UNIVERSITI PUTRA MALAYSIA

PARAMETRIC AND SEMIPARAMETRIC COMPETING RISKS MODELS FOR STATISTICAL PROCESS CONTROL WITH RELIABILITY ANALYSIS

FAIZ AHMED MOHAMED ELFAKI

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By

FAIZ AHMED MOHAMED ELFAKI

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of Requirements For the Degree of Doctor of Philosophy

July 2004

To my wife Salma,

To my love son Ahmed,

To my father, Ahmed

To my late mother, Eltayah

May Allah Rest Her Soul in heaven

Abstract of the thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of requirements for the degree of Doctor of Philosophy

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By

FAIZ AHMED MOHAMED ELFAKI

July 2004

Chairman: Associate Professor Isa Daud, Ph.D.

Faculty: Science and Environmental Studies

The work in this thesis is concerned with the development of techniques for the assessment of statistical process control in data that include censored observations. Various regression models with censored data are presented and we concentrate on four competing risks models namely, two parametric Cox's model that is, Cox's with Weibull distribution, Cox's with exponential distribution and two semiparametric Cox's model with subdistribution function that is, the weighted score function (W) and censoring complete (CC). The Expectation Maximization (EM) algorithm is utilized to obtain the estimate of the parameters in the models. A generated data where the failure times are taken as exponentially distributed are used to further compare these two parametric models. From the simulation study for this particular case, we can conclude that Weibull distribution describes well the nature of the model concerned as compared to the exponential distribution in terms of the mean value of parameter estimates, bias, and the root means square error. Plots of survival distribution function against failure time are used to examine the predicted survival patterns for the two types of failures.

In this thesis we develop a modified Fine and Gray methods to increase the sensitivity of the models and these methods are tested and compared. A simulation data using subdistribution function for the two types of failure are carried out to compare the performance of the modified model. The results of the study indicate the models show better result compared to Fine and Gray models. However, the weighted score function (W) shows better result compared to the censored complete data (CC). Residual-based approaches are used to assess the validity of the two models (MW, CC) assumptions. Plots of this residual against failure time are used to investigate whether important explanatory variables have been omitted from the model.

The study also carries out an investigation of the causes of failure for statistical process control. The $\bar{x}$ chart, Я chart and C_p, and C_{pk} are examined for the possibility of being used to detect the state of control of the covariates in the two competing risks models (Cox's with Weibull distribution (PHW2) and modification of weighted score function (MW)). The result of this study indicates that both models are successful in investigating the causes of failure for statistical process control. However, the results from the real data sets which involves the measurement of stress against three covariates (aluminum, wood and plastic) showed that the tubes wrapped on plastic mandrel have excellent crashworthiness performance with respect to the $\bar{x}$ chart, Я chart, C_p, and C_{pk}.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

MODEL RISIKO BERSAING PARAMETRIK DAN SEMIPARAMETRIK BAGI KAWALAN PROSES BERSTATISTIK DENGAN ANALISIS KEBOLEHPERCAYAAN

Oleh

FAIZ AHMED MOHAMED ELFAKI

Julai 2004

Pengerusi: **Profesor Madya Isa Daud, Ph.D.**

Fakulti: **Sains dan Pengajian Alam Sekitar**

Kajian di dalam tesis ini adalah berkaitan dengan pembangunan teknik bagi penilaian kawalan proses berstatistik bagi data yang mengandungi cerapan tertapis. Pelbagai model regresi dengan data tertapis dibincangkan dan tumpuan kajian adalah pada empat model risiko bersaing iaitu dua model berparameter Cox (Cox dengan taburan Weibull dan Cox dengan taburan eksponen) dan dua model semi-berparameter Cox (fungsi skor berpemberat (W) dan tapisan lengkap (CC)). Algoritma Pemaksimuman Jangkaan (EM) digunakan bagi menganggar parameter model tersebut. Data yang dijana dengan masa kegagalan bertaburan eksponen digunakan bagi tujuan perbandingan lanjut kedua-dua model parametrik. Daripada kajian simulasi untuk kes ini, dapat disimpulkan bahawa taburan Weibull menjelaskan dengan baik sifat model berbanding taburan eksponen dari segi nilai min anggaran parameter, kepincangan dan ralat punca min kuasa dua. Plot fungsi taburan kemandirian melawan masa kegagalan digunakan untuk melihat pola-pola kemandirian ramalan bagi dua jenis kegagalan.

Dalam tesis, kaedah Fine dan Gray terubahsuai dibangunkan untuk meningkatkan kepekaan model. Kaedah ini kemudiannya diuji dan dibandingkan. Data simulasi menggunakan fungsi subtaburan bagi dua jenis kegagalan dijalankan untuk membanding pencapaian model yang telah diubahsuai. Hasil kajian ini menunjukkan model ini menghasilkan keputusan yang lebih baik berbanding model Fine dan Gray. Walau bagaimanapun, fungsi skor berpemberat (W) menunjukkan keputusan yang lebih baik berbanding data penapisan lengkap (CC). Pendekatan berasaskan reja digunakan untuk menilai kesahihan anggapan dua model tersebut (MW dan CC). Plot reja melawan masa kegagalan digunakan untuk memeriksa samada pembolehubah penerang yang penting telah dikeluarkan daripada model.

Kajian dijalankan juga untuk mengetahui penyebab kegagalan bagi kawalan proses berstatistik. Carta $\bar{x}$, carta Я, C_p dan C_{pk} dipertimbangkan bagi kemungkinan digunakan untuk mengesan keadaan kawalan kovariat dua model risiko bersaing (Cox dengan taburan Weibull (PHW2) dan fungsi skor berpemberat terubahsuai (MW)). Keputusan kajian menunjukkan kedua-dua model dapat memeriksa penyebab kegagalan bagi kawalan proses berstatistik dengan jayanya. Walau bagaimanapun, hasil kajian daripada set data sebenar yang melibatkan ukuran tegangan melawan tiga kovariat (aluminum, kayu dan plastik) menunjukkan bahawa tiub-tiub yang dibaluti plastik ‘mandrel’ memberikan pencapaian ‘crashworthiness’ yang cemerlang berdasarkan carta $\bar{x}$, carta Я, C_p dan C_{pk}.

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I certify that an Examination Committee met on 20th July, 2004 to conduct the final examination of Faiz Ahmed Mohamed Elfaki on his Doctor of Philosophy thesis entitled "Parametric and semiparametric competing risks models for statistical process control with reliability analysis)" in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

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DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at Universiti Putra Malaysia or other institutions.

____

FAIZ AHMED MOHAMED ELFAKI

Date:

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