturer
ESR	Equivalent Series Resistor
f	Frequency
f_1	Natural Frequency of Transmitter Coil and Transmitter Capacitor
f_2	Natural Frequency of Receiver Coil and Transmitter Capacitor
f_3	Natural Frequency of Transmitter Coil and Receiver Capacitor
f_H	Upper Operational Frequency

f_L	Lower Optimal Frequency
F_m	Driving Term
$f_{natural}$	Natural frequency
HO	Output Voltage of How Side Gate Driver
$\hat{\imath}$	Vector in x-axis direction
I	Current
i_1	Current of the Transmitter Circuit in Time Domain
$i_{1\ peak}$	Peak Transmitter Current
$i_{2\ peak}$	Peak Receiver Current
i_2	Current of the Receiver Circuit in Time Domain
IC	Integrated Circuit
I_{peak}	Peak Current
$I_{steady\ state}$	Steady State Current
$\hat{\jmath}$	Vector in y-axis direction
$\hat{k}$	Vector in z-axis direction
k	Coupling Coefficient
k_1	First Residue of Partial Fraction
k_2	Second Residue of Partial Fraction
k_3	Third Residue of Partial Fraction
k_4	Fourth Residue of Partial Fraction
k_n	n-th Residue of Partial Fraction
L	Self-Inductance
l	Length of Conductor
L_1	Self-inductance of Transmitter Coil
L_2	Self-inductance of Receiver Coil
L-C	Inductor-Capacitor
LCR	Inductor-Capacitor-Resistor
LO	Output Voltage of Low Side Gate Driver
M	Mutual Inductance

m	Total number of current element
MATH	Mathematic Function of Oscilloscope
MIT	Massachusetts Institute of Technology
MKP	Metallized Polypropylene Film Capacitor
MFP	Metallized Polypropylene Film Capacitor (similar to MKP)
MKN	Metallized Polyethylene Naphthalate Film Capacitor
MKT	Metallized Polyester Film Capacitor
MFT	Metallized Polyester Film Capacitor(similar to MKT)
MOSFET	Metal Oxide Semiconductor Field Effect Transistor
N	Number of Coil Turns
n	Index of Discrete Point
N_1	Number of Turns of Transmitter Coil
N_2	Number of Turns of Receiver Coil
n_c	Number of Discrete Points in One Period of Oscillation
nF	NanoFarad
P	Power
p	Number of Mesh Element in the X-Axis
P_{loss}	Power Loss
P_{out}	Power Delivery to Load
$P_{radiation}$	Radiation Power
Q	Quality Factor
q	Number of Mesh Element in the Y-Axis
Q_{max}	Maximum Quality Factor
$\hat{r}$	Unit Vector of Relative Distance to Current element
R	Resistance
r	Distance of point P to current element
r	Radius of Loop
R_1	AC Resistance of Transmitter Circuit
r_1	First Root of Equation D(s)

R_2	AC Resistance Receiver Circuit
r_2	Second Root of Equation D(s)
r_3	Third Root of Equation D(s)
r_4	Fourth Root of Equation D(s)
R_{ac}	AC Resistance
$R_{ac_{Q_{max}}}$	AC resistance at Q_{max}
RC	Resistor-Capacitor
R_g	Gate Resistance
$R_{L\ optimum}$	Optimum Output Load
R_L	Output Resistive Load
RLC	Resistor-Inductor-Capacitor
$R_{L_{Q_{max}}}$	Optimal load at Q_{max}
RMS	Root Mean Squared
R_r	Radiation resistance
R_{skin}	Skin Resistance
S_{21}	Power Delivery in S-Parameter
SPWM	Sinusoidal Pulse Width Modulation
T	Period
t	Time
$\tan\delta$	Dissipation Factor
$u(t)$	Step Function
V	Voltage
V_{gs}	MOSFET Gate-Source Voltage
Vin	Input Voltage to Full Bridge Inverter
V_{peak}	Peak Voltage
Z	Impedance
Γ	Intrinsic Decay Rate
θ	Angle of Current Element
Φ	Magnetic Flux

α	Constant of Optimized Load
δ	Skin depth
ε_0	Permittivity constant
η	Efficiency
η_{21}	Efficiency in S-Parameter
λ	Wavelength of Radiation
μ	Magnetic permeability of material
μ_0	Permeability of Free Space
ρ	Conductivity of material
ω	Angular frequency

CHAPTER 1

INTRODUCTION

1.1 Background

Wireless Energy Transfer is a method of electrical energy transmission from a source to a receiver without the use of any physical connectors (wire or conductor). Nikola Tesla [1] is probably the first to demonstrate the wireless energy transfer via large electrical fields. His idea was not well received to then society as it was perceived to be hazardous [2]. On the other hand, wireless energy transfer through electromagnetic induction has been widely implemented in devices such as the transformer and electrical motor. However, they are subjected to very tight magnetic coupling [3]; hence the distance of wireless energy transfer is limited to small distances. Only recently, wireless energy transfer regained the interest from academics as well as the industry, partly due to the advancement of portable electronics such as tablets, smartphones and laptops.

In 2007, physicists at Massachusetts Institute of Technology (MIT) has demonstrated a high efficiency wireless energy transfer scheme at a relatively large distance; implemented via magnetic induction at resonant frequency. It is consisted of a transmitting coil and a receiving coil connected wirelessly through mutual inductance. The wireless energy transfer is achieved by oscillating current through the pair of coil whose natural frequency are identical to each other. Due to the nature of resonance, energy is transferred at high efficiency. This system demonstrated by the physicist has open up a new concept; such that in the near future, it is possible to truly live in a world without wires.

1.2 Problem Statement

The field of Wireless Energy Transfer via Resonance of Magnetic Induction is a relatively new field. It was first introduced in 2007; as such the physics to explain the behavior of the system is still at its infancy. The physicist at MIT has put forward the Coupled Mode Theory to explain the energy sharing behavior of the system. While this theory is widely accepted, it is more preferred in the physics world and is unfamiliar to electrical engineers [4], [5]. The reason is because; the equation presented introduced variables that do not directly correlate to the variables familiar to electrical engineer. For example, the Coupled Mode Theory only explained the energy of the system, and does not explain how the components variables that the system is consisted of are in relationship to the current and voltages of the system. Therefore, it takes huge amount of time for electrical engineers to study the Coupled Mode Theory, to understand the concept and behavior of the system. Moreover, the Coupled Mode Theory only solves the system in steady state condition and it does not give the behavior of the system in transient time. While the alternative model, the Impedance Analysis Model is more familiar to engineers, it only explains the behavior of the system in the frequency domain, not in the time domain. Thus, analyses of power delivery are often reported in the form of s-parameters and not in watts. Additionally, this model has to assume that voltages and currents are purely sinusoidal, which might not be the case in real world implementation. For example, the time varying real world occurrence (such as dead time and rise time) is not simulate-able in the Impedance Analysis Model. Furthermore, similar to the Coupled Mode Theory, the Impedance Analysis Model only solves the system in steady state condition. Hence, this thesis shall bridge these gaps, by providing a set of equations to solve the system in the time domain from an initial state condition to the steady state condition; by using variables that are familiar to electrical engineers.

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1.3 Importance of the Study

Apart from being able to better understand the underlying principle of operation of the system, studying the system's behavior, namely, the equation for current, power and efficiency of the system in the time domain is important because:

- It gives the behavior of the system's currents and voltages; the magnitude, and phase in relation to the time domain.
- It results in more precise simulation in comparison to actual hardware implementation by allowing real world occurrences such as dead time and rise time to be simulated. Moreover, the equation allows the system to be simulated under any type of input voltages defined by the user (as supposed to pure sinusoidal input voltage limited in other equation models).
- It allows the system's behavior to be simulated from an initial condition, transient condition and steady state condition.
- It allows for the optimal condition of the system to be determined based on the systems circuit's components and variables. These variables include:
    - Mutual Inductance (or Coupling Coefficient).
    - Frequency of Operation.
    - Series Tuning Capacitor.
    - Self-Inductance of the Coil.
    - Alternating Current Resistance of the Coil.
    - Output Load Resistance.

  By solving the equations of the system under these different variables conditions, the resonance frequency of operation, optimal load condition, and optimal self-inductance to capacitance ratio could be determined. Moreover, since the behavior of the system could be obtained under different mutual inductance (or coupling coefficient) condition, it is possible to obtain the optimal condition under a very large distance of energy transfer (extreme low coupling coefficient), as such that it would maximize the power delivery and efficiency of the energy transfer.

1.4 Objectives of Research

In line with the aforementioned problem statement, the main objective of this research is to provide a set of numerical equations that model and explain the behavior of the system, namely the current of the system in the time domain, by using variables that are more familiar to electrical engineers. To justify the significance of the presented equations, analysis is done on the equation as such it is to achieve the following:

- It should produce an agreement between experimental result and the simulation based on the equations presented.
- Analysis of the equation results in the determination of resonance frequency of operation, and optimal load condition, as such that the system under the optimal condition the system would produce the highest efficiency and power delivery
- Analysis of the equation results in the determination of optimal self-inductance-capacitance ratio, and optimal load as such it maximizes the efficiency for a system under extreme low coupling coefficient, k, where $0 < k < 0.01$.

In order to provide a comparison between the experimental result and the equation-based-simulation, the mutual inductance and the magnetic field around the coils need to be determined. Thus a secondary objective of this research is to provide a set of equations to

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calculate the magnetic field around the system's coils and hence calculation of the mutual inductance. This is done by applying the Biot-Savart equation to do finite elements calculations.

1.5 Scopes of Research

The scope of the research is limited to deriving the equation of the current of system in the time domain, using both analytical and numerical method. Then, the analysis of the equations is done by simulating the equations with MATLAB programming. The purpose of the simulation is to obtain the optimal conditions that maximize the efficiency and power delivery of the system. Upon obtaining the result of the analysis, experiment is designed where hardware is constructed. The result obtained from the hardware construction is compared to the result of the equations analysis; this is to justify the validity of the derived equations. Nonetheless, despite the fact that the thesis main contribution is the equation of the current of the system in the time domain, reader would find that most analysis is performed in the frequency domain. This is due to the fact that there is a lack of information in the literature regarding the current of the system in the time domain; in order to perform comparison analysis with the published work in the literature, the author has do the analysis in the frequency domain using the equation derived for the time domain; albeit reader would find that references shall be given in the time domain to explain the findings of the frequency domain results.

Moreover in order to do the comparison, the mutual inductance and magnetic field of the system has to be obtained; as such the equation governing the mutual inductance and magnetic field of the system shall also be derived. This research focuses on explaining the behavior and optimal conditions of the systems; thereby the health issues associated with the wireless energy transfer system shall not be investigated.

1.6 Research Contributions

This thesis claims the following to be its main contribution to the body of knowledge:

1. The equation of the transmitter current and receiver current of the Wireless Energy Transfer via Resonance of Magnetic Induction system in time domain solved using the technique of next discrete point approximation based on the circuit components and the input voltage to the system.

Moreover, the following are claimed to be the secondary contributions (contribution 2-4 are obtained by analysis of contribution 1):

2. A phenomenon unreported in the literature, obtained by analysis of the presented equation in contribution 1; as such that there exist un-symmetrically at high coupling coefficient, as such that the higher resonance frequency, f_H, and the lower resonance frequency, f_L are not centered at the natural frequency of the system, instead as the coupling coefficient increases, the difference of $\Delta f = f_H - f_L$ increases exponentially (as supposed to linear increment)
3. The maximization of efficiency obtained by analysis of the presented equation in contribution 1; as such that efficiency is maximized at the condition of optimal surd of self-inductance to capacitance ratio and the condition of optimal load for the system under extreme low coupling coefficient k, where $0 < k < 0.01$.
4. A phenomenon unreported in the literature, obtained by analysis of the presented equation in contribution 1; as such in contrast to the literature which only reports a

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single optimal load condition, it was shown that in over-coupled mode there exist more than one load and frequency conditions that give peak power delivery and efficiency.

5. The equation for magnetic field calculation is obtained by applying the Biot-Savart equation for finite element calculation to determine the mutual inductance and magnetic field around the system's coil. As such the result of the presented equation gives better precision compared to the well-known Grover's Method.

1.7 Thesis Layout

This thesis is structured into 5 chapters. Chapter One introduces the Wireless Energy Transfer via Resonance of Magnetic Induction, outline the problems, the research objectives and contributions. Chapter Two encompasses the review of the literature pertaining to the field of wireless energy transfer, and the objective of this section is to identify the gaps in the literature, hence justifying the relevancy of this thesis. Chapter Three discusses the methodology used to achieve the stated research objective, in such that the derivation of the equation (as in contribution 1) is presented, as well as the derivation of equations for magnetic field calculation, obtained by applying Biot –Savart Equation (as in contribution 5), also presented are the programming flowcharts for analysis of the optimal conditions of the system (as in contribution 2-4). Chapter Four presents the result and discussion in which the result obtained from the equation based simulation is compared to the experimental result, this chapter discusses the research findings and addresses the issues identified in Chapter Two. Finally Chapter Five gives the summary of the findings, and draws the conclusion.

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