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ADAPTIVE COMPLEX NEURO-FUZZY INFERENCE SYSTEM FOR NON LINEAR MODELING AND TIME SERIES PREDICTION

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By

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Chair: Associate Professor Mohammad Hamiruce Marhaban, PhD

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Complex fuzzy logic is an extension to traditional fuzzy logic where truth values are extended to complex unit disk. It can be interpreted as a generalization of infinite-valued Type-1 fuzzy logic. Due to growth of truth values, complex fuzzy logic has better ability to deal with nonlinearities. Moreover, humans' brain works adaptively based on amplitude and phase which indicate that complex numbers are natural as well and can represent human brain more accurate in comparison with real valued numbers. Extension of this notion into fuzzy logic shows that complex fuzzy logic has better ability to model nonlinear system more accurately with smaller rule base comparing to type-1 fuzzy logic. At present, most of the researches are theoretical in nature, and practical works are presented to apply complex fuzzy logic for signal processing and time-series forecasting where the system is univariate. However, real world systems mostly have multiple input and therefore complex fuzzy logic can be extended for modeling of these kind of systems. To this end, in this study a machine learning architecture based on complex fuzzy logic has been developed. This structure is developed to deal with nonlinear system modeling as well as time-series forecasting with multiple inputs. The development sequence of such method can be divided into two parts: Developing network structure to employ complex fuzzy

logic and proposing learning algorithm to train the system. For the first part, a structure based on a well-known adaptive network fuzzy inference system has been proposed and also Takagi-Sugeno inferential system has been modified. Second part has been done by proposing a novel learning rule containing genetic algorithm, Levenberg-Marquardt technique and least square estimation. In this regard, genetic algorithm generates different initial conditions of premise parameters to find the best one. Furthermore, Levenberg-Marquardt technique and least square estimation are employed to fine tune the premise and consequent parameters, respectively. To this end, a closed form derivative solution of error with respect to premise parameters has been derived. Afterward, the performance of proposed method has been evaluated using five different systems consisting of nonlinear function modelings, nonlinear dynamic modeling, chaotic dynamic prediction and sunspot dataset forecasting. The procedure of data selection for training tasks of these experiments have been taken from literature. This leads to conduct the experiments in the same condition as previous works and therefore be reasonable to compare with their results. The performance of the proposed method is then compared with other works in literature.

The first experiment is two dimensional Sinc function in which the proposed method with 4 rules has root mean square error of 0.017 while the root mean square error of adaptive network fuzzy inference system method is 0.074. This shows that proposed method has better ability to model this nonlinear system. The second experiment used proposed method to model a three input nonlinear function. The number of rules for this experiment is same for proposed method and adaptive network fuzzy inference system. However, the average percentage error of proposed method is 0.193% which is much lower than 1.066% for previous method. A nonlinear dynamic system has been modeled in the third experiment. The proposed method has root mean square error of 0.018 with 4 number of rules. In this regard, the error value is less than previous approaches while other approaches have same or higher number of rules. The other two experiments are done to predict two well known time series. Mackey-Glass is the first time series which is known as chaotic and aperiodic. Proposed method with 8 number of rules is able to predict this dataset with root mean square of 0.0007 and non dimensional error index of 0.0031 which is comparable with previous works in terms of error measure values and number of rules. The second time series database is annual sunspot dataset which is a natural and physical phenomena. The proposed method is able to predict this dataset with 8 number of rules. The values of error measures for this experiments are 2.2×10^{-3} for mean squared error and 0.209 for non dimensional error index. The performance of proposed method in terms of error measures is better than previous approaches.

The comparisons of performances in terms of number of rules and error measures for performed experiments between proposed method and previous approaches reveal the capability and effectiveness of proposed method in modeling nonlinear systems and forecasting of time series.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Master Sains

SISTEM INFEREN NEURAL KABUR UBAH SUAI KOMPLEKS BAGI MODEL TAK LINEAR DAN RAMALAN SIRI MASA

Oleh

MOHAMMADREZA SHOORANGIZ

Oktober, 2013

Pengerusi: Profesor Madya Mohammad Hamiruce Marhaban, PhD

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logik kabur kompleks adalah lanjutan logik kabur tradisional di mana nilai-nilai kebenaran dilanjutkan kepada unit cakera kompleks. Ia boleh ditafsirkan sebagai generalisasi tak terhingga bernilai Type- 1 logik kabur. Disebabkan oleh pertumbuhan nilai-nilai kebenaran, logik kabur kompleks mempunyai keupayaan lebih baik untuk berurusan dengan tak linear. Selain itu, otak manusia boleh menyesuaikan berdasarkan amplitud dan fasa yang menunjukkan bahawa nombor kompleks adalah semulajadi serta dan boleh mewakili otak manusia yang lebih tepat berbanding dengan nombor nyata. Lanjutan daripada idea ini ke dalam logik kabur menunjukkan bahawa logik kabur kompleks mempunyai keupayaan lebih baik untuk model sistem tak linear lebih tepat dengan peraturan asas yang lebih kecil yang membandingkan jenis-1 logik kabur. Pada masa ini, kebanyakan kajian adalah teori, dan kerja-kerja praktikal dikemukakan untuk menggunakan logik kabur kompleks untuk pemprosesan isyarat dan ramalan siri masa di mana sistem ini univariat. Walau bagaimanapun, sistem dunia sebenar kebanyakannya mempunyai pelbagai input dan oleh kerana itu logik

kabur kompleks boleh dilanjutkan untuk memodenkan sistem jenis ini. Untuk tujuan ini, dalam kajian ini seni bina mesin pembelajaran berdasarkan logik kabur kompleks telah dibangunkan. Struktur ini dibangunkan untuk berurusan dengan model sistem tak linear serta ramalan siri masa dengan pelbagai input. urutan pembangunan kaedah itu boleh dibahagikan kepada dua bahagian: Membangunkan struktur rangkaian untuk menggunakan logik kabur kompleks dan mencadangkan algoritma pembelajaran untuk melatih sistem tersebut. Untuk bahagian pertama, struktur berdasarkan penyesuaian rangkaian sistem kesimpulan terkenal kabur telah dicadangkan dan juga Takagi-Sugeno inferensi sistem telah diubah suai. Bahagian kedua telah dilakukan dengan mencadangkan satu peraturan pembelajaran novel yang mengandungi algoritma genetik, teknik Levenberg-Marquardt dan anggaran ganda dua terkecil. Dalam hal ini, algoritma genetik menjana keadaan awal yang berbeza parameter premis untuk mencari yang terbaik. Tambahan pula, teknik Levenberg-Marquardt dan anggaran ganda dua terkecil masing-masing digunakan untuk menalakan premis dan parameter konsekuen. Untuk tujuan ini, satu penyelesaian derivatif bentuk tertutup untuk ralat berkenaan dengan parameter premis telah diperolehi. Selepas itu, pelaksanaan kaedah yang dicadangkan telah dinilai menggunakan lima sistem yang berbeza yang terdiri daripada modeling fungsi tak linear, model dinamik tak linear, ramalan camuk dinamik dan ramalan dataset tompok pd matahari. Prosedur pemilihan data untuk tugas-tugas latihan eksperimen ini telah diambil dari literatur. Ini membawa kepada pelaksanaan eksperimen dalam keadaan yang sama seperti kerja-kerja sebelum ini dan oleh itu adalah munasabah untuk membandingkan dengan keputusan mereka. Prestasi kaedah yang dicadangkan itu kemudiannya dibandingkan dengan kerja-kerja lain dalam literatur.

Eksperimen pertama adalah dua dimensi fungsi sinc di mana kaedah yang dicadangkan dengan 4 peraturan yang mempunyai punca min ralat kuasa dua 0.017 manakala punca min ralat kuasa dua rangkaian penyesuaian kabur kaedah sistem kesimpulan adalah 0.074. Ini menunjukkan bahawa kaedah yang dicadangkan mempunyai keupayaan lebih baik untuk memodelkan sistem tak linear ini. Eksperimen kedua menggunakan kaedah yang dicadangkan untuk memodelkan fungsi tiga input tak linear. Bilangan peraturan untuk eksperimen ini adalah sama bagi kaedah yang dicadangkan dan penyesuaian sistem rangkaian kesimpulan kabur. Walau bagaimanapun, purata peratusan ralat bagi kaedah yang dicadangkan ialah 0.193 % yang jauh lebih rendah daripada 1.066 % untuk kaedah sebelumnya. Satu sistem dinamik tak linear telah dimodelkan dalam eksperimen ketiga. Kaedah yang dicadangkan mempunyai ralat punca min kuasa dua 0.018 dengan 4 peraturan. Dalam hal ini, nilai ralat adalah kurang daripada pendekatan sebelumnya manakala pendekatan yang lain mempunyai jumlah yang sama atau lebih tinggi peraturan. Dua eksperimen dilakukan untuk meramal dua siri masa terkenal. Mackey-Glass adalah siri masa yang pertama yang dikenali sebagai kelam-kabut dan aperiodik. Kaedah yang dicadangkan dengan 8 peraturan dapat meramalkan dataset ini dengan punca min kuasa dua 0.0007 dan indeks ralat bukan dimensi 0.0031 yang setanding dengan kerja-kerja sebelum ini dari segi nilai-nilai ukuran ralat dan bilangan peraturan. Pangkalan data siri masa kedua ialah dataset Tompok pada matahari tahunan yang merupakan fenomena semulajadi dan fizikal. Kaedah yang dicadangkan mampu untuk meramalkan dataset ini dengan 8 bilangan peraturan. Nilai-nilai langkah ralat untuk eksperimen ini adalah 2.2×10^{-3} untuk min ralat kuasa dua dan 0.209 bagi indeks ralat bukan dimensi. Prestasi kaedah yang dicadangkan dari segi pengukuran ralat adalah lebih baik daripada pendekatan yang sebelumnya.

Perbandingan prestasi dari segi bilangan peraturan dan pengukuran ralat untuk eksperimen yang telah dilaksanakan antara kaedah dan pendekatan yang dicadangkan sebelum ini mendedahkan keupayaan dan keberkesanan kaedah yang dicadangkan dalam permodelan sistem tak linear dan ramalan siri masa.

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I certify that a Thesis Examination Committee has met on 4 October 2013 to conduct the final examination of Mohammadreza Shoorangiz on his thesis entitled "Adaptive Complex Neuro-Fuzzy Inference System for Non Linear Modeling and Time Series Prediction" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

MOHAMMADREZA SHOORANGIZ

Date: 4 October 2013

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LIST OF ABBREVIATIONS

ACNFIS	Adaptive Complex Neuro-Fuzzy Inference System
ANFIS	Adaptive Network Fuzzy Inference System
ANCFIS	Adaptive Neuro-Complex Fuzzy Inference System
ABC	Artificial Bee Colony
ANN	Artificial Neural Network
ARIMA	AutoRegressive Integrated Moving Average
APE	Average Percentage Error
CFIS	Complex Fuzzy Inference System
CFL	Complex Fuzzy Logic
CFS	Complex Fuzzy Set
CMF	Complex Membership Function
GA	Genetic Algorithm
LSE	Least Square Estimate
LM	Levenberg Marquardt
MSE	Mean Squared Error
MF	Membership Function
NDEI	Non-Dimensional Error Index
PSO	Particle Swarm Optimization
RLSE	Recursive Least Square Estimate
RMSE	Root Mean Squared Error
SSE	Sum Squared Error
TS	Takagi-Sugeno

LIST OF SYMBOLS

ε	Amplitude Error Signal
Φ	Computational Complexity
ϵ	Error Signal
$\mathbf{e}$	Error Vector
η	Learning Rate
μ	Membership Function
O	Node Output
α	One Premise Parameter
E	Overall Error
ξ	Phase Error Signal
k	Step Size
T	Target Value
U	Universe of Discourse

CHAPTER 1

INTRODUCTION

1.1 Preface

System modeling is the process of creating a mathematical model to simulate, predict and understand a practical system in real life [1, 2]. A reliable model is capable of identifying the relationship between inputs and outputs accurately [3]. This concept plays an essential role in many disciplines such as control, expert systems, communications and so on [2, 4]. In last decades, system modeling received considerable attention due to large number of applications in different fields [5]. However, most of real world processes are nonlinear to some extent with individualistic nature and also physical insight about the system is generally unknown [1, 6]. This led researchers to develop black-box modeling which evolve a model only based on measured input-output data pairs [1]. Nevertheless, uncertainty in the collected data is inevitable and therefore traditional mathematical tools such as differential equations are not suitable to model the system [7].

In this regard, artificial neural networks (ANNs) and Takagi-Sugeno (TS) fuzzy systems are proven to be universal approximators [8, 9]. Each of these methods has its own advantages and has been used in various applications [10–12]. ANN mimics the process of biological neural network and is capable of learning functional relations based on input-output data pairs; however, the network itself is not intuitive [13–15]. Fuzzy systems have the ability to use human knowledge and assimilate human thinking and reasoning [16, 17]. In addition, fuzzy systems can qualitatively model ill-defined systems based on human knowledge without precise quantitative analysis [4, 7, 18]. On the other hand, designing fuzzy system needs knowledge and informations about the plant to maintain rule base [19]. These informations are generally extracted from an expert, however, defining suitable If-Then for a complicated system is a difficult task [20]. Therefore, neuro-fuzzy systems are developed to overcome limitations of aforementioned techniques [21]. Combination of ANN and fuzzy system can solve knowledge extraction problem [7, 22]. Also, the evolved neuro-fuzzy system after training can be interpreted. These systems benefit from learning ability of neural networks and human reasoning capability of fuzzy logic.

Adaptive Network Fuzzy Inference System (ANFIS) is one of the notable contributions in neuro-fuzzy systems [7]. Neural network has been utilized in ANFIS to find modifiable parameters and therefore tune the fuzzy system by employing a hybrid learning rule [7]. ANFIS creates If-Then rules based on grid partitioning of input-output space [23]. The number of rules depends on inputs and number of membership functions per each input. Nevertheless, the number of rules in ANFIS increases exponentially due to grid partitioning [23].

Complex fuzzy set (CFS) [24] has been proposed as an extension of standard fuzzy set theory. This notion extends the range of truth values from $[0, 1]$ to unit circle in complex plane. Consequently, complex fuzzy logic (CFL) [25] developed by utilizing CFSs instead of fuzzy sets in order to benefit from complex valued nature and other properties of CFSs. CFL maintains a unique framework which benefits from CFS properties in addition to advantages of traditional fuzzy logic. Therefore, CFL is a natural extension of fuzzy logic and capable of addressing problems which are difficult to be addressed by traditional fuzzy logic [25].

1.2 Motivation and Problem Statement

System modeling and identification is an important task which can be used for controller design, system analysis, simulation and so on. Mathematical approaches has been used for system identification; however, they require some physical insight about the system prior to modeling which is generally not available for nonlinear systems. On the other hand, neuro-fuzzy systems are capable of acquiring the input-output relationship based on sampled data even when the physical insight is not available.

Human brain functions adaptively based on amplitude and phase [26]. This implies that complex-valued numbers are natural like real-valued numbers and also they can better represent human brain. In this regard, Complex-valued neural network has more functionality, more generalization and smaller network in comparison with real-valued one [27]. This shows that CFSs and consequently CFL can bring a new view point for system modeling. Also, CFL has been applied for univariate time-series prediction [28] and has shown very good performance comparing to type-1 fuzzy methods. However, system modeling specially for multi input systems has not yet studied using CFL. In this regard, the structure presented in [28] cannot be extended to multi input nonlinear modeling due to its network structure and mathematical operators. Nevertheless, CFL has shown a very compromising performance for univariate time-series prediction and therefore is expected to handle imprecise data and ill-defined multi input systems better than traditional fuzzy with less number of rules.

Due to complex-valued nature of CFL, extracting rules and tuning membership functions are difficult, if not impossible, even for an expert. Complex-valued neural network, on the other hand, can learn and tune rule base of CFL based on input-output training data. Thus, combination of complex-valued neural network and CFL is expected to learn and model a system based on input output data. This research aims to investigate a new method of system modeling based on CFL which can learn the desired system based on input output data. It then followed by a comparison of performances, number of rules and parameters of proposed approach with ANFIS and other previous approaches.

1.3 Aim and Objectives

The main aim of this research is to develop a complex neuro-fuzzy architecture to implement complex fuzzy logic for system modeling. Complex fuzzy logic is the backbone of this architecture which will be tuned by complex-valued neural network based on input-output training data. This architecture is expected to be more compact in terms of number of rules as compared with similar neuro-fuzzy architectures. Since ANFIS has shown better performance for nonlinear modeling and time series prediction comparing to other neuro fuzzy techniques, its structure will be developed for the proposed method.

The objectives of this thesis are as follows:

1. To develop a new modeling and prediction method based on complex fuzzy logic and ANFIS approach.
2. To compare the performance of the proposed method with their established methods employing similar environment.

1.4 Scope of the Work

The proposed machine learning architecture is based on ANFIS where underlying procedures have been adapted to implement CFL. Also, membership functions for CFL are selected from two real-valued functions to represent amplitude and phase parts. These functions are selected as Gaussian and Sigmoid to represent amplitude and phase, respectively. The number of rules and type of membership functions are specified prior to learning task. The latter task is restricted to offline learning in this research. Performance of proposed systems, then, has been evaluated using five different examples. These examples consists of nonlinear functions modeling, nonlinear dynamic modeling, chaotic dynamic prediction and sunspot dataset forecasting. After each example, performance of proposed system is compared with previous approaches. These comparisons are made by means of appropriate error measure from literature.

1.5 Thesis Contributions

The proposed method is the first realization of CFL for system modeling with multiple inputs so far. In the proposed method, complex membership functions are built based on amplitude and phase parts as separate real-valued functions. Also, Takagi-Sugeno inference has been modified to be capable of translating complex-valued firing strengths of all rules into one real number. Moreover, a closed form derivative solution for gradient based optimizations has been derived for CFL based structures.

1.6 Thesis Organization

This thesis is made up of five chapters. Chapter 1 is Introduction which consisted of motivation, problem statement, objectives and thesis contributions. Chapter 2 contains introduction to type-1 fuzzy sets, ANFIS, CFSs and CFL. Chapter 3 starts with a description of the structure of the proposed method, it then follows by presenting the developed learning algorithm and then the computational complexity of the proposed method is analyzed. In chapter 4, five different experiments have been used to evaluate the performance of the proposed method and results have been compared to previous techniques. Finally, summary of this thesis and suggestions for the future work are given in chapter 5.

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