BLOCK MULTISTEP METHODS FOR SOLVING ORDINARY DIFFERENTIAL EQUATIONS

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By

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September 2006

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Multistep methods for the solution of systems of Ordinary Differential Equations (ODEs) were described. The first part of the thesis is about the construction and derivation of new Block Backward Differentiation Formula (BBDF) method of constant step size and variable step size for solving first order stiff Initial Value Problems (IVPs). Their regions of stability were presented and numerical results of the methods were compared with existing methods.

The second part of the thesis describes the derivation of the Adams type block method to solve second order nonstiff systems directly whilst a mixture of the Adams type formulae and the new implicit BBDF method were used to solve second order stiff problems directly. Partitioning strategies for the block method were discussed in detail and numerical results of the block partitioning

are compared with the nonblock Variable Step Variable Order (VSVO) Direct Integration method for solving second order ODEs directly.

Finally, this thesis deals with parallel numerical algorithms for the solution of systems of ODEs. The constructed BBDF methods are then tested and parallelism is obtained by using a Message Passing Insterface (MPI) library run on High Performance Computer (HPC). The parallel implementation of the new codes produced superlinear speedup as the dimension of the ODEs systems increased. Comparisons and illustrations with sequential codes are provided.

In conclusion, the numerical results clearly demonstrates the efficiency of using the new block multistep methods for solving ODEs. Application of these multistep block method to a widely used test problems reveals the reduction in the total number of steps and execution time when compared with sequential methods.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

KAEDAH BLOK MULTILANGKAH BAGI MENYELESAIKAN PERSAMAAN PEMBEZAAN BIASA

Oleh

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Kaedah multilangkah bagi menyelesaikan sistem Persamaan Pembezaan Biasa (PPB) dihuraikan. Bahagian pertama tesis ini berkenaan pembinaan dan pemerolehan kaedah baru Blok Formulasi Beza Ke Belakang (BFBB) menggunakan panjang langkah tetap dan panjang langkah berubah bagi menyelesaikan Masalah Nilai Awal (MNA) kaku peringkat pertama. Rantau kemantapan kaedah blok didedahkan dan keputusan berangka dibandingkan dengan kaedah sedia ada.

Bahagian kedua tesis ini menghuraikan pemerolehan kaedah blok Adams bagi menyelesaikan sistem tak kaku peringkat kedua secara terus manakala campuran daripada kaedah Adams dan kaedah tersirat BFBB yang baru digunakan bagi menyelesaikan masalah kaku peringkat kedua secara terus. Strategi pemetakan bagi kaedah blok dibincangkan secara terperinci dan keputusan berangka bagi pemetakan dalam blok dibandingkan dengan kaedah

bukan blok Saiz Langkah Berubah Peringkat Berubah (SBPB) bagi menyelesaikan PPB peringkat kedua secara terus.

Akhirnya, tesis ini membincangkan algoritma berangka selari bagi menyelesaikan PPB. Kaedah BFBB yang dibentuk diuji dan keselarian diperolehi dengan menggunakan perpustakaan Penghantaran Mesej Antaramuka (MPI) yang dilarikan atas komputer berkeupayaan tinggi (HPC). Pelaksanaan selari kod baru tersebut menunjukkan kecekapan superlinear apabila dimensi sistem PBB meningkat. Perbandingan dan ilustrasi dengan kod sesiri diberi.

Kesimpulannya, keputusan berangka dengan jelas menunjukkan kecekapan menggunakan kaedah blok bagi menyelesaikan PPB. Pemakaian kaedah blok multilangkah kepada masalah secara meluas mendedahkan pengurangan dalam jumlah langkah dan masa pelaksanaan apabila dibandingkan dengan kod kaedah sesiri.

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This thesis submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee are as follows:

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DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

ZARINA BIBI BT IBRAHIM

Date:

TABLE OF CONTENTS

CHAPTER