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UNIVERSITI PUTRA MALAYSIA

LINEAR AND NONLINEAR THERMOELASTIC ANALYSIS OF FUNCTIONALLY GRADED MATERIALS AXISYMMETRIC ROTATING DISKS

MEHDI BAYAT

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By
MEHDI BAYAT

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Doctor of Philosophy

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DEDICATION

To

My family members especially my beloved wife and my ever -encouraging father for his love

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Doctor of Philosophy

LINEAR AND NONLINEAR THERMOELASTIC ANALYSIS OF FUNCTIONALLY GRADED MATERIALS AXISYMMETRIC ROTATING DISKS

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Chairman: Professor Barkawi Bin Sahari, Ir. PhD

Faculty: Engineering

Functionally graded materials (FGMs) are non-homogeneous materials where the volume fraction of two or more materials is varied, as a power-law distribution, continuously as a function of position along certain dimension(s) of the structure. FGMs are usually made of a mixture of ceramic and metals. The ceramic constituent of the material provides the high temperature resistance due to its low thermal conductivity and the ductile metal constituent, on the other hand, prevents fracture caused by stress due to high temperature gradient in a very short period of time. These materials, usually designed to operate in high temperature environments, find their applications in automotive and aerospace as turbine rotors, flywheels, gears, tubes, disk brakes and energy storage devices. In all these applications, the performance of the components in terms of efficiency, service life and power transmission capacity depends on the material, thickness profile, speed of rotation and operating conditions. Normally, these components are fabricated by using

homogeneous metal. In the present work, components made of FGM are to be considered and they are axisymmetric disks subjected to body force, bending and thermal loads. The displacement and stress fields of these components are determined both analytically and numerically.

The effect of geometry and material-property nonlinearity on small and large deflections in functionally graded rotating disks is investigated by studying their elastic behavior under thermo mechanical loads. Six types of thickness profiles h, namely uniform, linear, concave, convex, hyperbolic convergent and hyperbolic divergent are considered. Material properties such as Young's modulus, E, mass density, ρ, and the thermal conductivity, α ,are assumed to be represented by two power law distributions along the radial direction. Material properties are also assumed to be temperature-dependent for more accurate and realistic results. A theoretical formulation for bending analysis of functionally graded (FG) rotating disks based on First Order Shear Deformation Theory (FSDT) is presented. A semi analytical solution for displacement field is obtained.

New linear and nonlinear equilibrium equations for FG axisymmetric rotating disk with bending and thermal loading are developed and presented. The disk has material properties varying through the thickness of the disk graded according to a power-law distribution of the volume fraction of the constituents. FSDT and von Karman theory are used and both small and large deflections are considered. In the case of small deflection, an exact solution for displacement field is given. For large deflection, power series solutions are employed to solve for displacement field. The results for displacement and stresses are normalized with respect to the corresponding disk with homogeneous material geometry and certain value of properties of disk with the same unit respectively. All the results shown are thus independent of the physical dimension of the component. As for practical applications, rotating disks with typical dimensions up to 2 meter diameter are considered.

The results for free-free FG rotating disk show that there exist combinations of values of parameters related to thickness profiles for which the radial stress can attain its maximum at radial distance greater than half of the radius, to be more specific at $r/R_o > 0.55$ if the ratio of inner to outer radius is assumed to be 0.2, and also the ratio of thickness to outer radius is 0.2 while material properties change in radial direction. The results for FG disk with variable thickness under thermomechanical loading show that an efficient and optimal design of the disk requires variable section thicker at the hub and tapering to smaller thickness at the periphery and also that the temperature-dependent material properties must be considered in high temperature environment. Applying FSDT, while material properties change in radial direction, it is seen that for the specific value of the grading index n ($n = 10$), the moment resultants in a FG solid disk with convex or constant thickness profile are lower throughout than those in pure material disk. In case of changing material properties in thickness direction by using large deflection theory, it is observed that the radial stresses in a full-metal disk due to thermal load, body force and vertical pressure are smaller than those in a full-ceramic disk. It is found that the small deflection theory gives large errors in the results for FG disks if the ratio of maximum deflection to thickness is close to 0.4 for a homogeneous (full-ceramic in this study) disk.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

ANALISIS KENYAL HABA LINEAR DAN TAK-LINEAR BAGI CAKERA BAHAN FUNGSIAN BERGRED SIMETRI SEPAKSI BERPUTAR

Oleh

MEHDI BAYAT

September 2008

Pengerusi: Professor Barkawi Bin Sahari, PhD

Fakulti: Kejuruteraan

Bahan Fungsian Bergred (FGMs) adalah bahan tak homogen di mana pecahan isipadu bagi dua atau lebih bahan adalah berubah, sebagai penyebaran hukum-kuasa, selanjar sebagai fungsi kedudukan di sepanjang dimensi struktur tersebut. FGM biasanya dihasilkan dengan mencampurkan seramik dan logam. Juzuk bahan seramik memberikan penebat suhu tinggi oleh kerana konduktiviti terma yang rendah. Juzuk mulur logam sebaliknya menghalang kepatahan yang mungkin berlaku akibat daripada tegasan oleh penurunan suhu tinggi pada kala masa yang sangat singkat. Bahan-bahan ini dihasilkan pada keadaan kendalian suhu tinggi, yang boleh diaplikasikan dalam aeroangkasa dan automobil sebagai roda turbin, roda tenaga, gear, tiub, brek cakera, dan peranti cakera penyimpan. Prestasi bagi kesemua aplikasi komponen dapat ditentukan melalui kecekapan, jangka hayat dan penghantaran kuasa. Ringkasnya, prestasi bergantung kepada bahan, profil ketebalan, laju putaran dan keadaan kendalian. Biasanya, komponen dibikin menggunakan sama ada logam atau bahan bertetulang gentian. Dalam kajian ini, komponen FGM adalah dianggap

cakera paksi simetri dengan bebanan daya jasad, kelenturan dan terma. Sesaran dan medan tekanan komponen ini ditentukan menggunakan kaedah analisis dan berangka.

Kesan geometri dan analisis bahan tak-linear ke atas pesongan kecil dan besar bagi putaran cakera dikenalpasti dengan mengkaji kelakuan kenyal akibat beban haba-mekanikal. Profil bagi enam jenis tebal h, dipertimbangkan iaitu seragam, linear, cekung, cembung, hiperbola menumpu dan hiperbola capah. Sifat bahan seperti modulus Young E, ketumpatan jisim ρ, dan konduktiviti terma, α diandaikan diwakili oleh dua jenis hukum kuasa yang berbeza. Seterusnya sifat bahan juga diandaikan bergantung kepada suhu bagi mendapatkan keputusan yang tepat dan realistik. Rumusan teori analisis lenturan bahan fungsian bergred (FG) bagi cakera berputar berasaskan Teori Ubahbentuk Ricih Tertib Pertama (FSDT) telah ditunjukkan. Penyelesaian separa analitik bagi medan anjakan diperolehi.

Persamaan keseimbangan linear dan tak linear baharu bagi cakera berputar paksi simetri FG dengan lenturan dan beban terma dibangunkan dan dibentangkan dengan ciri-ciri bahan berubah mengikut ketebalan cakera bergred berdasarkan penyebaran hukum-kuasa bagi isipadu pecahan juzuk tersebut. Teori Von Karman dan FSDT digunakan untuk pesongan kecil dan besar dipertimbangkan. Bagi kes pesongan yang kecil, penyelesaian bentuk tepat bagi medan anjakan diberi. Penyelesaian siri kuasa digunakan untuk menyelesaikon medan anjakan yang mempunyai pesongan besar. Keputusan pengalihan dan tekanan dinormalkan berdasarkan kepada geometri yang bersesuaian dan nilai tertentu bahan cakera tulen dengan unit yang sama masing-masing. Semua keputusan yang ditunjukkan adalah terdiri daripada dimensi fizikal komponen tersebut. Bagi aplikasi praktikal, cakera berputar dengan ukuran diameter sehingga 2 meter diambilkira.

Keputusan-keputusan cakera berputar FG menunjukkan kehadiran gabungan nilai-nilai parameter berkait dengan profil ketebalan di mana tekanan jejarian boleh mencapai sehingga maksimum pada jarak jejarian lebih besar daripada setengah jejarian, khususnya pada $r/R_o > 0.55$ jika nisbah jejarian dalam kepada jejori luar diandaikan kepada 0.2 dan nisbah jejarian luar kepada ketebalan adalah 0.2 sementara sifat bahan berubah. Keputusan untuk cakera FG dengan ketebalan yang berubah-ubah di bawah bebanan haba mekanikal menunjukkan rekabentuk terbaik dan optimum adalah cakera yang berubah ketebalan dengan tebal pada hab dan semakin menipis tirus pada sisi dan juga sifat bahan bergantung pada suhu perlu diambilkira pada keadaan suhu tinggi. Dengan menggunakan FSDT dan sifat bahan berubah dengan arah jejarian, didapati bahawa nilai indeks gred n khususnya $n = 10$, paduan momen dalam cakera FG tetal cembung atau tebal malar lebih rendah berbanding dengan cakera yang sama tetapi bahan tulen. Bagi kes sifat bahan berubah dengan arah tebal dan manggunakan teori anjakan besar, didapati bahawa tegasam jejarian cakera logam tulen oleh beban terma, beban jasad dan tekanan menegak lebih rendah berbanding dengan cakera seramik tulen. Ia juga didapati bahawa teori anjakan kecil memberikan ralat yang besar untuk cakera FG sekiranya nisbah anjakan maksimum dengan tebal menghampiri nilai 0.4 untuk cakera homogen seramik tulen.

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Last but not the least, my heart-full gratitude and love to my wife maryam, my (late) mother, my father and siblings whose unconditional support and love has made this dream comes true to me.

I certify that an Examination Committee has met on **date of viva** to conduct the final examination of **Mehdi Bayat** on his **Doctor of Philosophy** thesis entitled "**Linear and Nonlinear Thermoelastic Analysis of Functionally Graded Materials Axisymmetric Rotating Disks**" in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

MOHD SAPUAN SALIT, PROF. IR. DR.
Department of Mechanical and Manufacturing Engineering, Faculty of Engineering
Universiti Putra Malaysia
(Chairman)

SHAMSUDDIN SULAIMAN, PROF. DR.
Department of Mechanical and Manufacturing Engineering, Faculty of Engineering
Universiti Putra Malaysia
(Internal Examiner)

NOR MARIAH ADAM, PROF. MADYA IR. DR.
Department of Mechanical and Manufacturing Engineering, Faculty of Engineering
Universiti Putra Malaysia
(Internal Examiner)

AHMAD FAUZI MOHD NOOR, PROF. DR.
Department of Material & Mineral Resources Engineering, Faculty of Engineering
Universiti Sains Malaysia
(External Examiner)

HASANAH MOHD. GHAZALI, PhD
Professor/Deputy Dean
School of Graduate Studies
Universiti Putra Malaysia
Date:

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted in fulfillment of the requirements for the degree of Doctor of Philosophy. Members of the Supervisory Committee were as follows:

Barkawi Bin Sahari, PhD., P.Eng.
Professor
Faculty of Engineering
Universiti Putra Malaysia
(Chairman)

Mohammad Saleem, PhD
Associate Professor
College of Engineering and Technology A.M.U., 202002 INDIA
(Member)

Aidy Ali, PhD
Lecturer
Faculty of Engineering
Universiti Putra Malaysia
(Member)

Wong Shaw Voon, PhD
Associate Professor
Faculty of Engineering
Universiti Putra Malaysia
(Member)

HSANAH MOHD. GHAZALI, PhD
Professor and Dean
School of Graduate Studies
Universiti Putra Malaysia

Date: 15 January 2009

DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

Mehdi Bayat

Date:

TABLE OF CONTENTS

LIST OF ABBREVIATIONS

FG	Functionally graded
FGM	Functionally graded material
FGMs	Functionally graded materials
FE	Finite element
FEM	Finite element method
FSDT	First-order shear deformation theory
TSDT	Third-order shear deformation theory
HSDT	Higher-order shear deformation theory
CPT	Classical plate theory

LIST OF NOTATIONS

P	material property in FG structure
f	volume fraction
Q_{AB}	function that depends on material properties and volume fraction in component A, B as well as on quantities related to microstructures.
$\bar{A}$	an empirical parameter whose magnitude is of the order of unity
σ	normal / uniaxial stress distribution
$\bar{\sigma}_{11}$	longitudinal stress
γ	shear stress
ε	normal / uniaxial strain
H	hardening index
x	direction of the microstructural gradient
l	FGM thickness along the graded (i.e. x-direction)
ρ	density
λ	effective thermal conductivity of the mixture.
E	Young's modulus
α	coefficient of thermal expansion
ν	Poisson's ratio
T	temperature distribution
T_c	temperature at the cooler end of the FGM
t	thickness of structure
h	thickness distribution
r	variable in radial direction

u	radial displacement
v	hoop displacement
w	transverse displacement
$\bar{w}_o$	non-dimensional displacement at the center of the disk
ξ	non-dimensional radius
λ	Lama's constant
μ	Lama's constant
β_{sv}	semi-vertex angle
$g(z)$	volume fraction in P-FGM
$g_1(z)$	volume fraction in S-FGM (related to portion one)
$g_2(z)$	volume fraction in S-FGM (related to portion two)
K_e	kinematic energy
W	work done on the body
δu	virtual displacement
$\vec{f}$	body force vector
$\hat{t}$	surface traction vector
$\overset{\leftrightarrow}{\sigma}$	stress tensor
$\overset{\leftrightarrow}{\varepsilon}$	strain tensor
m	mass
$\vec{a}$	acceleration
U_0	strain energy density function
K	kinetic strain energy

U	strain energy
V_P	total potential energy of the structure
Π	total potential energy of the body
ω	angular velocity
U_1	total strain energy of the rotating disk
V_1	total potential energy of the rotating disk
V	total volume of structure
Ψ_x	rotation about y axis
Ψ_y	rotation about x axis
$\overset{\leftrightarrow}{L}$	strain tensor dyadic
k	coefficient of thermal expansion of the material
K_{con}	thermal conductivity
$p(z)$	generic material property at location z through the thickness
$t^{(k)}$	radial-width of the k^{th} sub-domain
u_r	radial displacement
M_r	stress couple
Q_r	transverse shear resultant
$q(r)$	transverse loading as a function of r
q_o	proportionality constant
$q_1(r)$	dependence of vertical load on r
N^T	resultant thermal stress
E_{FGC}	elastic modulus of FGCs

E_{YSZ}	elastic modulus of YSZ
$E_{NiCoCrAlY}$	elastic modulus of NiCoCrAlY
V_{YSZ}	volume fraction of the phase
E_{red}	reduced modulus
e_{ft}	thickness of material at the top of the sandwich panel
e_c	thickness of material at the center of the sandwich panel
q_{srs}	ratio of the stress to strain
λ^2	ratio of circumferential stiffness to radial stiffness

All notations shown below are considered as constants:

α_{con}, β_{con}, γ, β_{oj}, β_{-1j}, β_{1j}, β_{2j}, β_{3j}, κ, n_1, ρ, a_1, b_1, c_1, m_1, m_2, m_3, m_{Con}, β, A, B, E_{con}, ν_{con}, k_{con}, γ_{con}, δ_{con}, Δ_{con}, θ_{con}, n_o, p and q.

LIST OF SUPERSCRIPT

Unless stated otherwise, the superscripts will have the following meaning when used with the variable h, E and r.

*	the nominal value
'	the derivative with respect to radius

LIST OF SUBSCRIPTS

Unless stated otherwise, the subscripts connote the following meaning when applied with these variable: P, f, λ, E, σ, γ, ε, H, x, h, ρ, t, u, and r

A	related to component A
B	related to component B
c	Composite

1	related to phase 1
2	related to phase 2
Y	yield strength
i, in and a	inner surface
o, out and b	outer surface
θ, $\theta\theta$	hoop direction
r, rr	radial direction
z	Vertical or transverse direction
$r\theta$, θr	shear (Polar coordinate)
xy, yz and xz	Shear (Cartesian coordinate)
m, met	metal
cer	ceramic
al	aluminum
mat	matrix

All other symbols are defined in the text wherever used.

CHAPTER 1

INTRODUCTION

1.1 General Introduction

In many engineering structures or components, the property of the material is better changed at various points or directions depending on their applications and operating conditions (Parameswaran and Shukla 1999). For example, a gearwheel has a harder material at the surface of teeth than the base wheel and a turbine disk is designed in such way that it can withstand high thermal loads close to the outer radius and compressed stresses at the inner radius.

In the new millennium, composite materials have been widely used in aerospace, marine and automotive industry (Pagano and Reddy, 1994 and Jones, 1998) with a sole aim to improve the properties and behavior of the structures. Some of the properties that can be improved by using composite materials are stiffness, strength, weight reduction, and corrosion resistance, thermal properties, fatigue life, and wear resistance. In composite material structures, the ratios of the strength to weight and the modulus of elasticity to weight are high in comparison with single metal structures of similar size (Reddy, 2004).

It is known that many of the components in aerospace and nuclear structures are subjected to combination of mechanical and thermal loads. Obviously any abrupt change in material properties of these structures may lead to high temperature stress concentration due to mechanical and thermal loads. In laminated composite materials, properties such as the modulus of elasticity, mass, density, Poisson's ratio, thermal conductivity etc change abruptly across the interfaces and thus the constituents of the fiber-matrix composites remain prone to de-bonding at extremely high thermal loading with likely initiation of cracks at interfaces and growing into the weaker material sections. Such situations can be prevented from occurring by introducing a gradual change in the properties of the materials. Many structural components made of non-homogeneous materials have been used to withstand the severe thermal loads. Thermal barrier coating of super alloys made of ceramics used in jet engines, stainless steel cladding of pressure vessels in nuclear power plants and a great variety of diffusion bonded materials used in microelectronics can be some examples just to mention (Jeon et al. 1997).

In the near future, an ideal material may be made combining best properties of metals and ceramics such as toughness, electrical conductivity, machine-ability of metals, low density, high stiffness and high temperature resistance. There is already a demand for such materials from the automotive, electronics, telecommunications, aerospace and defense industries (Chakraborty et al. 2003).

1.2 Functionally Graded Materials (FGMs)

In laminated composite materials, the properties change from one layer to another. These materials are usually subjected to residual stresses due to the difference in coefficients of thermal expansion of the fiber and matrix during fabrication. To overcome these problems, the concept of FGMs was proposed in 1984 by Niino et al. (Koizumi, 1997) at the National Aerospace Laboratory in Japan as a means of preparing thermal barrier materials (also see, Yamanouchi et al., 1990; Koizumi, 1993; Koizumi, 1997). The FGMs are thus a new class of materials constructed to operate in high temperature conditions. In functionally graded (FG) structures the volume fraction of two or more materials changes gradually from one point to another as a function of position. FGMs are usually made of combination of metals and ceramic. The gradual change of material properties from one point to another in FGMs as shown in Figure 1.1 makes them preferable in many applications.

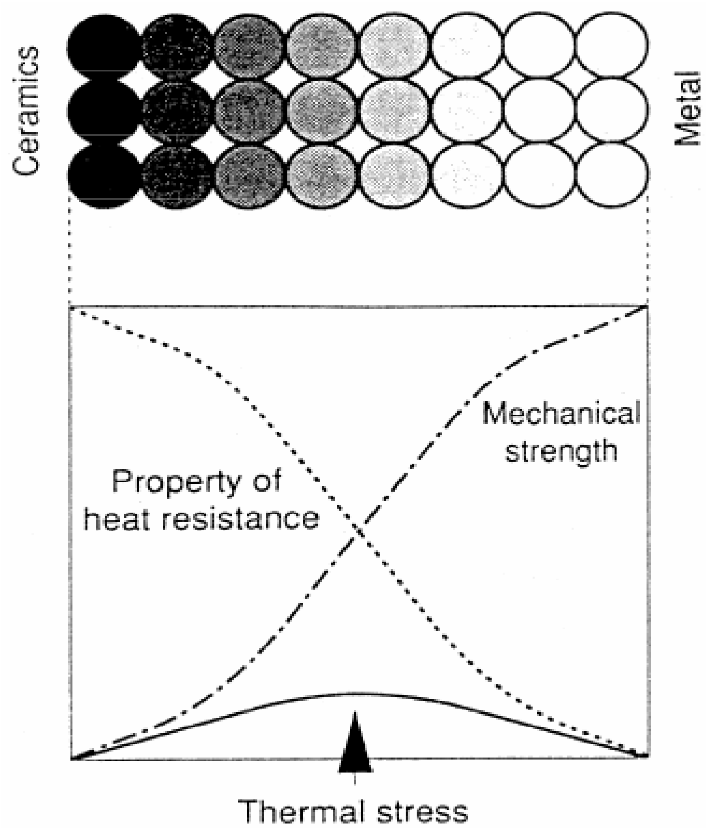

Figure 1.1: The concept of FGMs (Toshio et al., 1999).

Metal-matrix composites are gradually being used as structural materials in many aerospace (skin structure) and automotive (engine) applications. Usually, uniformly distributed FGMs are reinforcement in these composites (Nogata, 1995). Undoubtedly, FGMs have drawn international attention (Miyamoto et al. 1999) for their adaptability and viability in super-high-temperature environment.