UNIVERSITI PUTRA MALAYSIA

MODIFIED ALGORITHMS IN INTERVAL SYMMETRIC SINGLE-STEP PROCEDURE FOR SIMULTANEOUS INCLUSION OF POLYNOMIAL ZEROS.

ATIYAH BINTI WAN MOHD SHAM

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By

ATIYAH BINTI WAN MOHD SHAM

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science.

January 2014

DEDICATIONS

I would like to dedicate this thesis to

my father, Mr. Wan Mohd Sham bin Wan Bakar

my mother, Madam Azizah bt Hj. Abdul Rahman

my sisters,

Asma bt Wan Mohd Sham & Mastura bt Wan Mohd Sham

and

all my family members..

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

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Chairman: Mansor bin Monsi, PhD
Faculty: Science

Several modifications have been introduced in order to improve the problems in finding the zeros of polynomial simultaneously. They are named as Interval Symmetric Single-step 5-Delta procedure (ISS1-5D), Interval Midpoint Symmetric Single-step 5-Delta procedure (IMSS1-5D), Interval Zoro Symmetric Single-step 5-Delta procedure (IZSS1-5D) and Interval Midpoint Zoro Symmetric Single-step 5-Delta procedure (IMZSS1-5D) which were explained in details in this thesis. These four new procedures have been established from the previous symmetric single-step procedure.

Furthermore, we ensure that we start by choosing the suitable initial disjoint intervals which are guaranteed to contain one zero inside of each interval. The numerical results are given to validate the new modifications and the performances are being compared with the existing procedures in terms of number of iteration and computational time (CPU times).

In addition, the convergence properties for all new modifications are investigated to ensure that the procedures are useful for finding the zeros of polynomial. The convergence analysis of each procedure is also discussed. The programming codes are developed and implemented using Matlab R2012b incorporated with Intlab V5.5 toolbox and compared with the existing procedures. The efficiency of the procedures is justified by the numerical results given. The results generated showed that these new procedures produced less computational time, higher rate of convergence and achieved the desired accuracy.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk Ijazah Master Sains

PENGUBAHSUAIAN ALGORITMA DALAM PROSEDUR SELANG SIMETRI TUNGGAL UNTUK MEMERANGKAP SECARA SERENTAK PENSIFAR NYATA BAGI SUATU POLINOMIAL

Oleh

ATIYAH BT WAN MOHD SHAM

Januari 2014

Pengerusi : Mansor bin Monsi, PhD
Fakulti: Sains

Beberapa pengubahsuaian telah diperkenalkan untuk memperbaiki masalah dalam mencari pensifar bagi polinomial secara serentak . Mereka dinamakan sebagai prosedur Selang Simetri Tunggal 5 -Delta (ISS1 -5D), prosedur Selang Titik Tengah Simetri Tunggal 5 -Delta (IMSS1 -5D), prosedur Selang Zoro Simetri Tunggal 5 -Delta (IZSS1 -5D) dan prosedur Selang Zoro Titik Tengah Simetri Tunggal 5 -Delta (IMZSS1 -5D) yang telah diterangkan secara terperinci di dalam tesis ini . Keempat-empat prosedur baru telah diwujudkan berdasarkan dari prosedur selang simetri tunggal sebelumnya.

Tambahan pula, kita memastikan bahawa kita bermula dengan selang permulaan tidak bercantum yang dijamin mengandungi satu pensifar di dalam setiap selang. Keputusan berangka diberikan untuk mengesahkan pengubahsuaian baru tersebut dan perlaksanaannya yang dibandingkan dengan prosedur-prosedur yang sedia ada dari segi bilangan lelaran dan masa pengkomputeran (masa CPU).

Di samping itu, sifat-sifat penumpuan untuk semua pengubahsuaian baru diselidiki untuk memastikan bahawa prosedur ini berguna untuk mencari sifar polynomial. Analisis penumpuan setiap prosedur juga dibincangkan. Kod pengaturcaraan dibangunkan dan dilaksanakan dengan menggunakan Matlab R2012b digabungkan dengan Intlab V5.5 toolbox dan dibandingkan dengan prosedur yang sedia ada. Kecekapan prosedur ini dikuatkan dengan hasil berangka diberikan. Keputusan yang dihasilkan menunjukkan bahawa prosedur baru ini menghasilkan masa pengkomputeran yang lebih singkat, kadar penumpuan yang lebih tinggi dan mencapai ketepatan seperti yang dikehendaki.

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There would be no words to express my heartfelt gratitude towards my family especially to my parents, Wan Mohd Sham Wan Bakar and Azizah Hj Abd Rahman and the rest of my family for their prayers, love, endless supports and encouragement during the course of this study.

I certify that a Thesis Examination Committee has met on 16 January 2014 to conduct the final examination of Atiyah Bt Wan Mohd Sham on her thesis entitled "Modified Algorithms In Interval Symmetric Single-Step Procedure For Simultaneous Inclusion Of Polynomial Zeros" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U. (A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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DECLARATION

Declaration by graduate student

I hereby confirm that:

- this thesis is my original work
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This is to confirm that:

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LIST OF ABBREVIATIONS

$\mathbb{R}$	real numbers
$\mathbb{C}$	complex numbers
$I(\mathbb{R})$	real intervals
IA	interval analysis
p	polynomial
$R_p\left(w^{(k)}\right)$	R-factor
$\inf(x)$	infimum, or lower bound of **x**
$\sup(x)$	supremum, or upper bound of **x**
$O_R(I, x^*)$	R-order of convergence of an iterative process I with the limit point x^*
CPU	Central Processing Unit
IS	Interval Single-Step Method
PT	Point Total Step Method
ISS1	Interval Symmetric Single-Step Method
ISS1-5D	Interval Symmetric Single-Step 5-Delta Method
IMSS1-5D	Interval Midpoint Symmetric Single-Step 5-Delta Method
IZSS1	Interval Zoro Symmetric Single-Step Method
IZSS1-5D	Interval Zoro Symmetric Single-Step 5-Delta Method
IMZSS1-5D	Interval Midpoint Zoro Symmetric Single-Step 5-Delta Method

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CHAPTER 1

INTRODUCTION

1.1 Preliminaries

One of the significant fields in mathematics is numerical analysis. It is defined as the area of mathematics in solving the problems of continuous mathematics by creating, analyzing, and implementing algorithms. Almost as old as human civilization, numerical algorithms have been existed. During 1980's and 1990's, an area known as computational science was formed and started to take a serious view about the usage of numerical analysis (Atkinson K. E., 1855). The subject of numerical analysis provides computational methods for the study and solution of mathematical problems. (Atkinson, K. E., 1978)

In late 50's, interval analysis is a new and promising branch of applied mathematics which have been originated by Moore since 1959. Till now, interval analysis is widely used in order to solve a variety of mathematical problems. The main significance of interval analysis is that it can solve problems so that the results are guaranteed to be correct and can be computed with finitely precise floating point operations (Snyder, 1992). Thus, interval analysis is classified as one of the important tools in solving various kinds of problem such as problems of finding zeros in polynomials, optimization, differential equation and also integral equation where most mathematical models used in the natural sciences and engineering are based on those problems (Moore and Bierbaum, 1979).

1.2 Concepts and properties of interval

IA's mathematical definitions and notations are extended from set of theory and ordered numerical sets called interval (Schwartz, 1999). Our research considers closed interval analysis with the following definitions of an interval (using Matlab upper bound, lower bound style notation):

$$X = [\inf(x), \sup(x)] = \{x \mid \inf(x) \le x \le \sup(x), \inf(x), \sup(x), x \in \mathbb{R}\}$$

where $\inf(x)$ denotes the infimum, or lower bound of X and $\sup(x)$ denotes the supremum, or upper bound of X.

Another way to define number system is by using interval number system. Furthermore, the MATLAB programming (Rump, 1999) gives significant ideas of interval number system containing the answers that overcome all the possible errors made in any low-level computation. According to Moore (1966), an interval number corresponds to a pair of real numbers which representing the lower and upper bound of the parameter range.

The following are the main concepts and properties. In interval analysis, real numbers is denoted by $\mathbb{R}$ and members of $\mathbb{R}$ are denoted by lowercase letters $a, b, c, ..., x, y, z.$

A subject of $\mathbb{R}$ of the form

$$A = [a_1, a_2] = \{t \mid a_1 \le t \le a_2;\ a_1, a_2 \in \mathbb{R}\}$$

is called a closed real interval. The set off all closed real interval is denoted by $I(\mathbb{R})$ and the members of $I(\mathbb{R})$ is denoted by uppercase letters $A, B, C, ..., X, Y, Z.$ The following informations are available in Alefeld and Herzberger in 1983. (see also in Salim N.R., 2012)

Definition 1.1: Let $* \in \{+, -, \cdot, /\}$ be a binary operation on the set of real numbers $\mathbb{R}$ .If $A, B \in I(\mathbb{R})$ then

$$A * B = \{z = a * b \mid a \in A,\ b \in B\}$$

defines a binary operation on $I(\mathbb{R})$. Assume that $0 \notin B$ in this case of division. It is straightforward to define interval extensions of basic operations as

$a)$ $A + B = [a_1 + b_1, a_2 + b_2],$
$b)$ $A - B = [a_1 - b_2, a_2 - b_1],$
$c)$ $A \cdot B = [\min\{a_1b_1, a_1b_2, a_2b_1, a_2b_2\}, \max\{a_1b_1, a_1b_2, a_2b_1, a_2b_2\}],$
$d)$ $A/B = [a_1 \cdot a_2] \cdot [1/b_2, 1/b_1].$

Example 1.1: Let $A = [1, 2]$ and $B = [5, 6]$ therefore

$a)$ $A + B = [1,2] + [5,6] = [1+5, 6+2] = [6,8],$

$b)$ $A - B = [1,2] - [5,6] = [1-5, 2-6] = [-4, -4],$

$c)$ $A \cdot B = [1,2] \cdot [5,6] = [\min\{5,6,10,12\}, \max\{5,6,10,12\}] = [5,12],$

$d)$ $A/B = [1,2] \cdot \left[\frac{1}{6}, \frac{1}{5}\right] = \left[\min\left\{\frac{1}{6}, \frac{1}{5}, \frac{1}{3}, \frac{2}{5}\right\}, \max\left\{\frac{1}{6}, \frac{1}{5}, \frac{1}{3}, \frac{2}{5}\right\}\right] = \left[\frac{1}{6}, \frac{1}{3}\right].$

Example 1.2: Let $A=[2,4]$ and $B=[3,7]$ therefore

$a)$ $A+B=[2,4]+[3,7]=[2+3,4+7]=[5,11],$

$b)$ $A-B=[2,4]-[3,7]=[2-7,4-3]=[-5,1],$

$c)$ $A\cdot B=[2,4]\cdot[3,7]=[\min\{6,14,12,28\},\max\{6,14,12,28\}]=[6,28],$

$d)$ $A/B=[2,4]\cdot\left[\frac{1}{7},\frac{1}{3}\right]=\left[\min\left\{\frac{2}{7},\frac{2}{3},\frac{4}{7},\frac{4}{3}\right\},\max\left\{\frac{2}{7},\frac{2}{3},\frac{4}{7},\frac{4}{3}\right\}\right]=\left[\frac{2}{7},\frac{4}{3}\right].$

Definition 1.2: If $r(x)$ is a continuous unary operation on $\mathbb{R}$, then

$$r(X)=[\min_{x\in X} r(x),\max_{x\in X} r(x)]$$

defines a (subordinate) unary operation on $I(\mathbb{R})$. Examples of such unary operations on $I(\mathbb{R})$ are $X^k(k\in\mathbb{R}),\ e^x, \ln X$ and $\sin X$.

Example 1.3:

Let $r=\ln X$ and $X=\left[\frac{2}{3},3\right]$, then $r(X)=\left[\ln\frac{2}{3},\ln 3\right]=[-0.40547,1.09861]$

Theorem 1.1: Let $A,B\in I(\mathbb{R})$ with $a\in A, b\in B.$ Then it follows that

$$a*b\in A*B$$

for $*\in\{+,-,\cdot,/\}.$

The unary operations $r(X)$ of Definition 1.2 have the corresponding properties

$$X\subseteq Y\Rightarrow r(X)\subseteq r(Y),$$

$$x\in X\Rightarrow r(x)\in r(X).$$

Definition 1.3:

An interval $A\in I_d(\mathbb{R})$ is degenerate (or is point interval) if and only if $a_1=a_2$. The set $I_d(\mathbb{R})$ of degenerate intervals and the set $\mathbb{R}$ of real numbers are isomorphic. This permits a meaning to be given to

$$a * b = [a,b] * B(a \in \mathbb{R}, B \in I(\mathbb{R}),\ * \in \{+,-,\cdot,/\}).$$

Definition 1.4 : If $a \in \mathbb{R}$ and $B \in I(\mathbb{R})$ and $B = [b_1, b_2] \in I(\mathbb{R}),$ then

$a)\ a + B = [a,a] + B = [a + b_1, a + b_2],$

$b)\ a - B = [a,a] - B = [a - b_2, a - b_1],$

$c)\ a \cdot B = [a,a] \cdot B = [\min\{ab_1, ab_2\}, \max\{ab_1, ab_2\}],$

and if $0 \notin B,$ then

$d)\ a/B = [a,a] \cdot \left[\frac{1}{b_1}, \frac{1}{b_2}\right] = [\min\{a/b_2, a/b_1\}, \max\{a/b_2, a/b_1\}].$

Proposition 1.1:

Interval arithmetic is inclusion monotonic that is to say, if $A, B, C, D \in I(\mathbb{R})$ then $(\forall * \in \{+,-,\cdot,/\})$

$$(A \subseteq C, B \subseteq D) \Rightarrow (A * B \subseteq C * D).$$

Proof:By Definition 1.1.

$$\begin{aligned} A * B &= \{a * b \mid a \in A, b \in B\} \\ &\subseteq \{c * d \mid c \in C, d \in D\} \\ &= C * D. \end{aligned}$$

Definition 1.5: Let $A, B \in I(\mathbb{R})$ be given. Then the intersection $A \cap B$ of A and B is defined by

$$A \cap B = \{x \in \mathbb{R} \mid x \in A, x \in B\}.$$

Proposition 1.2:

a) $(\forall A, B \in I(\mathbb{R})) A \cap B = B \cap A;$

b) $(\forall A, B \in I(\mathbb{R})) A \cap B \subseteq A, A \cap B \subseteq B;$

c) $(A \cap B = A \Leftrightarrow A \subseteq B), (A \cap B = B \Leftrightarrow B \subseteq A).$

Proof: Of (a): By definition 1.5

$$A \cap B = \{X \in \mathbb{R} \mid x \in A, x \in B\}$$

$$\subseteq \{x \in \mathbb{R} \mid x \in B, d \in A\}$$

$$= B \cap A.$$

Of (b): By definition 1.5

$$A \cap B = \{x \in \mathbb{R} \mid x \in A, x \in B\}.$$

So

$$\{x \in A \cap B\} \Rightarrow x \in A,$$

then

$$(A \cap B) \subseteq A,$$

And

$$(x \in A \cap B) \Rightarrow x \in B.$$

Then

$$(A \cap B) \subseteq B.$$

Of (c): By $(A \cap B) \subseteq B,$ so

$$(A \cap B = A) \Rightarrow (A \subseteq B).$$

Conversely, if # C $then by Definition 1.5

$$A \cap B = \{x \mid x \in A, x \in B\}$$

$$\subseteq \{x \mid x \in A\}$$

$$= A.$$

Therefore

$$(A \cap B = A) \Leftrightarrow (A \subseteq B).$$

Interchanging #and $and using (1), it follows that

$$(A \cap B = A) \Leftrightarrow (B \subseteq A).$$

Definition 1.6:

The distance between two intervals $A = [a_1, a_2], B = [b_1, b_2] \in I(\mathbb{R})$ is defined as

$$q(A, B) = \max\{|a_1 - b_1|, |a_2 - b_2|\}$$

The sequence of intervals $\{A^{(k)}\}_{k=0}^{\infty}$ converges to an interval A if and only if the sequence of bounds of the individual members of the sequences converges to the corresponding bounds of $A = [a_1, a_2]$. We can therefore write

$$\lim_{k\to\infty} A^{(k)} = A \Leftrightarrow \left(\lim_{k\to\infty} a_1^{(k)} = a_1 \text{ and} \lim_{k\to\infty} a_2^{(k)} = a_2 \right).$$

Definition 1.7:

The $A, B \in I(\mathbb{R})$ then the absolute value of an interval $A = [a_1, a_2] \in I(\mathbb{R})$ and $B = [b_1, b_2] \in I(\mathbb{R})$ is defined as

$$|A| = q\left(A, [0,0]\right) = \max\left\{|a_1|, |a_2|\right\},$$

$$|B| = q\left(B, [0,0]\right) = \max\left\{|b_1|, |b_2|\right\},$$

and

$$A \subseteq B \Rightarrow d\,|A| \leq d\,|B|.$$

Theorem 1.2: Every sequence of intervals $\left\{A^{(k)}\right\}_{k=0}^{\infty}$ for which

$$A^{(0)} \supseteq A^{(1)} \supseteq A^{(2)} \supseteq \ldots$$

is valid converges to the intervals $A = \bigcap_{k=0}^{\infty} A^{(k)}$.

Proof: Let us consider the sequences of bounds

$$a_1^{(0)} \le a_1^{(1)} \le a_1^{(2)} \le a_1^{(3)} \le \ldots \le a_2^{(3)} \le a_2^{(2)} \le a_2^{(1)} \le a_2^{(0)}$$

Theorem 1.3: The operations $+,-,\cdot$ and $/$ introduced in Definition 1.1 for intervals are continuous.

Proof:

We carry through the proof only for the operation +. Let $\{A^{(k)}\}_{k=0}^{\infty}$ and $\{B^{(k)}\}_{k=0}^{\infty}$ be two sequences of intervals for which $\lim_{k\to\infty} A^{(k)} = A$ and $\lim_{k\to\infty} B^{(k)} = B$ The sequence of interval sums $\{A^{(k)} + B^{(k)}\}_{k=0}^{\infty}$ then from Definition 1.1, satisfies

$$\lim_{k\to\infty}\left(A^{(k)} + B^{(k)}\right) = \lim_{k\to\infty}\left(a_1^{(k)} + b_1^{(k)}, a_2^{(k)} + b_2^{(k)}\right)$$

$$= \left[\lim_{k\to\infty}\left(a_1^{(k)} + b_1^{(k)}\right), \lim_{k\to\infty}\left(a_2^{(k)} + b_2^{(k)}\right)\right].$$

Definition 1.8: The width of an interval $A = [a_1 + a_2]$ is defined to be

$$d(A) = a_2 - a_1 \ge 0$$

The set of point intervals may now be characterized as $\{A \in I_d(\mathbb{R}) \mid d(A) = 0\}$. From Definition 1.7 we get properties that lead to

$a)\ A \subseteq B \Rightarrow d(A) \le d(B),$

$b)\ d(A \pm B) = d(A) + d(B),$

$c)\ d(A) = \max_{a_1, a_2 \in \Re} |a_2 - a_1|.$

Definition 1.8:

The midpoint of mid (A) of $A \in I(R)$ is defined by $\operatorname{mid}(A) = \frac{1}{2}(a_1 + a_2).$

Theorem 1.4:

Let $A, B \in I(\mathbb{R})$ be real intervals, then

$$d(AB) \le d(A)|B| + |A|d(B),$$

$$d(AB) \ge \max\{|A|d(B), |B|d(A)\},$$

$$d(aB) = |a|d(B),\ a \in \mathbb{R},$$

$$d(A^n) \le n|A|^{n-1} d(A), n = 1, 2, ...,$$

$$\left(A^n := A \cdot A \cdot ... \cdot A,\ n \text{ times}\right)$$

$$d\left((X - x)^n\right) \ge 2\left(d(X)\right)^n, x \in X,\ n = 1, 2, ...,.$$

From an interval $C \in I(\mathbb{R})$ with $0 \in C$ it furthermore follows that

$$|C| \le d(C) \le 2|C|.$$

Theorem 1.5: Let $A, B \in I(\mathbb{R})$ be intervals, and assume that A is a symmetric interval; i.e, $A = -A$. The following properties then hold,

$$AB = |B|A,$$

$$d(AB) = |B|d(A),$$

The property (1.32) is also valid for $0 \in A$ if either $b_1 \ge 0$ or $b_2 \le 0$.

Theorem 1.6: If A, B and C are members of the real intervals $I(\mathbb{R})$. Then it follows that

a) $A + B = B + A,$
b) $A \cdot B = B \cdot A,$
c) $(A + B) + C = A + (B + C),$
d) $(A \cdot B) \cdot C = A \cdot (B \cdot C).$

The proof of the above theorem can be found in Alefeld and Herzberger (1983). It is easy to show that both interval addition and multiplication are commutative and associative; for any three intervals A, B and C. (Moore, 2009)

1.3 Theorem of convergence

The theorem can be referred to Alefeld and Herzberger(1983).

Theorem 1.7: Let I be an iteration procedure with the limit x^*, and let $\Omega(I,x^*)$ be the set of all sequences generated by I having the properties that $\lim_{k\to\infty} x^{(k)} = x^*$ and $x^* \subseteq x^{(k)}, k \geq 0$. If there exist a $p \geq 1$ and a constant γ such that for all $\left\{x^{(k)}\right\} \in \Omega\left(I,x^*\right)$ and for norm $\|\cdot\|$, it holds that $\| h^{(k+1)} \| \leq \gamma \| h^{(k)} \|^p, k \geq k\left(\left\{x^{(k)}\right\}\right)$, then follows that R-order of convergence of I satisfies the inequality $O_R(I,x^*) \geq p$.

The R-order of convergence is defined as the measurement of the asymptotic convergence rate of the procedure. Its concept is discussed precisely by (Ortega and Rheinboldt, 1970) meanwhile it has been explained sufficiently for this thesis. (Alefeld and Herzberger, 1983). TheR-order of procedure I is denoted by $O_R(I,x^*)$ and R-factor of a null sequence $(w^{(k)})$ is denoted by $R_p\left(w^{(k)}\right)$ which is generated from the procedure I. (Monsi, 1988).

Furthermore, if there exists a $p \geq 1$ such that for any null sequence $\left\{w^{(k)}\right\}$ generated from $\left\{x^{(k)}\right\}$, then the R-factor of the sequence is defined as,

$$R_p\left(w^{(k)}\right) = \begin{cases} \limsup_{k\to\infty} \| w^{(k)} \|^{1/k}, & p = 1 \\ \limsup_{k\to\infty} \| w^{(k)} \|^{1/p^k}, & p > 1 \end{cases}$$

where R_p is independent of the norm $\|\cdot\|$. Suppose that $R_p\left(w^{(k)}\right) < 1$ then it follows from Ortega and Rheinboldt (1970) that the R-order of I satisfies the inequality $O_R(I,x^*) \geq p$. We will use this result in order to calculate the rate of convergence of all modified methods in each chapters.

Definition 1.9: Let I be an iteration procedure converging to x^*. Then, according Alefeld and Herzberger in 1983, the R-order of the iteration I as

$$O_R(I,x^*) = \begin{cases} +\infty & \text{if } R_p(I,x^*) = 0 \text{ for } p = 1, \\ \inf\{p \mid p \in [1,\infty),\ R_p(I,x^*) = 1\} & \text{otherwise.} \end{cases}$$

1.4 Objectives of the research

The objectives of our research are:

1) To develop some new and efficient procedures based on symmetric single-step procedures for solving real and simple polynomials (Monsi, 1988) and (Rusli et. al, 2011).

2) To investigate the convergence properties of the modified procedures. We want to verify that the modified procedures are better in term of convergence rate and take less time to converge to the solution. These new procedures are supposed to have higher rate of convergences than does the existing procedures. These procedures are iterative in nature and produce a sequence of intervals $\{X_i^{(k)}\}(i=1,\ldots,n)$ that will converge faster to the solution compared to the existing procedures.

3) To compare the numerical performances; in term of number of iteration and CPU times. Matlab R2012b software (Rump, 1999) in associate with Intlab is employed to record the Central Processing Unit (CPU) time taken in seconds(s) for the existing procedures and the modified procedures.

1.5 Planning of the thesis

This thesis concerns mainly in finding the simple zeros of real polynomials using interval analysis approach. In Chapter 1, we provided some brief description about the mathematical background of the basic concept and properties of interval arithmetic. Some general theorems of local convergence which are referred along our research are presented.

Chapter 2 highlighted a detailed explanation about literature review based on interval analysis and simultaneous methods of finding polynomial zeros are given in this section.

Chapter 3, 4, 5, and 6 are the chapters that contain the precise and detailed discussions about all four advanced modified procedures. The new four modified procedures are Interval Symmetric Single-Step 5 Delta (ISS1-5D) procedures, Interval Midpoint Symmetric Single-Step 5 Delta (IMSS1-5D) procedures, Interval Zoro Symmetric Single-Step 5 Delta (IZSS1-5D) procedures, and Interval Midpoint Zoro Symmetric Single-Step 5 Delta (IMZSS1-5D) procedures.

We provide the algorithms and also the analysis of R-order of convergence for each of the new procedures respectively. In addition, we show the comparison between the new procedures with the original one in term of the number of iteration, CPU times and order of convergence.

The efficiency of these four new procedures will be shown clearly from the numerical results for each test polynomials which will be displayed in form of tables and graphs. Finally, we summarize in Chapter 7 by showing all the final numerical results from the previous chapters. In addition, some possible extensions of the work for future studies are also considered.

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