



**UNIVERSITI PUTRA MALAYSIA**

**PREDICTION OF CRITICAL VALUES FOR ONSET OF CONVECTION BY VERTICAL  
HEATED PLATE IN WATER UNDER CONSTANT HEAT FLUX CONDITION**

**NGEOW YEN WAN**

**T FK 2007 48**

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**MASTER OF SCIENCE  
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**By**

**NGEOW YEN WAN**

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia,  
in Fulfilment of the Requirements for the Degree of Master of Science**

**June 2007**



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment  
of the requirements for the Degree of Master of Science

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**June 2007**

**Chairman: Associate Professor Thomas Choong Shean Yaw, PhD**

**Faculty: Engineering**

The local onset of convection in water generated by vertical heated plate is examined. It is generally accepted that the occurrence of buoyancy convection can be predicted using well known conventional critical Rayleigh number derived by Lord Rayleigh (1916). However, the development of local transient instability is less well understood for fluids suddenly heated by vertical heating plate. In this work, the correlation between local onset of convection and distance from the leading-edge has been derived and has allowed the tracking of local critical time along the heating plate.

Patterson *et al.* (2002) experiments have been reproduced based on Constant Heat Flux (CHF) boundary condition. Experiments of Patterson *et al.* (2002) have shown that the local onset of convection occurred at the departure from the initial heat

conduction temperature profile. The characteristic of temperature profile along the vertical heated plate for x-axis and y-axis have been studied to determine the correlation between the local onset of convection and the distance from the leading-edge along the heating plate. Subsequently, simulations under different vertical heated plate lengths and heat fluxes have been simulated to study the effect for both of these conditions.

A computational fluid dynamics (CFD) software, Fluent 6.0 is used in this study to solve the governing partial differential equations for heat transfer using finite volume technique under various heat fluxes and plate lengths. 2D-time simulations were conducted for constant heat flux (CHF) boundary conditions. Various heat fluxes and plate lengths were applied and the effects were investigated. The mechanism of the local onset of convection by the vertical heating plate was observed. The temperature profiles, velocity magnitude and heat transfer coefficient versus time were plotted to detect the local onset of convection. The newly derived correlation of local onset of convection was incorporated in this study to predict the local critical time and compare with the simulated results.

It is observed that the local critical Rayleigh number is consistent with respect to their location and is independent of heating plate length. The local critical time is earlier as the heat flux is increased under the same heating plate length.

Approximately 280 simulations were conducted and most of these simulated local critical Rayleigh number were in good agreement with the predicted value using the newly derived equation.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

**PERAMALAN NILAI KRITIKAL BAGI PERMULAAN KONVEKSI OLEH  
PLAT TEGAK YANG DIPANASKAN DI DALAM AIR DALAM KEADAAN  
PEMANASAN HABA ADALAH MALAR**

Oleh

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Permulaan konveksi tempatan di dalam air disebabkan pemanasan plat tegak telah dikaji. Adalah diterima secara keseluruhan bahawa pengapungan konveksi boleh diketahui dengan penggunaan nombor genting Rayleigh daripada Lord Rayleigh (1916). Tetapi, pembangunan kestabilan tempatan masih belum difahami dengan sepenuhnya terutamanya bagi cecair yang tiba-tiba dipanasi plat tegak panas. Dalam kajian ini, perhubungan antara permulaan konveksi tempatan dengan jarak dari “leading-edge” telah dikaji dan dengan ini masa genting tempatan di atas plat panas dapat ditentukan.

Simulasi eksperimen Patterson *et al.* (2002) telah diulangi dalam keadaan pemanasan haba adalah malar (CHF) telah dijalankan. Eksperimen yang dilakukan oleh Patterson *et al.* (2002) menunjukkan konveksi bermula apabila konduksi berlepas daripada profil suhu. Corak suhu mengikut masa telah dijalankan pada tempat-tempat



berdekatan plat tegak iaitu paksi-x dan paksi-y. Akhirnya, simulasi telah dilakukan untuk mengkaji pengaruh kepanjangan plat dan flux haba yang berbeza.

Perisian komputer bendalir, Fluent 6.0 telah digunakan dalam kajian ini untuk menyelesaikan masalah matematik pemindahan haba menggunakan cara had isipadu dalam keadaan flux haba dan plat yang berbeza. Simulasi dalam bentuk 2-D telah dijalankan dalam bentuk berkeadaan tidak stabil di mana pemanasan haba adalah malar (CHF) di sempadan. Pengaruh bagi simulasi bagi berbagai jenis flux haba dan panjang plat yang berbeza telah telah dikaji dan mekanisme permulaan konveksi dalam simulasi telah diperhatikan. Profil suhu digunakan untuk menentukan masa di mana konveksi bermula. Pembentukan persamaan baru untuk menentukan permulaan konveksi tempatan telah digunakan dalam kajian ini dan dibanding dengan nilai yang dihasil daripada simulasi.

Adalah didapati, simulasi pemanasan plat tegak menunjukkan nilai kritikal tempatan Rayleigh adalah sama mengikut lokasi dan tidak bergantung pada kepanjangan plat yang digunakan. Selain itu, masa kritikal adalah lebih cepat apabila flux haba ditingkatkan bagi panjang plat pemanas yang sama.

Sebanyak 280 simulasi telah dijalankan dan purata hasil simulasi yang diperolehi memberi keputusan yang memuaskan berbanding dengan nilai yang dihasilkan daripada persamaan yang baru dibentuk.

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I certify that an Examination Committee has met on 15<sup>th</sup> June 2007 to conduct the final examination of Ngeow Yen Wan on his Master of Science thesis entitled "Prediction of Critical Values for Onset of Convection by Vertical Heated Plate in Water Under Constant Heat Flux Condition" in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the student be awarded the degree of Master of Science.

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## **DECLARATION**

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

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**NGEOW YEN WAN**

Date: 1<sup>st</sup> September 2007

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## LIST OF ABBREVIATIONS

### Nomenclature

$B_i$	Biot number
$c_p$	Specific heat, J/ kg. K
$d$	Lateral depth, m
$F$	External force, N
$g$	Gravity acceleration, m/s <sup>2</sup>
$h$	Heat transfer coefficient, W/ m. K
$k$	Thermal conductivity, W.m/ K
$L$	Plate Length, m
$Pr$	Prandtl number
$q^o$	Constant surface heat flux (W/ m <sup>2</sup> )
$Ra$	Rayleigh number
$Ra_c$	Critical Rayleigh number
$Ra_{max}$	Maximum transient Rayleigh number
$S_h$	Enthalpy, J
$T$	Temperature, K
$T_0$	Initial temperature, K
$T_s$	Surface Temperature, K
$\Delta T$	Temperature different, K
$t$	Time, s
$t_c$	Critical time, s
$\Delta t$	Time step, s
$u$	Velocity, m/s

$\hat{v}$	Velocity Scale of Traveling Wave, m/s
$v_{adv}$	Boundary layer velocity, m/s
$x$	Distance from surface of heating plate, cm
$X$	Horizontal length of computational domain, m
$y$	Distance from leading-edge, m
$Y$	Vertical height of computational domain, m
$z$	Penetration depth of thermal layer, m
$z_{max}$	Maximum depth of penetration, m

### **Greek symbols**

$\alpha$	Volumetric coefficient of thermal expansion, K <sup>-1</sup>
$\beta$	Constant temperature gradient, K/ m
$\delta$	Boundary layer thickness, m
$\delta_e$	Thickness of effective thermal layer, m
$\kappa$	Thermal diffusivity, m <sup>2</sup> / s
$\mu$	Viscosity, Pa. s
$\nu$	Kinematics viscosity, m <sup>2</sup> / s
$\rho$	Density, kg/ m <sup>3</sup>
$\tau$	non-dimensional time

### **Subscripts**

c	Critical
0	Initial
s	Surface
max	maximum

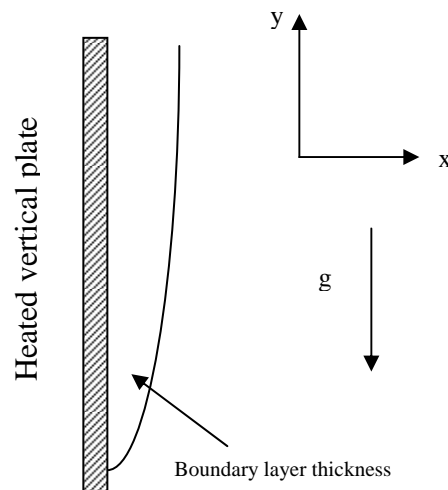
## **Abbreviations**

CFD	Computational Fluid Dynamics
CHF	Constant Heat Flux
FST	Fixed Surface Temperature
LEE	Leading-Edge Effect
SIMPLE	Semi-Implicit Method for Pressure-Linked Equations

## CHAPTER 1

### INTRODUCTION

Heat transfer continues to be a fertile area due to its application in several fields, which includes the cooling system for electronics appliances, refrigerators, packaging for electronic industries, chiller system, geotechnical engineering and solar collector. Free convection plays a significant role as one of the mode of heat transfer. The classical interest of free convection is the study of a semi-infinite wall which is initially having the same temperature as the ambient fluid; the wall is suddenly heated, either by imposing of constant heat flux or rise in the wall temperature. Figure 1.1 shows the schematic diagram of a vertical heated wall boundary layer.



**Figure 1.1: Schematic diagram of a vertical heated wall boundary layer.**



The local onset of convection induced by transient heat conduction has been described in the following terms: at any fixed position on the plate, the flow is initially described as one dimensional and unsteady, as though the plate is doubly infinite; at some time later, over a non zero period of time, which depends on the position, a transition occurs in the flow, known as the leading-edge effect (LEE), and the flow becomes two dimensional and steady (Ostrach, 1964).

Lord Rayleigh (1916) derived a criterion for the onset of buoyancy convection in a fluid layer bounded by two free surfaces. Spangenberg and Rowland (1961) through their experimental studies have found that the onset of convection is independent of the depth of the water which is also confirmed by Foster (1965). Tan and Thorpe (1996, 1999a) have shown that the local onset of the transient instability and convection in horizontal deep fluids can be characterized by transient Rayleigh number that is dependent upon the Biot number of the interface. They derived equations for the prediction of the critical time and critical depth which were successfully applied for horizontal heating plate. This research aims to derive the correlation of local onset of convection for a vertical heating plate under constant heat flux (CHF) boundary condition and to verify the theory using Computational Fluids Dynamics (CFD) simulation.

Patterson *et al.* (2002) attempted to determine the transition of the unsteady-state heat conduction in a semi-infinite fluid from a vertical plate and the LEE by relating it to the traveling waves on the boundary layer. They claimed that their experimental findings at