



## On Soft Expert Metric Spaces

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### ABSTRACT

In this paper, we first introduce soft expert real number and soft expert point. Following this, the notion of soft expert metric over soft expert real sets has been defined and then soft expert metric spaces are investigated. Later, soft expert open ball, soft expert closed ball, soft expert neighborhood, soft expert open set and soft expert closed set are defined and studied some properties of these concepts.

**Keywords:** Soft expert set, soft expert point, soft expert open set, soft expert closed set, soft expert metric spaces

## 1. Introduction

Many of the real life problems in environment, social sciences, medical science and engineering etc. include varying uncertainties. In the related literature before theory of soft set fuzzy real numbers and it was studied the basic properties of it (1). In (2) Gerla studied necessary and sufficient conditions for the existence of a good definition of fuzzy point. Gottwald and Siegfried (3) show that three theorems of Wong on local properties of fuzzy topology are wrong. In (4) Murali show the equivalence to the set of fuzzy points and study its effect on the relationship between fuzzy points and fuzzy subsets. Neog, Jyoti, Sut and Hazarika in (5) studied some properties related to fuzzy soft topological spaces. Ming and Ming (6) investigated structure of a fuzzy point. Fuzzy open set and fuzzy closed set are explored in (7, 8), soft open set and soft closed set are investigated in(9–12), fuzzy soft open set, fuzzy soft closed set in (13, 14) and fuzzy metric space were studied by many researchers such as (15–22). For these subject uncertainties principle firstly studied by Molodtsov (23). In his study Molodtsov initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties. Studies related to the soft theory and applications in soft sets were done by Chen et al. (24) and Maji et al.(25, 26). Following this, soft metric spaces (27–30), neighborhood properties of soft topological spaces were studied in (31). However, there are many points to study related to the soft set idea, see (33–35). Further the subject of soft expert metric spaces has also not been studied yet. For this reason, using the definition of soft expert set given by (32), we introduce a notion of soft expert metric and soft expert metric space based on soft expert point of soft expert sets. In this way we obtain soft expert metric which is more generalized set definition than soft metric. In Section 1 many studies from fuzzy sets to soft expert sets are mention. In section 2 well-known results of some preliminaries are given. In section 3 notions of soft expert real number and soft expert point are explored. In section 4the definition of soft expert metric and some properties of soft expert metric spaces with theorems and examples are given. In Section 5 give conclusions.

## 2. Preliminaries

In this section we recall some basic notions in soft experts set theory (32). Let  $U$  be universe,  $E$  a set of parameters and  $X$  a set of experts. Let  $O$  be a set opinions,  $Z = E \times O \times X$  and  $A \subseteq Z$ .

**Definition 2.1.** (32) A pair  $(F, A)$  is called a soft expert set over  $U$ , where  $F$  is mapping given by  $F : A \rightarrow P(U)$  where  $P(U)$  denotes the power set of  $U$ .

We assume in this paper, two-valued opinions only in set  $O$  that is

$$O = \{0 = \text{disagree}, 1 = \text{agree}\}$$

but multi-valued opinions may be assumed as well.

**Definition 2.2.** (32) For two soft expert sets  $(F, A)$  and  $(G, B)$  over  $U$ ,  $(F, A)$  is called a soft expert subset of  $(G, B)$  if

1.  $A \subseteq B$
2.  $\forall \varepsilon \in B, G(\varepsilon) \subseteq F(\varepsilon)$ .

This relationship is denoted by  $(F, A) \subseteq (G, B)$ . In this case  $(G, B)$  is called a soft expert superset of  $(F, A)$ .

**Definition 2.3.** (32) Two soft expert sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$ , are said to be equal if  $(F, A)$  is a soft expert subsets of  $(G, B)$  and  $(G, B)$  is a soft expert subset of  $(F, A)$ .

**Definition 2.4.** (32) Let  $E$  be a set of parameters and  $X$  a set of experts. The NOT set of  $Z = E \times O \times X$  denoted by  $IZ$  is defined by

$$IZ = \{(I_{e_i, x_j, o_k}), \forall i, j, k\}$$

where,  $I_{e_i}$  is not  $e_i$ .

**Theorem 2.1.** (32) The complement of a soft expert set  $(F, A)$  is denoted by  $(F, A)^c$  and is defined by  $(F, A)^c = (F^c, IA)$ , where  $F^c : IA \rightarrow P(U)$  is mapping given by

$$F(\alpha)^c = U - F(I\alpha) \quad \forall \alpha \in IA.$$

**Definition 2.5.** (32) An agree soft expert set  $(F, A)_1$  over  $U$  is a soft expert subset of  $(F, A)$  defined as follow:

$$(F, A)_1 = \{F_1(\alpha) : \alpha \in E \times X \times \{1\}\}.$$

**Definition 2.6.** (32) An disagree soft expert set  $(F, A)_0$  over  $U$  is a soft expert subset of  $(F, A)$  defined as follow:

$$(F, A)_0 = \{F_0(\alpha) : \alpha \in E \times X \times \{0\}\}.$$

**Definition 2.7.** (32) The union of two soft expert sets  $(F, A)$  and  $(G, B)$  over  $U$  denoted by " $(F, A) \cup (G, B)$ " is the soft expert set  $(H, C)$ , where  $C = A \cup B$  and  $\forall e \in C$ ,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cup G(e), & \text{if } e \in A \cap B \end{cases}$$

we express it as  $(F, A) \cup (G, B) = (H, C)$ .

The following definition of intersection of two soft expert sets is given as that of the bi intersection in.

**Definition 2.8.** (32) The intersection of two soft expert sets  $(F, A)$  and  $(G, B)$  over  $U$  denoted by " $(F, A)\tilde{\cap}(G, B)$ " is the soft expert set  $(H, C)$ , where  $C = A \cap B$  and  $\forall e \in C$ ,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cap G(e), & \text{if } e \in A \cap B \end{cases}$$

we express it as  $(F, A)\tilde{\cap}(G, B) = (H, C)$ .

**Definition 2.9.** (32) If  $(F, A)$  and  $(G, B)$  are two soft expert sets over  $U$  then  $(F, A)$  AND  $(G, B)$  denoted by  $(F, A) \wedge (G, B)$  is defined by

$$(F, A) \wedge (G, B) = (H, A \times B)$$

where  $H(\alpha) = F(\alpha)\tilde{\cap}G(\alpha) \quad \forall(\alpha, \beta) \in A \times B$ .

**Definition 2.10.** (32) If  $(F, A)$  and  $(G, B)$  are two soft expert sets over  $U$  then  $(F, A)$  OR  $(G, B)$  denoted by  $(F, A) \vee (G, B)$  is defined by

$$(F, A) \vee (G, B) = (O, A \times B)$$

where  $O(\alpha) = F(\alpha)\tilde{\cup}G(\alpha) \quad \forall(\alpha, \beta) \in A \times B$ .

### 3. Soft Expert Real Number and Soft Expert Point

**Definition 3.1.** Let  $R$  be the set of real numbers and  $B(R)$  be collection of all non-empties bounded subsets of  $R$  and  $E$  a set parameters and  $X$  a set of expert. Let  $O$  be a set opinions,  $Z = E \times O \times X$  and  $A \subseteq Z$ . Then a mapping  $F : A \rightarrow B(R)$  is called a soft expert real set. It is denoted by  $(F, A)$ . If specifically  $(F, A)$  is a singleton soft expert set, then identifying  $(F, A)$  with the corresponding soft expert element, it will be called a soft expert real number and denoted  $\tilde{k}, \tilde{m}, \tilde{n}$  such that  $\tilde{k}(e) = e, \forall e \in A$  etc. For example  $\bar{1}$  soft expert real number  $\bar{1}(e) = 1 \forall e \in A$ .

**Definition 3.2.** For two soft expert real numbers;

1.  $\tilde{k} \lesssim \tilde{m}$  if  $\tilde{k}(a) \lesssim \tilde{m}(a) \forall a \in A$ ;
2.  $\tilde{k} \gtrsim \tilde{m}$  if  $\tilde{k}(a) \gtrsim \tilde{m}(a) \forall a \in A$ ;

3.  $\tilde{k} \tilde{<} \tilde{m}$  if  $\tilde{k}(a) \tilde{<} \tilde{m}(a) \forall a \in A$ ;

4.  $\tilde{k} \tilde{>} \tilde{m}$  if  $\tilde{k}(a) \tilde{>} \tilde{m}(a) \forall a \in A$ .

**Definition 3.3.** A soft expert set  $(P, A)$  over  $U$  is said to be soft expert point if there is exactly one  $e \in A$ , such that  $P(a) = \{\tilde{x}\}$  for some  $\tilde{x} \in U$  and  $P(a') = \emptyset \forall a' \in A \setminus \{a\}$ . It will be denoted by  $\tilde{x}_a$ .

**Definition 3.4.** Two soft expert points  $\tilde{x}_a, \tilde{y}_a$  are said to be equal if  $a' = a$  and  $P(a) = P(a')$  i.e.  $\tilde{x} = \tilde{y}$ . Thus  $\tilde{x}_a = \tilde{y}_a \Leftrightarrow \tilde{x} = \tilde{y}$ , or  $a' \neq a$ .

**Theorem 3.1.** The union of any collection of soft expert points can be considered as soft expert set and every soft expert set can express as union of all soft expert points belonging to it:

$$(F, A) = \bigcup_{\tilde{x}_a \in (F, A)} \tilde{x}_a$$

Let  $\tilde{U}$  be absolute soft expert set i.e.  $F(a) = U, a \in A$ , where,  $(F, A) = \tilde{U}$  and  $SEP\tilde{U}$  be collection of all soft expert point  $\tilde{U}$  and  $R(A)^*$  denoted the set of all non-negative soft expert real numbers.

## 4. Soft Expert Metric and Soft Expert Metric Spaces

**Definition 4.1.** A mapping  $\tilde{d} : SEP(\tilde{U}) \times SEP(\tilde{U}) \rightarrow R(A)^*$  is said to be a soft expert metric on the soft expert set  $\tilde{U}$  if  $\tilde{d}$  satisfies the following conditions:

$$SEM1 \quad \tilde{d}(\tilde{x}_{a_1}, \tilde{y}_{a_2}) \tilde{\geq} \bar{0} \quad \forall \tilde{x}_{a_1}, \tilde{y}_{a_2} \in \tilde{U},$$

$$SEM2 \quad \tilde{d}(\tilde{x}_{a_1}, \tilde{y}_{a_2}) = \bar{0} \Leftrightarrow \tilde{x}_{a_1} = \tilde{y}_{a_2},$$

$$SEM3 \quad \tilde{d}(\tilde{x}_{a_1}, \tilde{y}_{a_2}) = \tilde{d}(\tilde{y}_{a_2}, \tilde{x}_{a_1}) \quad \forall \tilde{x}_{a_1}, \tilde{y}_{a_2} \in \tilde{U},$$

$$SEM4 \quad \forall \tilde{x}_{a_1}, \tilde{y}_{a_2}, \tilde{z}_{a_3} \in \tilde{U}, \tilde{d}(\tilde{x}_{a_1}, \tilde{z}_{a_3}) \tilde{\leq} \tilde{d}(\tilde{x}_{a_1}, \tilde{y}_{a_2}) + \tilde{d}(\tilde{y}_{a_2}, \tilde{z}_{a_3}).$$

The soft expert set  $\tilde{U}$  with soft expert metric  $\tilde{d}$  on  $\tilde{U}$  is called a soft expert metric space and denoted by  $(\tilde{U}, \tilde{d}, A)$ .

**Definition 4.2.** Let  $(F, A) (\neq) \in s(\tilde{U})$ , then the collection of all soft expert elements of  $(F, A)$  will be denoted by  $SEE(F, A)$ . For a collection  $B$  of soft expert elements of  $\tilde{U}$ , the soft expert set generated by  $B$  is denoted by  $SES(B)$ .

**Example 4.1.** Let  $\tilde{U}$  is being a non- empty set and  $E$  a set of parameters and  $X$  a set of experts. Let  $O$  be a set opinions,  $Z = E \times O \times X$  and  $A \subseteq Z$ . Let

$\tilde{U}$  be the absolute soft expert set i.e.  $F(r) = \tilde{U}, \forall r \in A$ , where  $(F, A) = \tilde{U}$ . We define  $\forall \tilde{x}_{a_1}, \tilde{y}_{a_2} \in \tilde{U}, \tilde{d} : SEP(\tilde{U}) \times SEP(\tilde{U}) \rightarrow R(A)^*$  by

$$\tilde{d}(\tilde{x}_{a_1}, \tilde{y}_{a_2}) = \begin{cases} \bar{0}, & \tilde{x}_{a_1} = \tilde{y}_{a_2} \\ \bar{1}, & \tilde{x}_{a_1} \neq \tilde{y}_{a_2} \end{cases}$$

Normally  $\tilde{d}$  satisfies all the soft expert metric axioms. So,  $\tilde{d}$  is soft expert metric on the soft expert set  $\tilde{U}$  and  $(\tilde{U}, \tilde{d}, A)$ .  $\tilde{d}$  is called the discrete soft expert metric on the soft expert set  $\tilde{U}$  and  $(\tilde{U}, \tilde{d}, A)$  is said to be the discrete soft expert metric space.

**Example 4.2.** Let  $\tilde{U} \subseteq R$  be a non-empty set and  $E \subseteq R$  be the non-empty set of parameters and  $X$  a set of experts. Let  $O$  be a set opinion,  $Z = E \times O \times X$  and  $A \subseteq Z$ . Let  $\tilde{U}$  be the absolute soft expert set i.e.  $F(r) = \tilde{U}, \forall r \in A$  where  $(F, A) = \tilde{U}$ . Let  $\tilde{a}$  denote the soft expert real number such that  $\tilde{a}(r) = r, \forall r \in A$ . We define  $\tilde{d} : SEP(\tilde{U}) \times SEP(\tilde{U}) \rightarrow R(A)^*$  by

$$\tilde{d}(\tilde{x}_{a_1}(\gamma), \tilde{y}_{a_2}(\mu)) = |\tilde{x}_{a_1} - \tilde{y}_{a_2}| - |\tilde{\gamma} - \tilde{\mu}|,$$

for  $\forall \tilde{x}_{a_1}, \tilde{y}_{a_2} \in \tilde{U}$ ; where  $|\cdot|$  denotes the modules of soft expert real numbers. Then  $\tilde{d}$  is a soft expert metric on  $\tilde{U}$ . Let us satisfies (SEM1), (SEM2), (SEM3) and (SEM4) for soft expert metric.

SEM1) It is obvious from the above definition that;

$$\tilde{d}(\tilde{x}_{a_1}(\gamma), \tilde{y}_{a_2}(\mu)) \geq \bar{0}, \quad \forall \tilde{x}_{a_1}(\gamma), \tilde{y}_{a_2}(\mu) \in \tilde{U}.$$

SEM2) we have

$$\begin{aligned} \tilde{d}(\tilde{x}_{a_1}(\gamma), \tilde{y}_{a_2}(\mu)) = 0 &\Leftrightarrow |\tilde{x}_{a_1} - \tilde{y}_{a_2}| - |\tilde{\gamma} - \tilde{\mu}| = 0 \\ &\Leftrightarrow |\tilde{x}_{a_1} - \tilde{y}_{a_2}| = 0 \text{ and } |\tilde{\gamma} - \tilde{\mu}| = 0 \\ &\Leftrightarrow x = y \text{ and } \gamma = \mu \\ &\Leftrightarrow \tilde{x}_{a_1}(\gamma) = \tilde{y}_{a_2}(\mu) \end{aligned}$$

SEM3) we have

$$\begin{aligned} \tilde{d}(\tilde{x}_{a_1}(\gamma), \tilde{y}_{a_2}(\mu)) &= |\tilde{x}_{a_1} - \tilde{y}_{a_2}| - |\tilde{\gamma} - \tilde{\mu}| \\ &= |\tilde{y}_{a_2} - \tilde{x}_{a_1}| - |\tilde{\mu} - \tilde{\gamma}| \\ &= \tilde{d}(\tilde{y}_{a_2}(\mu), \tilde{x}_{a_1}(\gamma)), \quad \forall \tilde{x}_{a_1}(\gamma), \tilde{y}_{a_2}(\mu) \in \tilde{U}, \end{aligned}$$

SEM4) we have  $\forall \tilde{x}_{a_1}(\gamma), \tilde{y}_{a_2}(\mu), \tilde{z}_{a_3}(\delta) \in \tilde{U}$ ;

$$\begin{aligned} \tilde{d}(\tilde{x}_{a_1}(\gamma), \tilde{z}_{a_3}(\delta)) &= |\tilde{x}_{a_1} - \tilde{z}_{a_3}| - |\tilde{\gamma} - \tilde{\delta}| \\ &= |\tilde{x}_{a_1} - \tilde{y}_{a_2} + \tilde{y}_{a_2} - \tilde{z}_{a_3}| - |\tilde{\gamma} - \tilde{\mu} + \tilde{\mu} - \tilde{\delta}| \\ &\leq |\tilde{x}_{a_1} - \tilde{y}_{a_2}| + |\tilde{y}_{a_2} - \tilde{z}_{a_3}| - |\tilde{\gamma} - \tilde{\mu}| + |\tilde{\mu} - \tilde{\delta}| \\ &\leq \tilde{d}(\tilde{x}_{a_1}(\gamma), \tilde{y}_{a_2}(\mu)) + \tilde{d}(\tilde{y}_{a_2}(\mu), \tilde{z}_{a_3}(\delta)) \end{aligned}$$

Thus  $\tilde{d}$  is soft expert metric on  $\tilde{U}$ .

**Definition 4.3.** Let  $(\tilde{U}, \tilde{d}, A)$  be a soft expert metric space and  $\tilde{\varepsilon}$  be a non-negative soft expert real number

$$D(\tilde{x}_{a_1}, \tilde{\varepsilon}) = \{\tilde{y}_{a_2} \in \tilde{U} : \tilde{d}(\tilde{x}_{a_1}, \tilde{y}_{a_2}) \tilde{<} \tilde{\varepsilon}\} \tilde{C}SEP(\tilde{U})$$

is called the soft expert open ball with center  $\tilde{x}_{a_1}$  an radius  $\tilde{\varepsilon}$  and

$$D[\tilde{x}_{a_1}, \tilde{\varepsilon}] = \{\tilde{y}_{a_2} \in \tilde{U} : \tilde{d}(\tilde{x}_{a_1}, \tilde{y}_{a_2}) \tilde{\leq} \tilde{\varepsilon}\} \tilde{C}SEP(\tilde{U})$$

is called the soft expert closed ball with center  $\tilde{x}_{a_1}$  an radius  $\tilde{\varepsilon}$ .

**Example 4.3.** Consider the discrete soft expert metric space  $(\tilde{U}, \tilde{d}, A)$  as in Example 4.2. Then for any  $\tilde{x}_{a_1} \in \tilde{U}$ ,  $D(\tilde{x}_{a_1}, \tilde{\varepsilon}) = SEP(\tilde{U})$ ,  $\tilde{1} \tilde{<} \tilde{\varepsilon}$  and  $D(\tilde{x}_{a_1}, \tilde{\varepsilon}) = \{\tilde{x}_{a_1}\}$  if  $\tilde{\varepsilon} \tilde{<} \tilde{1}$  also  $D[\tilde{x}_{a_1}, \tilde{\varepsilon}] = SEP(\tilde{U})$ ,  $\tilde{1} \tilde{\leq} \tilde{\varepsilon}$  and  $D[\tilde{x}_{a_1}, \tilde{\varepsilon}] = \{\tilde{x}_{a_1}\}$ .

**Definition 4.4.** Let  $(\tilde{U}, \tilde{d}, A)$  be a soft expert metric space  $\tilde{x}_a \in \tilde{U}$ . A collection  $N(\tilde{x}_a)$  of soft expert points containing the soft expert point  $\tilde{x}_a$  is said to be neighbourhood of the soft expert point  $\tilde{x}_a$ , if there exists a non-negative soft expert real number  $\tilde{\varepsilon}$  such that  $\tilde{x}_a \in D(\tilde{x}_a, \tilde{\varepsilon}) \tilde{C}N(\tilde{x}_a)$ .  $SES(N(\tilde{x}_a))$  will be called a soft expert neighbourhood of the soft expert point  $\tilde{x}_a$ .

**Theorem 4.1.** Every soft expert open ball is a soft expert neighborhood of each of its soft experts points.

*Proof.* Let  $(\tilde{U}, \tilde{d}, A)$  be a soft expert metric space. Let us consider any soft expert open ball  $SES(D(\tilde{x}_a, \tilde{\varepsilon}))$  with centre  $\tilde{x}_a$  and radius  $\tilde{\varepsilon}$ . From the definition of soft expert neighbourhood it follows that  $SES(D(\tilde{x}_a, \tilde{\varepsilon}))$  is a soft expert neighbourhood of  $\tilde{x}_a$  in  $(\tilde{U}, \tilde{d}, A)$ . Let us consider any soft expert points  $\tilde{y}_{a_2} \in SES(D(\tilde{x}_{a_1}, \tilde{\varepsilon})) \tilde{<} \tilde{\varepsilon}$  other than  $\tilde{x}_{a_1}$  then we have must  $\tilde{0} \neq \tilde{d}(\tilde{x}_{a_1}, \tilde{y}_{a_2}) \tilde{<} \tilde{\varepsilon}$ . Choose  $\tilde{\varepsilon}_1$  with  $\tilde{0} \tilde{<} \tilde{\varepsilon}_1 \tilde{<} \tilde{\varepsilon} - \tilde{d}(\tilde{x}_{a_1}, \tilde{y}_{a_2})$ . Then  $\tilde{\varepsilon}_1$  is a non-negative soft expert real number. Now each  $\tilde{z}_{a_3} \in SES(D(\tilde{y}_{a_2}, \tilde{\varepsilon}))$ , we have  $\tilde{d}(\tilde{y}_{a_2}, \tilde{z}_{a_3})$ . Now by SEM4), we get,

$$\begin{aligned} \tilde{d}(\tilde{x}_{a_1}, \tilde{z}_{a_3}) &\tilde{\leq} \tilde{d}(\tilde{x}_{a_1}, \tilde{y}_{a_2}) + \tilde{d}(\tilde{y}_{a_2}, \tilde{z}_{a_3}) \\ &\tilde{\leq} \tilde{d}(\tilde{x}_{a_1}, \tilde{y}_{a_2}) + \tilde{\varepsilon}_1 \\ &\tilde{\leq} \tilde{\varepsilon}. \end{aligned}$$

$SES(D(\tilde{x}_{a_1}, \tilde{\varepsilon}))$  is a soft expert neighbourhood of each of its soft expert points.  $\square$

**Definition 4.5.** Let  $(F, B)$  be a soft expert subset in a soft expert metric space  $(\tilde{U}, \tilde{d}, A)$ . Then a soft expert point  $\tilde{x}_a$  is said to be interior point of the soft expert set  $(F, B)$ . If  $\exists$  a non-negative soft expert real number  $\tilde{\varepsilon}$  such that  $\tilde{x}_a \in D(\tilde{x}_a, \tilde{\varepsilon}) \tilde{C}SEP(F, B)$ .

**Definition 4.6.** Let  $(F, B)$  be a soft expert subset in a soft expert metric space  $(\tilde{U}, \tilde{d}, A)$ . Then the interior of the soft expert set  $(F, B)$  is defined to be the set consisting of all interior points  $(F, B)$ . The interior of the soft expert set  $(F, B)$  is denoted by  $(F, B)^\circ$ .

**Definition 4.7.** Let  $(\tilde{U}, \tilde{d}, A)$  be a soft expert metric space and  $(F, B)$  be a soft expert subset of  $\tilde{U}$ .  $(F, B)$  is called a soft expert open subset of  $\tilde{U}$  and  $\tilde{\varepsilon}$  non-negative soft expert real number if;  $D(\tilde{x}_a, \tilde{\varepsilon}) \tilde{C}(F, B) \quad \forall \tilde{x}_a \in (F, B)$ .

**Definition 4.8.** Let  $(\tilde{U}, \tilde{d}, A)$  be a soft expert metric space. A soft expert set  $(G, R) \tilde{C} \tilde{U}$ , is said to be "soft expert closed in  $\tilde{U}$  with respect to  $\tilde{d}$  " if its complement  $(G, R)^c$  is soft expert open set in  $(\tilde{U}, \tilde{d}, A)$ .

**Theorem 4.2.** In a soft expert metric space every soft expert open ball is a soft expert open set.

*Proof.* Let  $D(\tilde{x}_a, \tilde{\varepsilon})$  be soft expert open ball with center  $\tilde{x}_a$  and radius  $\tilde{\varepsilon}$  in a soft expert metric space  $(\tilde{U}, \tilde{d}, A)$ . We are proving that  $D(\tilde{x}_a, \tilde{\varepsilon})$  is a soft expert open set. Clearly  $SES(D(\tilde{x}_a, \tilde{\varepsilon}))$  is generated by the set of all soft expert point of  $D(\tilde{x}_a, \tilde{\varepsilon})$  and all of them are interior points of  $SES(D(\tilde{x}_a, \tilde{\varepsilon}))$ . Thus  $SES(D(\tilde{x}_a, \tilde{\varepsilon}))$  is a soft expert open set in the soft expert metric space  $(\tilde{U}, \tilde{d}, A)$ .

□

## 5. Conclusion

In this study firstly, we have introduced the definition of soft expert point in soft expert sets and using this we have given the definition of soft expert metric. Following this soft expert open ball, soft expert closed ball, soft expert neighborhood, theorem of soft expert open sets and soft expert closed sets are studied. This may lead to an ample scope on soft expert metric spaces in soft expert set setting.

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