

UNIVERSITI PUTRA MALAYSIA

FAST FINITE DIFFERENCE TIME DOMAIN ALGORITHMS FOR SOLVING ANTENNA APPLICATION PROBLEM

MOHAMMAD KHATIM BIN HASAN

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FAST FINITE DIFFERENCE TIME DOMAIN ALGORITHMS FOR SOLVING ANTENNA APPLICATION PROBLEM

By

MOHAMMAD KHATIM BIN HASAN

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DEDICATION

To My Mother, Aishah Arshaad, My Late Father, Hasan Salleh, My Wife, Rini Roslina Amir, To ALL My Brothers and Sisters, My Mother-in-law, Bahia Samion, My Father-in-law, Amir Husin and My Grandfather, Samingon...Thank You for everything





Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

FAST FINITE DIFFERENCE TIME DOMAIN ALGORITHMS FOR SOLVING ANTENNA APPLICATION PROBLEM

 $\mathbf{B}\mathbf{y}$

MOHAMMAD KHATIM BIN HASAN

March 2008

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Faculty: Computer Science and Information Technology

This thesis describes the implementations of new parallel and sequential algorithms for electromagnetic wave propagation from a monopole antenna. Existing method, known as FDTD needs a very long processing time to solve this problem. The objective of the thesis is to develop new sequential and parallel algorithms that are faster than the standard Finite Difference Time Domain method. In this thesis, a SMP machine, the Sun Fire V1280 using six existing processors is used to solve 1D and 2D free space Maxwell equations with perfectly conducting boundary and absorbing boundary conditions. Complexity reduction approach concept is used to develop these algorithms. This approach split the solution domain into $\frac{1}{3}$ and $\frac{2}{3}$ compartments in 1D case and $\frac{1}{9}$ and $\frac{8}{9}$ compartments in 2D cases. Only $\frac{1}{3}$ and $\frac{1}{9}$ parts of the solution domain are solved in the main looping construct for problem in 1D and 2D, while the remaining points are solved outside the loop. The



solutions to both parts are discussed in details in this thesis. These new parallel and sequential finite difference time domain (FDTD) algorithms yield from $O(h^2)$, ordinary $O(h^4)$ and weighted average $O(h^4)$ centered difference discretization using direct-domain and temporary-domain are used to solve problems mentioned above. In parallel implementation, techniques such as static scheduling, data decomposition and load balancing is used. Based on experimental results and complexity analysis, these new sequential and parallel algorithms are compared with the standard sequential and parallel FDTD algorithms, respectively. Results show that these new sequential and parallel algorithms run faster than the standard sequential and parallel FDTD algorithms. Beside that, formulation of a new higher accuracy second order method, which is called improved high speed low order finite difference time domain (IHSLO-FDTD) with direct-domain and temporary-domain are also proposed to solve the same problem are also described. Results show that, the IHSLO-FDTD with direct-domain and temporary-domain approaches are more efficient and economical. In general, almost all new proposed methods are more economical and run faster (except the Weighted Average High Speed High Order Finite Difference Time Domain (WAHSHO-FDTD) in directdomain and temporary-domain for 1D case) compared to the standard FDTD method for 1D and 2D case especially for IHSLO-FDTD.



Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

ALGORITMA-ALGORITMA DOMAIN MASA BEZA TERHINGGA PANTAS UNTUK MENYELESAIKAN MASALAH APLIKASI ANTENA

Oleh

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Tesis ini menerangkan tentang implementasi beberapa algoritma selari dan berjujukan terbaru khususnya untuk menyelesaikan masalah perambatan gelombang elektromagnet dari sebuah antena monopol. Kaedah sedia ada yang dikenali sebagai Domain Masa Beza Terhingga (DMBT) memerlukan masa yang panjang untuk menyelesaikan masalah yang dinyatakan tadi. Objektif tesis ini adalah untuk menghasilkan algoritma berjujukan dan selari terbaru yang lebih pantas dari algoritma DMBT piawai. Dalam tesis ini, Sistem komputer multipempropses simetri, Sun Fire V1280 menggunakan enam buah pemproses sedia ada untuk menyelesaikan persamaan Maxwell ruangan bebas satu dan dua dimensi dengan syarat sempadan pengkonduksi sempurna dan sempadan menyerap. Pendekatan pengurangan kompleksiti digunakan di dalam pembinaan algoritma-algoritma ini. Pendekatan ini memisahkan sebahagian penyelesaian kepada $\frac{1}{3}$ dan $\frac{2}{3}$ dalam do-



main penyelesaian 1D, manakala $\frac{1}{9}$ dan $\frac{8}{9}$ dalam domain penyelesaian 2D . Hanya $\frac{1}{3}$ dan $\frac{1}{9}$ bahagian penyelesaian diselesaikan di dalam gegelung penyelesaian utama masing-masing untuk masalah 1D dan 2D, dan yang selebih bahagian penyelesaian diselesaikan diluar dari gegelung penyelesaian utama tersebut. Kaedah penyelesaian bagi kedua-dua bahagian ada dibincangkan dengan terperinci di dalam tesis ini. Beberapa algoritma selari dan berjujukan DMBT terbaru hasil dari pendiskretan beza pusatan dengan peringkat pangkasan $O(h^2)$, $O(h^4)$ biasa dan $O(h^4)$ purata berpemberat menggunaan pendekatan domain-terus dan domainsementara diimplementasi dalam menyelesaikan masalah yang diterangkan tadi. Pada implementasi secara selari, beberapa teknik seperti penskedulan statik, pempartisian data dan pengimbang beban digunakan. Berasaskan kepada keputusan eksperimen termasuk kompleksiti pengiraan, beberapa algoritma berjujukan dan selari DMBT terbaru dibandingkan dengan algoritma berjujukan dan selari DMBT piawai. Perumusan satu kaedah peringkat dua yang lebih jitu terbaru Peringkat Rendah Berkelajuan Tinggi Domain Masa Beza Terhingga yang diperbaiki (PRBT-DMBT diperbaiki) dengan pendekatan domain-terus dan domainsementara juga diperkenalkan dan digunakan untuk menyelesaikan persamaan Maxwell yang sama juga diterangkan. Keputusan eksperimen menunjukkan bahawa kaedah PRBT-DMBT diperbaiki adalah lebih berkesan dan ekonomi berbanding kaedah-kaedah yang sebelumnya. Keputusan ini disokong oleh analisis kekompleksan pengiraannya. Secara keseluruhan, dapatlah dikatakan bahawa hampir kesemua kaedah terbaru adalah lebih ekonomi (kecuali kaedah Purata Berpemberat Peringkat Tinggi Berkelajuan Tinggi Domain Masa Beza Terhingga (PBPTBT-DMBT) domain-terus dan domain-sementara bagi masalah 1D) berbanding kaedah DMBT bagi kes 1D dan 2D terutamanya kaedah PRBT-DMBT diperbaiki.



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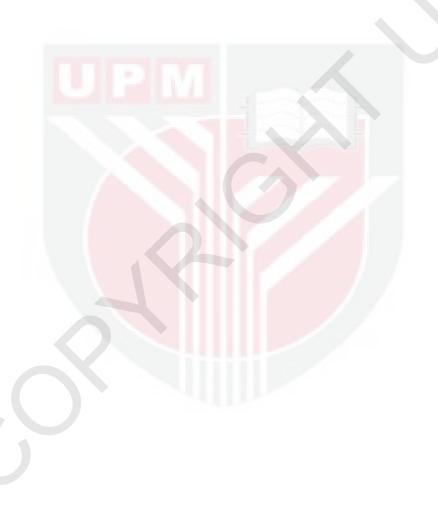
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DECLARATION

I declare that the thesis is my original work, except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

MOHAMMAD KHATIM BIN HASAN

Date: 7 APRIL 2008



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LIST OF ABBREVIATIONS

 $O(h^m)$ Truncation error of order *m* Electric permittivity ε Magnetic permeability μ Electric permittivity for free space \mathcal{E}_0 Magnetic permeability for free space μ_0 Conductivity ρ_{v} EElectric field В Magnetic flux Magnetic field HVelocity of light cAngular frequency Wave frequency Wave number k Weighted Average parameter Imaginary part of a complex number \mathfrak{I} Processor SIMD Single instruction multiple data Multiple instruction multiple data **MIMD** Finite Difference Time Domain **FDTD** Parallel implementation of Finite Difference Time Domain P-FDTD **HO-FDTD** High Order Finite Difference Time Domain



PHO-FDTD	Parallel implementation of High Order Finite Difference Time Domain
HSLO-FDTD (DD)	High Speed Low Order Finite Difference Time Domain using direct domain approach
IHSLO-FDTD (DD)	Improved High Speed Low Order Finite Difference Time Domain using direct domain approach
HSHO-FDTD (DD)	High Speed High Order Finite Difference Time Domain using direct domain approach
WAHSHO-FDTD (DD)	Weighted Average High Speed High Order Finite Difference Time Domain using direct domain approach
HSLO-FDTD (TD)	High Speed Low Order Finite Difference Time Domain using temporary domain approach
IHSLO-FDTD (TD)	Improved High Speed Low Order Finite Difference Time Domain using temporary domain approach
HSHO-FDTD (TD)	High Speed High Order Finite Difference Time Domain using temporary domain approach
WAHSHO-FDTD (TD)	Weighted Average High Speed High Order Finite Difference Time Domain using temporary domain approach
UHSLO-FDTD (DD)	Ultra High Speed Low Order Finite Difference Time Domain using direct domain approach
UIHSLO-FDTD (DD)	Ultra Improved High Speed Low Order Finite Difference Time Domain using direct domain approach
UHSHO-FDTD (DD)	Ultra High Speed High Order Finite Difference Time Domain using direct domain approach
WAUHSHO-FDTD (DD)	Weighted Average Ultra High Speed High Order Finite Difference Time Domain using direct domain approach
UHSLO-FDTD (TD)	Ultra High Speed Low Order Finite Difference Time Domain using temporary domain approach
UIHSLO-FDTD (TD)	Ultra Improved High Speed Low Order Finite Difference Time Domain using temporary domain approach



UHSHO-FDTD (TD) Ultra High Speed High Order Finite Difference Time

Domain using temporary domain approach

WAUHSHO-FDTD (TD) Weighted Average Ultra High Speed High Order Finite

Difference Time Domain using temporary domain

approach

RPA Remaining Point Approximation





CHAPTER 1

INTRODUCTION

1.1 Overview

Many advanced technologies rely on electromagnetic fields. The fields contribute a lot to a modern lifestyle of living. The transmission of electrical power for the purpose of communication is carried out by means of electromagnetic waves. The electrical power may be transmitted via free space or guiding conductors. When a quantity of electromagnetic wave is generated in unbounded space, it cannot remain at rest, but must travel as a wave until the energy is dissipated.

Advancement in computer technology has revolutionized the design of pilot products from "classical trial and error" method to "soft" and low-cost method, which is popularly known as the computer simulation method (Rice, 1995). This scenario has highlighted the importance of numerical simulation in most research and development in the area of science and technology.

Most physical phenomenon can be simulated via differential equation. The differential equation can be classified into two groups, which are Ordinary Differential Equations (ODEs) and Partial Differential Equations (PDEs). Analytical solutions to these equations are sometimes hard or impossible to determine. Therefore, numerical solution is the best alternative to approximate the solutions.



Electromagnetic wave propagation can be simulated via Maxwell equations. The equations consist of two components that exist alternately; namely the magnetic and electric fields. The computer simulation of electromagnetic field problems often requires powerful numerical solver due to the geometrical and physical complexities. The availability of fast and efficient solvers is crucial especially in such cases.

A well-known numerical solver that uses second order central difference approximation of Taylor series as the main ingredient has been proposed by Yee (1996), which is now popularly known as Finite Difference Time Domain (FDTD) method (Taflove, 1995). The method derives the "king crab couple" of Maxwell equations into the most simple form of approximate equation. These equations clearly exhibit a low "discrete" complexity characteristic of its algorithm. This scenario awards the method as the most practical method to be used in approximating Maxwell equations (Taflove & Hagness, 2005). However, the method needs a long processing time to produce results.

Over the last few decades, increasing number of various types of multiprocessor technology machines have been developed. With such systems, it is possible to design and develop algorithm that exploits the advantage of multiprocessor architecture (Rozita, 1994). User of such system tends to solve large problems, with the ambition to speed-up the program execution time but still produces accurate results.



To exploit such powerful machine, more researches are done to develop parallel algorithms that are suitable for such architecture. Following that scenario, existing serial algorithms are continuously converted into parallel algorithms. This conversion creates new researchable issues that do not exist previously in single processor machine architecture. This is because the conversion from serial algorithm to parallel algorithm is not always straightforward and in some cases the efficient parallel algorithms are completely different from the best serial algorithms for the same problems.

This computing paradigm has been used by several researchers on multiprocessor machines to develop parallel FDTD algorithm (Araujo et al., 2003; Fijany et al., 1995; Nguyen et al., 1994; Perlik et al., 1984; Varadarajan & Mittra, 1994; Zhenghui et al., 2002). This action has off course succeeded in speeding up the FDTD computational speed.

In recent decade, some researchers in finite difference area of research proposed some complexity reduction approach focusing mainly to speed-up the computational execution time, such as the Reduced Iterative Alternating Decomposition Explicit (RIADE) method (Sahimi & Khatim, 2001). The method succeeds in speeding up the Iterative Alternating Decomposition Explicit (IADE) method to solve heat conduction equation. Since then, RIADE method have been implemented in various applications (Mohammad Khatim et al., 2001; Mohammad Khatim et al., 2003; Mohammad Khatim & Bahari, 2003).



Meanwhile, another extraordinary simple concept which also uses complexity reduction approach has been widely used to speed-up computational speed of various finite difference and finite element methods through half and quarter sweep approaches. These approaches have improved the computational speed of the methods. The motivation of developing the half and quarter sweep has been inspired by Abdullah (1991) with his Explicit Decouple Group (EDG) method. The extension of this method, is however, developed by Othman and Abdullah (2000) via their Modified Explicit Group (MEG) method. Both EDG and MEG methods have successfully speeded-up the computational speed since they have reduced the complexity of the original methods by half and three-quarter, respectively. Since then, various half and quarter sweep methods have arose in numerical fields of research (Jumat & Abdul Rahman, 1999; Mohamed, 1999; Sulaiman et al., 2004).

Inspired by these findings, and since the objective of this research is to develop efficient (fast with tolerable accuracy) algorithms for free space electromagnetic wave propagation, some sequential algorithms utilizing the quarter sweep concept will be developed. Besides developing new sequential algorithms that are fast in computing, the opportunity to utilize the computing power offered by multiprocessor technology should also be taken to further enhance the speed of the new algorithms.

1.2 Problem Statement

Most problems in free space radio wave data transmission cannot be solved analytically and require a numerical solution. This is because the solution of the

