UNIVERSITI PUTRA MALAYSIA

ACCELERATING COSMOLOGIES WITH EXTENDED PRODUCT SPACES

CH’NG HAN SIONG

ITMA 2006 8

ACCELERATING COSMOLOGIES WITH EXTENDED PRODUCT SPACES

By

CH'NG HAN SIONG

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

November 2006

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

ACCELERATING COSMOLOGIES WITH EXTENDED PRODUCT SPACES

By

CH'NG HAN SIONG

November 2006

Chairman: Associate Professor Hishamuddin Zainuddin, PhD

Institute: Advanced Technology

Accelerating cosmologies in extra dimensional spaces have been studied. These extra dimensional spaces are products of many spaces. The physical behaviors of accelerating cosmologies are investigated from Einstein's field equation in higher dimensional Friedmann-Robertson-Walker (FRW) universe and superstring/M theory points of view. It is found that if some assumptions of flatness are made for sector of the FRW universe, the remaining sector needs to be hyperbolic. These properties are in parallel with those found in the model of superstring/M theory. The extended product made for the superstring model did not show any more new features other than those already found. A similar accelerating phase of this product space cosmology was found with difference in numerical values of the accelerating period.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

KOSMOLOGI MEMECUT DENGAN TAMBAHAN RUANG HASIL DARAB

Oleh

CH'NG HAN SIONG

November 2006

Pengerusi: Profesor Madya Hishamuddin Zainuddin, PhD

Institut: Teknologi Maju

Kosmologi memecut yang mempunyai lebihan dimensi telah dikaji. Lebihan dimensi ini adalah hasil darab daripada ruang yang berlainan. Sifat fizikal untuk kosmologi memecut ini adalah dikaji daripada pandangan persamaan Einstein untuk alam semesta Friedmann-Robertson-Walker (FRW) lebihan dimensi dan juga daripada pandangan teori bebenang/M. Didapati apabila beberapa andaian rata dibuat ke atas beberapa sektor alam semesta FRW, sektor bakinya adalah hiperbolik. Sifat ini selaras dengan apa yang ditemui dalam teori bebenang/M. Lebihan hasil darab untuk model bebenang tidak menunjukkan ciri-ciri yang baru selain daripada apa yang telah ditemui. Fasa pemecutan bagi kosmologi ruang hasil darab serupa ada ditemui dengan pembezaan dalam nilai angka tempoh pemecutan.

ACKNOWLEDGEMENTS

I would like to thank my supervisor, Assoc. Prof. Dr. Hishamuddin Zainuddin, for his introduction of the field of accelerating cosmologies to me. I am very grateful for the freedom he has given me in doing the research. He is undoubtedly a wise researcher knows well the ways to conduct open and sincere discussions with me, and he master well the skills in fine tuning guidance from himself and opinions from me. I also truly appreciate with his positive assessment about my research ability greatly strengthens my endurance to face challenges in doing this research. I would also like to thank Dr. Zainul Abidin Hassan, who has been my co-supervisor for this work.

I am also grateful to the Institute of Advance Technology and Theoretical Studies Laboratory, where this work was initiated. This research was financially supported by the Ministry of Science and Innovation under Pasca Siswazah Scheme.

I certify that an Examination committee has met on 28th November 2006 to conduct the final examination of Ch'ng Han Siong on his Master of Science thesis entitled "Accelerating Cosmologies with Extended Product Spaces" in accordance with Universiti Putra Malaysia (Higher Degree) Act 1980 and Universiti Putra Malaysia (Higher Degree) Regulations 1981. The committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

MOHD. YUSOF SULAIMAN, PhD
Professor
Faculty of Science
Universiti Putra Malaysia
(Chairman)

JUMIAH HASSAN, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Internal Examiner)

ADEM KILICMAN, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Internal Examiner)

ROSY THE CHOOI GIM, PhD
Associate Professor
Faculty of Science
Universiti Sains Malaysia
(External Examiner)

____

HASANAH MOHD. GHAZALI, PhD
Professor/Deputy Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:

This thesis submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of the Supervisory Committee are as follows:

Hishamuddin Zainuddin, PhD
Associate Professor
Institute of Advanced Technology
Universiti Putra Malaysia
(Chairman)

Zainul Abidin Hassan, PhD
Lecturer
Faculty of Science
Universiti Putra Malaysia
(Member)

AINI IDERIS, PhD
Professor/Dean
School of Graduate Studies
Universiti Putra Malaysia

Date: 8 MARCH 2007

DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

CH'NG HAN SIONG

Date: 26 JANUARY 2007

TABLE OF CONTENTS

LIST OF NOTATIONS

$R(t)$	Scale factor (if it is function of t)
R	Ricci scalar (if it is not function of t)
$R_{\mu\nu}$	Ricci tensor
$R^{\sigma}_{\mu\nu\lambda}$	Riemann tensor
$T_{\mu\nu}$	Energy-momentum tensor
$g_{\mu\nu}$	Metric tensor
G	Newton's gravitational constant
$\bar{G}$	Higher dimensional Newton's gravitational constant
Λ	Cosmological constant
$G_{\mu\nu}$	Einstein tensor
k_i	Curvature of i-th sector of product space
m_i	Number of dimensions of the i-th sector of product space
R_i	Scale factor of i-th sector of product space
ε	Mass-energy density
p_i	Pressure of i-th sector of product space
$\theta_1^{(i)}, \theta_2^{(i)}, \theta_3^{(i)}, ...$	Coordinate of spherical coordinate system for i-th sector of product space
$R_{\mu\nu}^{(i)}$	Ricci tensor for i-th sector of product space
$g_{\mu\nu}^{(i)}$	Metric tensor for i-th sector of product space

CHAPTER 1

INTRODUCTION

1.1 History of Accelerating Cosmologies

The Wilkinson Microwave Anisotropy Probe (WMAP) data and the observations of the dimming of type Ia supernovae in distant galaxies showed that the universe is undergoing accelerated expansion at the present epoch [1,2]. Although it is not difficult to find cosmological models that exhibit these features, but usually one will hope that such model can be derivable from a fundamental theory that incorporates both gravity and the standard model of particle physics [3]. Most such efforts are from superstring or M theory framework [3,4,5,6,7,8]. In superstring theory or M theory, one needs a compactification of six or seven internal spaces. There is however no solution for accelerating universe if the internal space is time-independent [4]. Besides this, Einstein's equations in higher dimensions are of interest as generalizing known solutions. There are some of these earlier works that were done in Ref. [9,10,11,12], which considered the extra dimensions in single space. Besides this, extra dimensional space was treated as a source for generating acceleration of the universe.

The no-go theorem is due to the strong energy condition not being violated by either eleven dimensional supergravity or any of the ten dimensional supergravity theories. If the higher dimensional stress tensor satisfies the strong energy condition, so will the lower dimensional stress tensor. But this is not the case in Friedmann-Lemaître-

Robertson-Walker (FLRW) (homogeneous and isotropic) universe, because it needs a violation of the strong-energy condition.

The metric of the FLRW form:

$$ds^2 = -d\tau^2 + a(\tau)^2\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right) \quad , \tag{1.1-1}$$

where k = -1,0,+1;

$a\left(\tau\right)$ = scale factor;

$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$.

The time-time component of 4D Ricci tensor for FLRW is given in [4]

$$R_{00} = -3\frac{\ddot{a}}{a} \quad , \tag{1.1-2}$$

where a = scale factor;

$\ddot{a}$ = second order time derivative of the scale factor.

So, in four-dimensional FLRW universe, the universe is accelerating if strong energy condition is violated.

The strong energy condition $R_{00} \geq 0$, is however necessary for lower dimensional stress tensor of supergravity theory. Therefore, we must circumvent this no-go theorem in order to get a viable cosmology from string or M theory. Townsend and N. R. Wohlfarth found that this can be achieved if the condition of time-independence of the internal space is given up [3].

Recently, Townsend and Wohlfarth [3] have worked on a single hyperbolic internal space. Subsequently, this work was generalized by proposing the product of internal spaces. So now, we are interested on product space $R^{3+1} \times R^{m_1} \times R^{m_2} \times H^{m_3}$. These product internal spaces are useful because they provide the situation similar to the hybrid model of inflation [5].

This works can be divided into two parts. For part one, we generalized Einstein's equation to include extra dimensions in the form of product spaces and give some interpretations. Before doing these interpretations, we will make some assumptions for physical behavior of the scale factor. In addition, we assume the universe to be matter-dominated and the spatial curvature for our ordinary three dimensional space is zero. Later, we will turn to consider the accelerating universe in superstring/M-theory point of view. The extra dimensional spaces that will be considered in this context are also product spaces.

In this part two, we need to find a scale factor for the evolution of the universe and from the scale factor, there is a time interval in which the universe is undergoing accelerated expansion.

The objective of this work is to determine the spatial curvature of extra dimensions that is responsible for obtaining accelerated expanding universe. These extra dimensions can be in the form of product spaces.

1.2 No-go Theorem for String Theory

The Einstein equation can produce the two following equations [4]:

Raychaudhuri equation: $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right)$; (1.2-1)

Friedmann equations : $\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G\rho}{3}$; (1.2-2)

where ρ = energy density,

p = pressure.

For an accelerating cosmology, $\frac{\ddot{a}}{a}$ must be positive. Therefore, $\rho + 3p$ must be negative. This violates the strong energy condition [13,7].

From Ref. [4], $\rho + 3p = 2\left(T_{00} - \frac{1}{2} g_{00} T_\lambda^\lambda\right)$

$$= \frac{1}{4\pi G} R_{00} \ . \quad (1.2\text{-}3)$$

Raychaudhuri equation becomes:

$$\frac{\ddot{a}}{a} = -\frac{1}{3} R_{00} \, . \quad (1.2\text{-}4)$$

If we want to get inflation, R_{00} must be negative. But, as it was already mentioned in section 1.1, strong energy condition is not violated by eleven dimensional supergravities or any ten dimensional supergravity theories. Therefore, if the higher dimensional stress tensor satisfies the strong energy condition, then so will the lower dimensional stress tensor.

This implies that [7]

$R_{00}^{(D)} \geq 0$ only if $R_{00}^{(4)} \geq 0$.

Actually, this is a consequence from the following deduction [6,7,9]:

We consider a D dimensional metric ansatz:

$$ds_D^2 = A^2\left(y\right)ds_4^2\left(x\right) + ds_m^2\left(y\right), \tag{1.2-5}$$

where ds_m^2 is the metric of some compact, non-singular m-manifold M without boundary with coordinates y and ds_4^2 is the metric of the four dimensional space-time with coordinates x. A is a smooth, non-vanishing function, known as a warp factor [7,14].

For the compact extra dimensions, we know that [6,7]

$$\left[\int_M A^2\right] R_{00}^{(4)} = \int_M A^2 R_{00}^{(D)} \quad . \tag{1.2-6}$$

This shows that $R_{00}^{(D)} \geq 0$ only if $R_{00}^{(4)} \geq 0$.

The question now arises on how to circumvent the no-go theorem in the hope of explaining the accelerating cosmology. This solution was given in [3]. There is a time-dependent scalar fields, ϕ and a metric g , one can consider

$$\tilde{g}_{\mu\nu} = e^{2\phi} g_{\mu\nu} \tag{1.2-7}$$

as a new, conformally rescaled metric.

If $g_{\mu\nu}$ is the metric of the FLRW cosmology and scalar field, ϕ depend only on time, then

$$\tilde{R}_{00} = R_{00} - 3\left[\ddot{\phi}(t) + H(t)\dot{\phi}(t)\right], \qquad (1.2\text{-}8)$$

where $H(t)$ is Hubble ‘constant’.

The key point is that positivity of R_{00} does not indicate positivity of $\tilde{R}_{00}$ [3,7].

1.3 A Brief of Superstring/ M-Theory

Nowadays, one major problem considered by physicists is that the quantum field theory is incompatible with general relativity. One particular way to reconcile these theories is to modify the quantum field theory in order that it can give rise to the explanation of gravity [15, 16]. This can be done by giving up one of the basic notions of quantum field theory that is the mathematical point description of elementary particles becomes one-dimensional extended objects. The extended objects are called strings. String theory gives a very interesting result that the theory of general relativity is essentially an outcome in this theory. When the distance is very short, the theory can explain very well what is expected, and when the distance is at ordinary range, it yields the explanation as in general relativity. In other words, string theory requires the existence of gravity. In quantum field theory, the point particle forms the world line in a space-time diagram. While for string theory, the corresponding part is called world sheet. The idea is that the fundamental element, strings are of the order of Planck length (about 10^{-35} m) and vibrate at resonant frequencies. Every string vibrates at different resonance frequencies and these different resonance frequencies determine the certain types of force. String theory needs 10,11 or 26 space-time dimensions, which is more than the number of dimensions that can be observed. This is called extra dimensions, and the extra dimensions are compactified to a very small size. Physicists also incorporate supersymmetry into the string theory, and then the theory is called superstring theory. There are five different types of superstring theories and they are “Type I”, “Type IIA”, “Type IIB”, “Heterotic-O” and “Heterotic-E”. This poses a problem, since one desired property of a unified theory is just a single theory explaining all physical forces. In the 1990s, this problem has however been addressed in the second superstring revolution. These five different superstring theories were found to be different limits of a yet to be known single fundamental theory called M-theory.

CHAPTER 2

LITERATURE REVIEW

The need of extra dimensions is common in theories that are unifying gravity and other forces. We could not see these extra dimensions because the scales of these extra dimensions arenot within the present limits of physical detection. Since the unified theory in principle gives all physical forces, scientists thus hope that these theories can explain the phenomena of accelerating universe. In this section, different proposals of the accelerating universe with extra dimensions are reviewed.

2.1 Evolution of the Universe with Flat Extra Dimensions

Gu et al. [17] proposed that the evolution of our universe in different eras, including the present accelerating expansion era, is governed not wholly by the matter contents (excepting dark energy), but also by the curvature of the ordinary 3-space and the evolution of extra spaces. They investigate the possibility of this model to accommodate an accelerated expansion and at the same time conforming to observational constraints. They also present several significant features of their model, such as the automatic stabilization of extra dimensions and the explanation to the cosmic coincidence problem (the energy densities of dark energy and dark matter are comparable now).

In their work, they consider a $(3+n+1)$-dimensional space-time where n is the number of extra spatial dimensions. They make an assumption that the three-dimensional ordinary space and the n-dimensional extra space are homogeneous and isotropic, and represent this space-time by using two spatial parts of the Robertson-Walker metric as follows:

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr_a^2}{1-k_a r_a^2} + r_a^2 d\Omega_a^2\right) - b^2(t)\left(\frac{dr_b^2}{1-k_b r_b^2} + r_b^2 d\Omega_b^2\right), \qquad (2.1\text{-}1)$$

where $a(t)$ and $b(t)$ are scale factors, and k_a and k_b are curvatures of the ordinary 3-space and the extra space, respectively.

2.2 Cosmological Evolution of Homogeneous Universal Extra Dimensions

Bringmann et al. [18] proposed a cosmological model called Homogeneous Universal Extra Dimensions (UED) theory. They present the cosmological solutions of Einstein's field equations for a $(3+n+1)$ dimensional homogeneous and anisotropic universe. They also explain on the interpretation of pressure in higher dimensions and obtain a general relation between pressure and energy density in universal extra dimensions (UED) cosmology. They investigate the solutions with constant extra dimensions during radiation and matter domination era and considered a possible transition between these two states.

For the UED model, the lightest Kaluza-Klein (KK) particle (LKP) could still be present today as a thermal relic, because of KK parity conservation. In addition, it has all the properties of a weakly interacting massive particle (WIMP) if it is neutral and nonbaryonic and is one of the dark matter candidates. The KK photon and the KK neutrino could account for dark matter with $\Omega_M \sim 0.3$, as proposed by the current cosmological concordance model [10], for which one takes the compactification scale of about $R \sim 1TeV^{-1}$ size.

Their work is to study the evolution of the universe as modeled by a suitable extension of the usual Friedmann equations that is the corresponding Friedmann equations for higher dimensional universe. They concern on solutions with constant or slowly varying extra dimensions without any stabilization mechanism. Despite this, their work is to investigate whether the extra dimensions are varying with time or otherwise. From this perspective, they also did a numerical study on the transition regime between the radiation dominated and matter dominated eras.

2.3 Problems with Time-Varying Extra Dimensions or "Cardassian Expansion" as Alternatives to Dark Energy

Cline and Vinet have discussed in Ref. [19] on the mechanisms where the acceleration is due to the presence of extra dimensions, which is proposed in Ref. [12] and Ref. [20]. In [12], the main role is played by a new component, which is some kind of bulk stress energy. This kind of new component could change the form of the Friedmann equations at later times. While in Ref. [20], the acceleration is due to the time-variation of the size of the extra dimensions. Then, they look for accelerated expansion universe by applying these ideas and at the same time, complying with well known physical constraints, which are on the time variation of the four dimensional Newton's gravitational constant or on the possible equation of state of the late form of stress energy.

They tested the idea in [20] against experimental and observational constraints on the constancy over time of the gravitational force, indicating that it goes into a problem. They also examine the relationship between this model and Brans-Dicke theory. They also showed some problems can be overcome by assuming that the extra dimensions are not isotropic. They also show that the Cardassian acceleration model does not obey the weak energy condition in the bulk.

2.4 Accelerating Cosmologies from Compactification

Townsend and Wohlfarth [3] have shown that if we give up the condition of time-independence of the internal space, the no-go theorem for M/superstring theory could be removed. Firstly, the accelerated expansion needs a violation of the strong-energy condition, which requires that $R_{00} \geq 0$ under normal circumstances. However R_{00} must be negative to accommodate to the acceleration of a FLRW (homogeneous and isotropic) universe. Secondly, many four dimensional supergravity theories violated the strong energy condition, except eleven dimensional supergravity or any of the ten dimensional supergravity theories that serve as effective field theories for a superstring theory. The third observation is that if the higher-dimensional stress tensor satisfies the strong energy condition then so will the lower dimensional stress tensor and is applicable to the types of compactification mentioned in [3].

In the paper, they showed there exist cosmological compactifications on Einstein spaces of negative curvature that can give an accelerating four-dimensional FLRW cosmology in Einstein frame. Their vacuum solutions could produce accelerating cosmologies if one assumes compact hyperbolic internal space that is time dependent. The space-time, considered is in the form of $R^{3+1} \times H_n$.

2.5 A Note on Acceleration from Product Space Compactification

Chen et al. [5] have derived a general formulation of Einstein equations for vacuum solutions for product spaces of flat, spherical and hyperbolic spaces. They also examine a simpler case, which is a product of two compact spaces, $R^{3+1} \times K_1 \times K_2$. However, these cases could not give sufficient inflation, but they showed that in principle, one could get an eternal inflation from the similar models with only coefficients of order one. They also analyzed general product spaces $K_0 \times K_1 \times ...... \times K_n$, where the 0-sector of space does not need to be flat and its number of dimensions does not have to be 3. They then obtained the exact solutions for the product spaces in which $K_1 = K_2 = ... = K_n$, where K_i can be a spherical, hyperbolic or flat space.

As a summary, they generalized the Townsend and Wohlfarth's work in Ref. [3] and examined the compactifications of product spaces of flat, spherical and hyperbolic spaces. Finally, the acceleration phases are still not sufficient in all the cases they have examined.

2.6 Hyperbolic Space Cosmologies

Chen et al. have investigated the models compactified on a product of hyperbolic and flat spaces [8] in order to get a model that give sufficient inflation. Generally, solutions are difficult to obtain for general product space compactifications. Therefore, they try to use a specific ansatz and then found a solution for a new class of vacuum space-times, which is a product of flat and hyperbolic spaces.

In addition to the above, they also considered solutions for the cases of hyperbolic external dimensions, external hyperbolic space with internal flat space, and the case of both external and internal spaces being hyperbolic with the hope of getting a sufficient inflation. In the final case, it gives solutions whose late time behavior is like a Milne space-time, expanding with constant rate. From perturbative expansions of such Milne solutions, they found that eternally accelerating expansion can be obtained for which the number of internal dimensions is greater than or equal to seven.

CHAPTER 3

METHODOLOGY AND THEORY

3.1 Einstein's Field Equation

In 1915, Albert Einstein developed his general theory of relativity. The important equation which play the main role in this theory is what we call Einstein's field equation or just Einstein's equation:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} , \qquad (3.1\text{-}1)$$

where $R_{\mu\nu}$ = Ricci tensor,

$g_{\mu\nu}$ = Metric tensor,

$T_{\mu\nu}$ = Energy-momentum tensor,

R = Ricci scalar.

Einstein's field equation determines how the metric (left hand side) responds to energy and momentum of matter. This metric dually describes the geometry of space-time and the gravitational field of the matter source. The solutions of the Einstein's field equation are metrics of space-time.

The equation (3.1-1) can also be rewritten as

$$R_{\mu\nu} = \frac{8\pi G}{c^4} \left(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right), \qquad (3.1\text{-}2)$$

where T = Trace of the energy-momentum tensor, to substantiate the second perspective.