

Memory Polynomial with Binomial Reduction in Digital Pre-distortion for Wireless Communication Systems

Hong Ning Choo^{1*}, Nurul Adilah Abdul Latiff¹, Pooria Varahram² and Borhanuddin Mohd Ali¹

¹*Department of Computer and Communication Systems Engineering, Faculty of Engineering, Universiti Putra Malaysia, 43400 UPM, Serdang, Selangor, Malaysia*

²*Department of Electronic Engineering, National University of Ireland Maynooth, Maynooth, Co. Kildare, Ireland*

ABSTRACT

One of the biggest power consuming devices in wireless communications system is the Power Amplifier (PA) which amplifies signals non-linearly when operating in real-world systems. The negative effects of PA non-linearity are energy inefficiency, amplitude and phase distortion. The increases in transmission speed in present day communication technology introduces Memory Effects, where signal spreading happens at the PA output, thus causing overhead in signal processing at the receiver side. PA Linearization is therefore required to counter the non-linearity and Memory Effects. Digital Pre-distortion (DPD) is one of the outstanding PA Linearization methods in terms of its strengths in implementation simplicity, bandwidth, efficiency, flexibility and cost. DPD pre-distorts the input signal, using an inversed model function of the PA. Modelling of the PA is therefore vital in DPD, where the Memory Polynomial Method (MP) is used to model the PA with memory effects. In this paper, the MP method is improved in Memory Polynomial using Binomial Reduction method (MPB-imag-2k). The method is simulated using a modelled ZVE-8G Power Amplifier and sampled 4G (LTE) signals. It was found MPB-imag-2k is capable of achieving comparable anti-scattering/anti-distortion in MP for non-linearity order of 3, memory depth of 3 and pre-amplifier gain of 2.

Keywords: Power Amplifier, PA Linearization, Digital Pre-Distortion, 4G, Memory Polynomial

ARTICLE INFO

Article history:

Received: 24 August 2016

Accepted: 02 December 2016

E-mail addresses:

eddiechoohn@gmail.com (Hong Ning Choo),
nuruladilah@upm.edu.com (Nurul Adilah Abdul Latiff),
pooria.varahram@nuim.ie (Pooria Varahram),
borhan@upm.edu.com (Borhanuddin Mohd Ali)

*Corresponding Author

INTRODUCTION

The non-linearity of the Power Amplifier (PA) causes output signal distortion in amplitude and phase (Chen et al., 2014; Choo, 2012; Choo et al., 2013; Ding et al., 2004; Ding, 2004; Liu et al., 2014; Morgan et al., 2006; Parta et al., 2014; Pinal & Pere, 2007). The

simplistic solution of avoiding the PA's non-linearity is to back-off the PA to operate only in the linear region. Besides resulting in poor energy efficiency, the backed-off solution causes higher operational cost. Today's high speed transmission technology such as Wideband Code Division Multiple Access (WCDMA) and Orthogonal Frequency Division Multiplexing (OFDM) causes high Peak to Average Power Ratios (PAPR) in communication systems, which encourages the PA to be backed off further from its saturation point. Low efficiency of the PA contributes to waste of power, which is an undesired additional cost in the telecommunication industry. The business needs of the industry justify the demand of linearizing the PA, to eliminate the undesired effects of inefficiency, distortions and power wastage.

The Digital Pre-Distortion (DPD) technique linearizes the cheaper non-linear power amplifiers, resulting in a lower overall cost of the communication system (Choo, 2012; Choo et al., 2013; Techsource-asia, 2012; Varahram et al., 2010). DPD offers many advantages in terms of cost, power management, reliability and handling if compared to the other analog methods (Choo, 2012; Choo et al., 2013; Varahram et al., 2009; Varahram et al., 2010).

Nomenclature	
DPD	Digital Pre-distortion
OFDM	Orthogonal Frequency Division Multiplexing
MOR	Multiplication Operations Reduction
MP	Memory Polynomial
MPB	Memory Polynomial with Binomial Reduction
PA	Power Amplifier
PAPR	Peak to Average Power Ratios
WCDMA	Wideband Code Division Multiple Access

MODEL DESCRIPTION

Digital Pre-distortion (DPD)

The pre-distorter behaves as an inversed function of the PA. The PA input signal is first pre-distorted at the pre-distorter, and then directed into the PA. The two connecting sequential functions: inversed non-linear pre-distorter and the non-linear PA, results in a linear output (Pinal & Pere, 2007; Techsource-asia, 2012; Varahram et al., 2009; Varahram et al., 2010).

Digital Pre-distortion (DPD) is where the pre-distortion is conducted at baseband digital domain. DPD is one of the most cost effective PA linearizing methods with the least compromise on efficiency (Ding et al., 2004; Ding, 2004; Varahram et al., 2009; Varahram et al., 2010; Choo, 2012; Techsource-asia, 2012; Choo et al., 2013; Chen et al., 2014). Accurate modelling of the non-linearity PA is required in-order to calculate the pre-distorter function as an inversed model of the PA.

Memory Polynomial (MP)

Volterra Series is traditionally used to model non-linear systems. However, the complexity increases exponentially when the PA non-linearity order increases 0-0. The Memory Polynomial (MP) method utilizes the diagonal kernels of the Volterra Series, resulting in a reduced number of coefficients. MP is widely explored and built on top in (Chen et al., 2014; Liu et al., 2014; Morgan et al., 2006; Xie & Zeng, 2012; Yu & Jiang 2013). The MP method is shown below (Ding et al., 2004; Ding, 2004):

$$z(n) = \sum_{\substack{k=1 \\ k \text{ odd}}}^K \sum_{q=0}^Q a_{kq} x(n-q) |x(n-q)|^{k-1} \quad (1)$$

Where Q is the memory depth, K is the non-linearity order. $x(n)$ is the PA Input Signal, and is a_{ka} the inversed of the PA coefficients, which is also the MP coefficients.

The Least Squares (LS) method is used to obtain the MP coefficients (Ding et al., 2004; Ding, 2004). The input signal $x(n)$ is replaced with the output signal $y(n)$, yields the following:

$$z(n) = \sum_{\substack{k=1 \\ k \text{ odd}}}^K \sum_{q=0}^Q a_{kq} y(n-q) |y(n-q)|^{k-1} \quad (2)$$

(2) in matrix form:

$$z = Y \cdot a \quad (3)$$

Where

$$z = [z(0), z(1), \dots, z(N-1)]^T \quad (4)$$

$$Y = [y_{10}, \dots, y_{K0}, \dots, y_{1Q}, \dots, y_{KQ}] \quad (5)$$

$$y_{KQ} = [y_{KQ}(0), y_{KQ}(1), \dots, y_{KQ}(N-1)]^T \quad (6)$$

$$a = [a_{10}, \dots, a_{K0}, \dots, a_{1Q}, \dots, a_{KQ}]^T \quad (7)$$

The least square solutions in (3) could be rewrite as:

$$a = (Y^{conj} \cdot Y)^{-1} Y^{conj} z \quad (8)$$

Memory Polynomial with Binomial Reduction (MPB)

The MP equation in (1) is rephrased, where non-linearity order, k , and linearity order, q starts from 0:

$$z(n) = \sum_{k=0}^K \sum_{q=0}^Q a_{kq} x(n-q) [x(n-q)_{real}^2 + x(n-q)_{imag}^2]^k \tag{9}$$

Using the binomial theorem below

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^k a^{n-k} = \sum_{k=0}^n \binom{n}{k} a^k x^{n-k} \tag{10}$$

The non-linear portion of the basis function is restructured as:

$$[x(n-q)_{real}^2 + x(n-q)_{imag}^2]^k = x(n-q)_{imag}^{2k} \sum_{h=0}^k \binom{k}{h} \left[\frac{x(n-q)_{real}}{x(n-q)_{imag}} \right]^{2h} \tag{11}$$

$$[x(n-q)_{real}^2 + x(n-q)_{imag}^2]^k = x(n-q)_{real}^{2k} \sum_{h=0}^k \binom{k}{h} \left[\frac{x(n-q)_{imag}}{x(n-q)_{real}} \right]^{2h} \tag{12}$$

Let the binomial basis function of (11) represented as

$$y = \sum_{h=0}^k \binom{k}{h} \left[\frac{x(n-q)_{real}}{x(n-q)_{imag}} \right]^{2h} = \sum_{h=0}^k \binom{k}{h} x^{2h} \tag{13}$$

Similarly, let the binomial basis function of (12) represented as

$$y = \sum_{h=0}^k \binom{k}{h} \left[\frac{x(n-q)_{imag}}{x(n-q)_{real}} \right]^{2h} = \sum_{h=0}^k \binom{k}{h} x^{2h} \tag{14}$$

Let

$$y = \sum_{h=0}^k \binom{k}{h} x^{2h} \approx x^j \tag{15}$$

where $-5 \leq x \leq 5$ and $3 \leq k \leq 5$

The value of j is explored by using the macro-matching graph method as shown in Figure 1, Figure 2 and Figure 3.

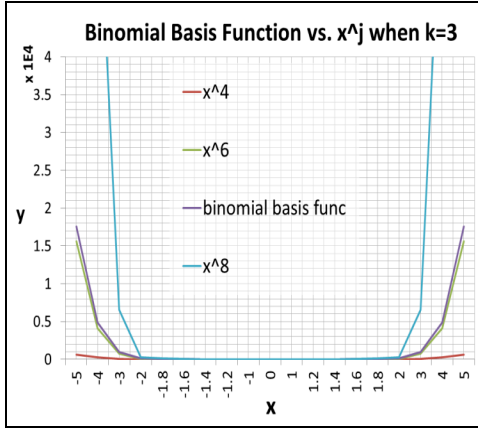


Figure 1. Binomial Basis Function vs. x^j when $k = 3$

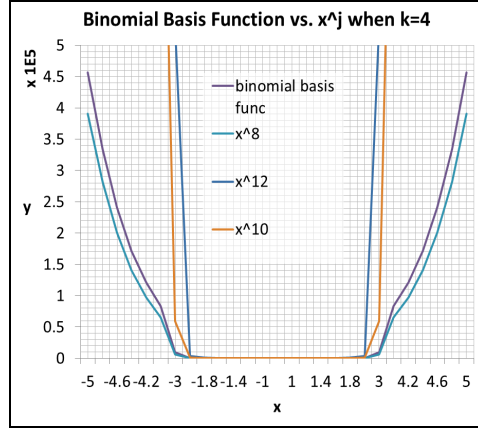


Figure 2. Binomial Basis Function vs. x^j when $k = 4$

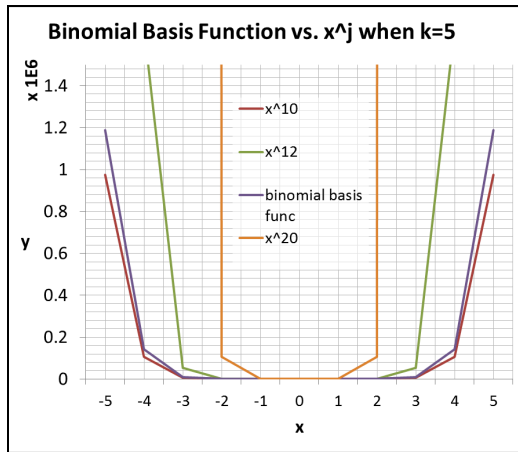


Figure 3. Binomial Basis Function vs. x^j when $k = 5$

The binomial basis function equivalent is shown in Table 1 below:

Table 1
Binomial basis function equivalent

Non-linearity order, k =3 (Figure 1)	k =4 (Figure 2)	k =5 (Figure 3)
$y = \sum_{h=0}^3 \binom{3}{h} x^{2h} \approx x^6$	$y = \sum_{h=0}^4 \binom{4}{h} x^{2h} \approx x^8$	$y = \sum_{h=0}^5 \binom{5}{h} x^{2h} \approx x^{10}$

The following could be concluded:

$$\sum_{h=0}^k \binom{k}{h} x^{2h} \approx x^{2k} \tag{16}$$

Substituting (16) into (13), (11) and (9) gives MPB-real-2k equation below:

$$z(n) = \sum_{k=0}^K \sum_{q=0}^Q a_{kq} x(n-q) x(n-q)_{real}^{2k} \tag{17}$$

Similarly, substituting (16) into (14), (12) and (9) yields MPB-imag-2k:

$$z(n) = \sum_{k=0}^K \sum_{q=0}^Q a_{kq} x(n-q) x(n-q)_{imag}^{2k} \tag{18}$$

RESULTS AND DISCUSSION

Amplitude and Phase Distortion Correction

Figure 4 shows the AMAM graph for MPB-imag-2k vs. MP with Non-linearity Order (K) = 3; Memory Depth (M) = 3, and Pre-amp Gain = 2. Pre-distortion (MPB) is capable of resolving the scattering of PA output signal observed.

Figure 5 shows the AM/PM graph for MPB-imag-2k vs. MP with Non-linearity Order (K) = 3, Memory Depth (M) = 3, and Pre-amp Gain = 2. The pre-distorted PA output is capable of resisting phase distortion, where the phase difference between output and input signal is close to zero.

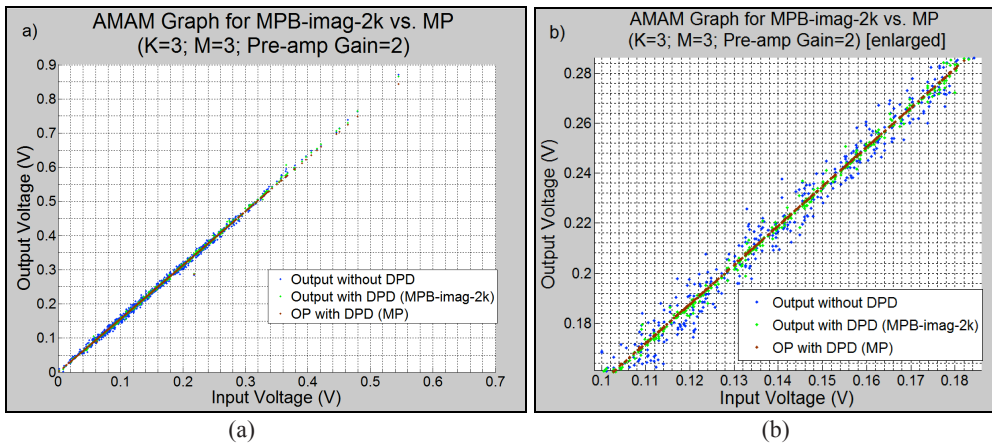


Figure 4. (a) AMAM Graph for MPB-imag-2k vs. MP with Non-linearity order=3, Memory Depth=3, and Pre-amplifier Gain=2; (b) Enlarged image of the AMAM graph

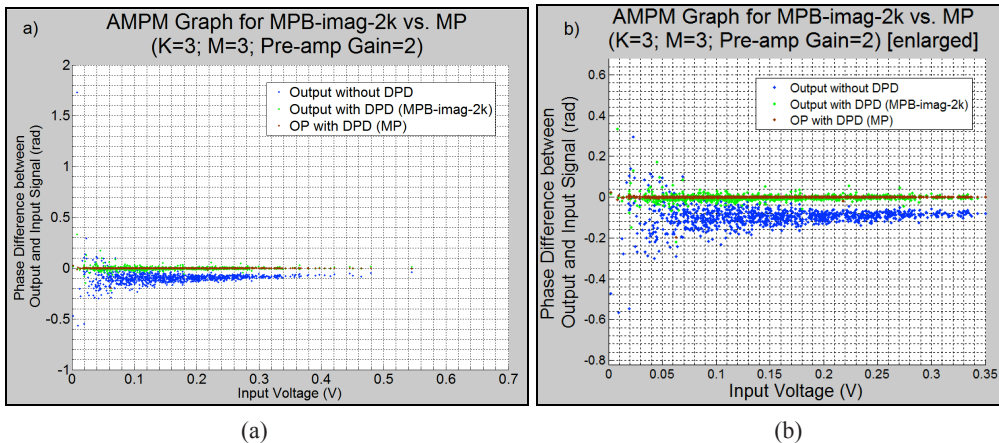


Figure 5. Figure 5. (a) AMPM Graph for MPB-imag-2k vs. MP with Non-linearity Order=3, Memory Depth=3, and Pre-amplifier Gain=2; (b) Enlarged image of the AMPM graph

Resource Optimization on Multiplication Operations Reduction

MPB is compared with MP on resources reduction in terms of multiplication operations. Table 2 and 3 shows the respective formulas used to calculate the required multiplication operations.

Table 2
Multiplication Operations Calculation Formula for MP

Method Formula	$z(n) = \sum_{k=0}^K \sum_{q=0}^Q a_{kq} x(n-q) \sqrt{x(n-q)_{real}^2 + x(n-q)_{imag}^2}^{-2k}$
Multiplications Operations Calculation Formula	$Q(1 + 3(2K))$

Table 3
 Multiplication Operations Calculation Formula for MP

Method Formula	$z(n) = \sum_{k=0}^K \sum_{q=0}^Q \alpha_{kq} x(n-q) x(n-q)_{imag}^{2k}$
Multiplications Operations Calculation Formula	$Q(1 + 2(K))$

The net reduction of multiplication operations are calculated by finding the difference between the two formulas, which results in Multiplication Operations Reduction (MOR) in Table 4.

Table 4

Table 4
 MOR of MPB (MPB-imag-2k) against MP

MPB (MPB-imag-2k) from Table 2	MP from Table 3	MOR
$Q(1 + 2(K))$	$Q(1 + 3(2K))$	$4(K)$

CONCLUSION

MPB is an improved MP Model which uses the Binomial Reduction Method at the MP Basis Function. This results in linearly reduced multiplication operations but with matching PA Linearization Performance in MP, especially in anti-scattering and anti-phase-distortion.

REFERENCES

- Chen, W., Zhang, S., Liu, Y. J., Ghannouchi, F. M., Feng, Z., & Liu, Y. (2014). Efficient Pruning Technique of Memory Polynomial Models Suitable for PA Behavioral Modeling and Digital Predistortion. *IEEE Transactions on Microwave Theory and Techniques*, 62(10), 2290-2299.
- Choo, H. N. (2012). *DSP implementation of digital pre-distortion in wireless systems*. (Degree Dissertation). Universiti Putra Malaysia.
- Choo, H. N., Varahram, P., & Ali, B. M. (2013, November). DSP implementation of Digital Pre-distortion in wireless communication systems. In *Communications (MICC), 2013 IEEE Malaysia International Conference on* (pp. 207-212). IEEE.
- Ding, L., Zhou, G. T., Morgan, D. R., Ma, Z., Kenney, J. S., Kim, J., & Giardina, C. R. (2004). A robust digital baseband predistorter constructed using memory polynomials. *IEEE Transactions on Communications*, 52(1), 159-165.
- Ding, L. (2004). *Digital predistortion of power amplifiers for wireless applications* (Doctoral dissertation). Georgia Institute of Technology. Retrieved from http://www.academia.edu/download/34086448/ding_lei_200405_phd.pdf
- Liu, Y. J., Zhou, J., Chen, W., & Zhou, B. H. (2014). A robust augmented complexity-reduced generalized memory polynomial for wideband RF power amplifiers. *IEEE Transactions on Industrial Electronics*, 61(5), 2389-2401.

- Morgan, D. R., Ma, Z., Kim, J., Zierdt, M. G., & Pastalan, J. (2006). A generalized memory polynomial model for digital predistortion of RF power amplifiers. *IEEE Transactions on Signal Processing*, 54(10), 3852-3860.
- Parta, H., Ercegovic, M. D., & Pamarti, S. (2014, August). RF digital predistorter implementation using polynomial optimization. In *2014 IEEE 57th International Midwest Symposium on Circuits and Systems (MWSCAS)* (pp. 981-984). IEEE.
- Pinal, P. L. G., & Pere, L. (2007). *Multi Look-Up Table Digital Predistortion for RF Power Amplifier Linearization*. (Doctoral dissertation). Universitat Politècnica de Catalunya. Retrieved from <http://upcommons.upc.edu/handle/10803/6915>
- Techsource-asia. (2012). *Digital predistortion: Seeing clearly in the house of mirrors*. Retrieved from <http://techsource-asia.com/news/press-room/detail/25-digital-predistortion--seeing-clearly-in-the-house-of-mirrors.html>
- Varahram, P., Mohammady, S., Hamidon, M. N., Sidek, R. M., & Khatun, S. (2009). Digital Predistortion Technique for Compensating Memory Effects of Power Amplifiers in Wideband Applications. *Journal of Electrical Engineering*, 60(3), 129-135.
- Varahram, P., Mohammady, S., Hamidon, M., Sidek, R. M., & Khatun, S. (2010). Complex Gain Predistortion in WCDMA Power Amplifiers with Memory Effects. *The International Arab Journal of Information Technology*, 7(3), 333-341.
- Vella-Coleiro, G. (2011). Improving the Efficiency of RF Power Amplifiers with Digital Predistortion. In *26th Annual MTT-AP Symposium and Mini Show*.
- Xie, D., & Zeng, Z. (2012, March). An Improved Adaptive Algorithm for Digital Predistortion. In *Proc. IEEE International Conference on Computer Science and Electronics Engineering*. [S. 1.]: IEEE Press (pp. 340-343).
- Yu, X., & Jiang, H. (2013). Digital predistortion using adaptive basis functions. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 60(12), 3317-3327.

