UNIVERSITI PUTRA MALAYSIA

STABILITY OF MARANGONI CONVECTION IN A FLUID LAYER WITH TEMPERATURE-DEPENDENT VISCOSITY

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By

NURUL HAFIZAH BINTI ZAINAL ABIDIN

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

OCTOBER 2007

To My Family and Friends.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

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NURUL HAFIZAH BINTI ZAINAL ABIDIN

October 2007

Chair : **Norihan Md Arifin, PhD**

Institute : **Institute for Mathematical Research**

The onset of steady Marangoni convection induced by surface tension gradients along the upper surface of a horizontal layer of fluid have been the subject of a great deal of theoretical work since the pioneering theoretical investigations of Pearson (1958). The system is heated from below and cooled from above. The purpose of the thesis is to study in detail the onset of Marangoni convection with temperature-dependent viscosity. Few cases of boundary conditions at the bottom surface are studied which are conducting with no-slip, conducting with free-slip and insulating with no-slip. We perform a detailed numerical calculation of the marginal stability curves. We showed that the effect of a temperature-dependent viscosity may be either stabilizing or destabilizing depending on the measurement of the relative variation of viscosity in the fluid volume (viscosity group, R_v). We also present the problem to a case where the fluid layer is overlying a solid layer and here, we undertake a detailed investigation to look at the effect of the thickness or the

conductivity of the solid layer to the onset of Marangoni convection with temperature-dependent viscosity. Again we perform a detailed numerical calculation of marginal stability curves. We showed that the coupled effect of the solid layer depth (or its conductivity) and a temperature-dependent viscosity (with negative sign of variation of viscosity with temperature) is to stabilize the fluid layer.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Master Sains

KESTABILAN OLAKAN MARANGONI DALAM LAPISAN BENDALIR DENGAN KELIKATAN BERSANDAR PADA SUHU

Oleh

NURUL HAFIZAH BINTI ZAINAL ABIDIN

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Pengerusi : Norihan Md Arifin, PhD

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Permulaan olakan Marangoni mantap yang wujud disebabkan kecerunan tegangan permukaan diatas permukaan bebas menjadi penting sejak Pearson (1958) mengkajinya secara teori. Sistem pemodelan ini dipanaskan dari bawah dan disejukkan daripada atas. Tujuan tesis ini adalah untuk mengkaji secara terperinci kesan kelikatan bersandar pada suhu ke atas permulaan olakan Marangoni. Beberapa syarat sempadan pada permukaan bawah telah dikaji iaitu berkonduksi dengan tak gelincir, berkonduksi dengan bebas gelincir, dan berpenebat dengan tak gelincir. Kami menunjukkan secara berangka lengkung kestabilan sut dan mengkaji samada kelikatan bersandar kepada suhu akan menstabilkan atau menyahstabilkan sistem yang mana ia bergantung pada pekali variasi relatif kelikatan di dalam ruang bendalir (kumpulan kelikatan, R_v). Seterusnya, kami mengkaji kesan ketebalan atau kekonduksian satu lapisan pejal terhadap permulaan olakan Marangoni mantap dengan kelikatan bersandar kepada suhu sekiranya ia berada di bawah satu lapisan

bendalir. Sekali lagi, kami tunjukkan secara terperinci pengiraan berangka lengkung kestabilan sut. Kami tunjukkan bahawa kedua-dua faktor ketebalan (atau kekonduksian) lapisan pejal dan kelikatan bersandar kepada suhu (dimana pekali variasi kumpulan kelikatan bernilai negatif) akan menstabilkan lapisan bendalir.

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I certify that an Examination Committee has met on 5th October 2007 to conduct the final examination of Nurul Hafizah binti Zainal Abidin on her Master of Science thesis entitled "Stability of Marangoni Convection in a Fluid Layer with Temperature-dependant Viscosity" in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the student be awarded the degree of Master of Science.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

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Date: 22 January 2008

DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations, which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

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NURUL HAFIZAH BINTI ZAINAL ABIDIN

Date:

TABLE OF CONTENTS

LIST OF TABLES

Table **Page**

LIST OF FIGURES

Figure **Page**

LIST OF ABBREVIATIONS

a	thermal expansion coefficient of density
B_i	Biot number
B_o	Bond number
c	specific heat
C_r	Crispation number
d	depth of fluid layer
d_r	depth ratio, d_s/d
d_s	depth of solid layer
f	shear stress
g	gravity
h	heat transfer coefficient
H	mean curvature of the surface
k	thermal conductivity of fluid
k_r	thermal conductivity ratio, k_s/k
k_s	thermal conductivity of solid layer
M	Marangoni number
p	pressure
p_g	gas pressure
P_r	Prandtl number
R	Rayleigh number
R_v	Viscosity group
s	growth rate

t	time
T	temperature of fluid
T_g	temperature of the ambient gas
T_s	temperature of solid layer
u	velocity
x, y, z	Cartesian coordinates

Greek Abbreviations

α	wavenumber
γ	thermal expansion coefficient of dynamic viscosity
δ_d	surface deflection
ε	thermal expansion coefficient of surface tension
ζ	thermal expansion coefficient of kinematic viscosity
θ	temperature disturbance
κ	thermal diffusivity of fluid
κ_s	thermal diffusivity of solid layer
μ	dynamic viscosity of fluid
ν	kinematic viscosity of fluid
ρ	density of fluid
σ	surface tension

Subscript

$_0$	reference quantity
c	critical state
s	property of the solid plate
r	ratio of the solid plate property to the fluid property

CHAPTER 1

INTRODUCTION

Convection is the transfer of heat by the motion of or within a fluid. It may arise from the temperature differences either within the fluid or between the fluid and its boundary, other sources of density variations (such as variable salinity) or from the application of an external motive force. It is one of the three primary mechanisms of heat transfer, the others being conduction and radiation. Convection has a wide range of applications, including calculating forces and moments on aircraft, determining the mass flow rate of petroleum through pipelines and to predict weather patterns. Some of its principles are even used in traffic engineering, where the traffic is treated as a continuous fluid. Generally, convection means fluid motion caused by temperature difference with the temperature gradient pointing in any direction (Chapmen 1984).

In this thesis, the stability on convection in a plane horizontal fluid layer heated from below is considered as it is one of the most common types of convection. It was studied for the first time by Pellew and Southwell (1940) and then was summarized by Chandrasekhar's (1961) book, where a nearly complete picture of the linear analysis called Rayleigh-Bénard convection was presented. Usually, the term Rayleigh-Bénard convection is attributed to convection due to buoyancy mechanism, while the term Marangoni convection refers to surface tension gradient mechanism.

(a) (b)

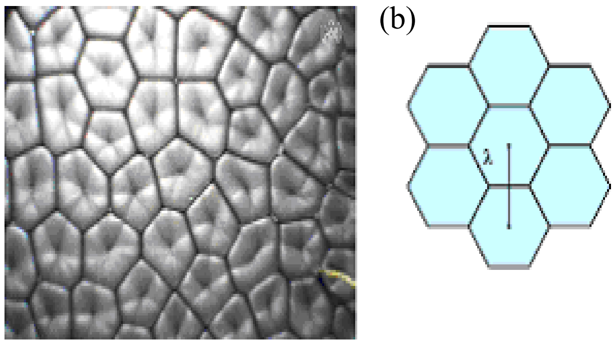

Figure 1.1: (a) Honeycomb like pattern observed in Bénard convection and (b) Hexagon geometry showing the wavelength λ (Maroto et al. 2007)

1.1 Convection in a Fluid Layer

When a fluid layer in a horizontal plane is heated from below, instabilities arise when the temperature gradient across the layer exceeds a certain critical value which is caused by buoyancy or by the surface tension gradient. Bénard (1900) and Block (1959) were the pioneers of the comprehensive investigation of convection for both mechanism.

Bénard (1900), carrying his experiment on a thin horizontal layer of molten spermaceti with a free surface observed the establishment of a regular, steady pattern of flow cells when the layer is heated from below. These cells, which are known as Bénard cells, were mainly hexagonal, and the pattern resembled a honeycomb as shown in Figure 1.1. The distance between the central points of neighbouring hexagonal cells defines the wavelength of the system. Bénard (1900) believed that the phenomenon was caused by buoyancy and therefore, this experiment was regarded as the starting point of convection instabilities. But later, Block (1956) and

Pearson (1958) suggested that the convection was caused by surface tension gradient as the case studied used a thin fluid layer with a free surface where buoyancy may not exist.

Block (1956) is the pioneer of convection that is caused by a surface tension gradient and the study is known as Marangoni convection. The experimental investigation of the convection led to the conclusion that the hexagonal pattern observed by Bénard (1900) was due to the surface tension gradient and not the buoyancy. In the experiment, a thin hydrocarbon film less than one millimeter with a free surface was used and Block (1956) observed that convection would stop when the fluid layer was covered on the top by a thin silicone layer. Hence, Block (1956) concluded that the convection in a fluid layer with a free surface was caused by surface tension gradient. Later, Pearson (1958) furthers the study of Marangoni convection theoretically.

1.1.1 Physical Mechanism for Marangoni Convection

Consider a horizontal fluid layer bounded from below by a rigid boundary and from above by a free surface. An infinitesimal disturbance of the temperature on the surface, say a warm spot, creates surface tension traction on the surface if the surface tension coefficient of the fluid is a function of temperature. Since surface tension usually decreases with temperature, a warm spot will be a soft spot at the surface from which the fluid will be pulled away laterally as shown in figure 1.2. The fluid underneath the warm spot must consequently rise. The fluid motion at the surface

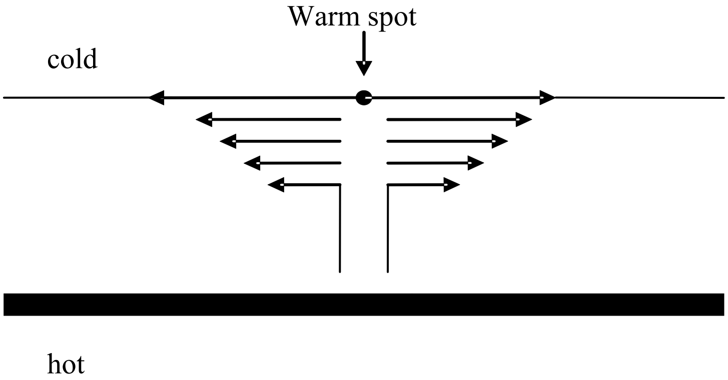

Figure 1.2: Convection mechanism caused by surface tension gradient

will be transmitted by viscosity to the interior of the fluid layer, but viscosity will also dampen the motion. The motion in the fluid can be sustained only if energy is provided to overcome the frictional losses. The required energy can be provided by a vertical temperature differences across the fluid layer, which can be heated from below. If sufficient energy is supplied, and if a critical vertical temperature difference is reached, convection caused by surface tension gradients will commence and will be sustained.

1.1.2 Convection with Variable Viscosity Effect

Viscosity is a measure of the resistance of a fluid to deform under shear stress. Realistically, viscosity in a fluid depends on density. The higher the density of the fluid, the higher the viscosity will be. Fluid also possess a temperature-dependent viscosity which influences heat transport and the spatial structure of the fluid. When the temperature increases, the viscosity decreases, but for gases, the viscosity will increase. Viscosity varies widely with temperature, but temperature variation has the opposite effect on the viscosities of fluids because of their fundamentally different