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MATHEMATICAL MODELING FOR BOUNDARY LAYER FLOW OVER A STRETCHING OR SHRINKING CYLINDER

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By

NAJWA BINTI MOHD NAJIB

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Master of Science

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To My Beloved Husband, Adorable Son, Lecturers and Friends

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Master of Science

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August 2014

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In this study, the steady boundary layer flow, mass transfer and heat transfer of a cylinder near the stagnation point over a stretching or shrinking sheet are investigated numerically. The governing nonlinear partial differential equations are transformed into a system of nonlinear ordinary differential equations using a similarity transformation which is then solved numerically using a shooting method. The numerical results are presented in tables or graphs for the skin friction coefficient, the local Nusselt number and the local Sherwood number as well as the velocity, temperature and concentration profiles for a range of various parameters such as stretching or shrinking parameter ε, curvature parameter γ, Prandtl number Pr, Schmidt number Sc, reaction rate parameter β and suction or injection parameter f_0. It is observed that the skin friction coefficient, the local Nusselt number which represents the heat transfer rate at the surface and the local Sherwood number are significantly influenced by these parameters. The results indicate that dual solutions exist for a shrinking cylinder. The results also indicate that suction increases the range in which the solution exists, while injection acts in the opposite manner. On the other hand, the increase of the curvature parameter cause the skin friction coefficient, the heat and mass transfer rate at the surface to increase.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

PEMODELAN BERMATEMATIK BAGI ALIRAN LAPISAN SEMPADAN MELALUI SILINDER YANG MEREGANG ATAU MENGECUT

Oleh

NAJWA BINTI MOHD NAJIB

Ogos 2014

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Dalam kajian ini, aliran lapisan sempadan mantap, pemindahan jisim dan pemindahan haba bagi silinder berdekatan titik genangan pada permukaan meregang atau mengecut dikaji secara berangka. Persamaan pembezaan separa tak linear dijelmakan kepada sistem persamaan pembezaan biasa tak linear menggunakan penjelmaan keserupaan yang seterusnya diselesaikan secara berangka dengan menggunakan kaedah tembakan. Keputusan berangka dipersembahkan dalam bentuk jadual atau graf bagi pekali geseran kulit, nombor Nusselt setempat dan nombor Sherwood setempat serta profil halaju, suhu dan kepekatan dalam julat tertentu pelbagai parameter, seperti parameter regangan atau mengecut ε, parameter kelengkungan γ, nombor Prandtl *Pr*, nombor Schmidt *Sc*, parameter kadar tindak balas β dan parameter sedutan atau semburan f_0. Didapati bahawa pekali geseran kulit, nombor Nusselt setempat yang mewakili kadar pemindahan haba pada permukaan dan nombor Sherwood setempat dipengaruhi oleh parameter-parameter tersebut. Keputusan menunjukkan bahawa wujudnya penyelesaian dual bagi silinder yang mengecut. Keputusan juga menunjukkan sedutan meningkatkan julat kewujudan penyelesaian mengecut, manakala semburan bertindak balas sebaliknya. Selain itu, peningkatan parameter kelengkungan menyebabkan pekali geseran kulit, kadar pemindahan haba dan jisim pada permukaan adalah meningkat.

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Last but not least, I would like to thank all for their helping in completing my thesis. May God bless all. Thank you.

I certify that a Thesis Examination Committee has met on 21 August 2014 to conduct the final examination of Najwa binti Mohd Najib on her thesis entitled "Mathematical Modeling for Boundary Layer Flow Over a Stretching or Shrinking Cylinder" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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This is to confirm that:

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LIST OF ABBREVIATIONS

C	concentration coefficient
C_∞	constant concentration in the free stream
C_f	skin friction coefficient
D	diffusion coefficient
e	exponential function
f	non-dimensional stream function
f_0	constant mass transfer parameter
k	thermal conductivity
L	characteristics length
Nu_x	local Nusselt number coefficient
Pr	Prandtl number
R	radius of cylinder
R_r	reaction-rate of the solute
Re	Reynolds number
Sc	Schmidt number
Sh	Sherwood number
T	fluid temperature in the boundary layer
T_0	ambient temperature at the leading edge
T_w	surface temperature
T_∞	free-stream temperature
u	velocity component in the x direction
u_e, u_w	free-stream fluid velocity
U, U_e , U_∞	constant
v	velocity component in the r direction
x	non-dimensional Cartesian coordinate along the surface
r	non-dimensional Cartesian coordinate normal to the surface

Greek symbols

α	thermal diffusivity
β	coefficient of thermal expansion
η	similarity variable
θ	non-dimensional temperature
ν	kinematic viscosity
μ	dynamic viscosity
ρ	fluid density
τ_w	shear stress at the surface
q_w	heat flux at the surface
ψ	stream function
ε	stretching or shrinking parameter
γ	curvature parameter
ϕ	non-dimensional mass

Subscripts

c	critical value
w	condition at the surface
∞	free-stream condition

Superscripts

′	differentiation with respect to η or r

CHAPTER 1

INTRODUCTION

1.1 Introduction

In this thesis, we consider the research theoretically without making the experiment. This thesis is focusing on the mathematical model, which models resemble reality by using mathematical language. Moreover, for example a model can be fixed, modified or sometimes might be use in research or being a reference to build a good model.

Boundary layer is the layer of fluid in the immediate vicinity of the bounding surface where the effects of viscosity cannot be ignored. The theory of viscosity was introduced by Ludwig Prandtl in 1904. Prandtl says that the flow can be divided by two which are inviscid flow at the main section and thin layers adjacent to body surface (boundary layer). In the thin layers, friction force need to be considered but for the outer thin layers, the friction forced can be neglected (Schlichting, 1979).

Heat is the thermal energy that flows between two samples of matter due to their difference in temperature. Heat transfer is the energy transfer caused by the difference temperature in a medium or across a medium. The heat transfer between two objects must be from higher temperature to lower temperature. Heat transfer occurs in three different phenomenon which are conduction, convection and radiation (Özisik, 1985). Heat conduction happened through solid, liquid and gas. The heat conduction happens when direct collision between higher and lower molecular kinetics energy. Heat convection involved heat transfer affected by fluid flow. On the other hand, radiation is the transfer of heat through the air (space) in wave electromagnet. However, this thesis only considered the heat transfer in free convection.

Mass transfer is the net movement of mass from one location to another location. This type of movement usually called a stream, phase, fraction or component. Mass transfer occurs in many processes, such as evaporation, drying, precipitation, adsorption, membrane filtration and distillation. Nonetheless, mass transfer is being used for different processes and mechanisms. For example, the phrase is commonly used in engineering processes while membrane filtration and distillation is used by the biologist. Some common examples of mass transfer processes are the evaporation of water from a pond to the atmosphere, the distillation of alcohol and the purification of blood in the kidneys and liver. On the other hand, the engineering processes that involves mass transfer operation such as separation of chemical components in distillation columns, absorbers i.e scrubbers, activated carbon beds and liquid extraction.

Prandtl number, Pr is a dimensionless number represented the ratio of momentum diffusivity (kinematic viscosity) to thermal diffusivity. It was proposed by Ludwig Prandtl in 1904. Pr is defined as

$$Pr = \frac{\mu c_\rho}{k} = \frac{\nu}{\alpha} = \frac{\text{viscous diffusion rate}}{\text{thermal diffusion rate}},$$

where ν is the kinematic viscosity, α is thermal diffusivity, μ is the dynamic viscosity, k is the thermal conductivity, c_ρ is the specific heat and ρ is the density. If the fluid is more viscous, the *Pr* is greater and the heat transfer will be less convective. *Pr* is directly influenced the boundary layer thickness and velocity. Specifically, the values of *Pr* = 0.7,1 and 7 are representing the air, electrolyte fluid and water respectively. The mass transfer of *Pr* is called Schmidt number, *Sc*.

1.2 Research Background

Fluid dynamics is about an object that travel or moving along the fluid. The fluid is either a liquid or a gas. A fluid will be deformed continuously under a shear stress. There are two types of fluid flow which are laminar flow and turbulent flow. However, in this thesis only laminar flow will be taken into consideration. In laminar flow the particles in the fluid will follow the streamlines, and the motion of the particles in the fluid will be predicted.

1.2.1 Stagnation Point Flow

Stagnation point is a point in a flow field where the local velocity of the fluid is zero. At the surface of the plate there exist stagnation points, where the fluids come to the rest by the plate.

Stagnation flow, describing the motion of fluid near the stagnation point where the fluid pressure, heat transfer and rates of mass deposition are highest. There is one streamline that divides the flow by half which are above and below the streamline. Above this streamline, all the flow goes over the plate while below this streamline the flow goes under the plate. Along the dividing streamline, the fluid motion is towards the plate.

1.2.2 Stretching or Shrinking Sheet

Crane (1970) was pioneering the research about boundary layer flow over a stretching sheet. This research is being investigated from many aspects such as stagnation point flow, the slip flow, effects of suction and injection that involved vertical and horizontal surface. Stretching sheet occurs when velocity at the boundary is away from a fixed point. The application of stretching sheet in engineering processes such as paper production and glass blowing. The final quality of the product is depending on rates of heat transfer at the stretching surface.

Recently, the flow over a shrinking sheet has attracted many researchers due to its applications in processes. Apart from that, Wang (2008) studied the viscous flow over a shrinking sheet. Shrinking sheet occurs when velocity at the boundary is moving towards a fixed point. The shrinking sheet problem is very important to the agriculture field. Some of the applications are the effects of capillary in small pores, behaviour of shrinking or stretching and hydraulic characteristics of clay for agriculture purposes.

1.2.3 Cylinder

The flow of an incompressible fluid past a cylinder is one of the mathematical models that exist in fluid dynamics. This concept of flow can be found in mathematical physics such as vector fields, coordinate transformations and the most important, the physical interpretations of mathematical results.

The potential flow around a circular cylinder is a solution for the flow of an inviscid, incompressible fluid around a cylinder that is transverse to the flow. A viscous flow past a cylinder will acquire vorticity in a thin boundary layer adjacent to the cylinder. Boundary layer separation can occur behind the cylinder and cause the lower pressure to drag the flow to the downstream of the cylinder.

1.3 Problem Statements

There are many researches that have been done about boundary layer flow in cylindrical case. Some of the issues about the cylinder are:

i. How the effects of chemical reaction on the stagnation point and mass transfer flow in stretching or shrinking cylinder?

ii. What are the effects of the stagnation point and heat transfer if surface heat flux is prescribed?

iii. What are the effects of cylinder parameter on the wall shear stress (skin friction) and surface heat flux (heat transfer from surface) as well as fluid velocity and temperature profiles when we use exponential similarity?

iv. How does the suction or injection affect the flow and heat transfer characteristics?

1.4 Objectives and Scope of the Thesis

To analyse the boundary layer and heat transfer flows over a stretching or shrinking cylinder near the stagnation point by solving the mathematical models for the following problems:

i. Stagnation point flow and mass transfer with chemical reaction past a stretching or shrinking cylinder

ii. Stagnation point flow and heat transfer over a stretching or shrinking cylinder with prescribed surface heat flux

iii. Stagnation point flow and heat transfer over an exponentially shrinking cylinder

iv. Stagnation point flow and heat transfer over an exponentially shrinking cylinder with suction and injection

1.5 Significance of Study

The stretching and shrinking cylinder have significant impact on the technology applications in fluid dynamics. The technological applications due to the flow and heat transfer along a stretching cylinder included hot rolling, wire drawing, drawing of plastic films, paper and glass fibre production. Process of manufacturing in the polymer industries such as a polymer is extruded continuously from a die with a tacit assumption that the fibre is inextensible. The cooling of a long metallic wire in a bath (an electrolyte) is another physical application to the stretching sheet cases. On the other hand, the situation towards a shrinking sheet occurs on a rising or shrinking balloon.

The mass transfer problems have great importance in extending the theory of separation processes and chemical reaction. Also, the addition of mass transfer phenomenon to the heat transfer has received various attentions of new researchers for its enormous applications in chemical industries, reservoir engineering and many other technological processes. There are some transport processes with surface mass transfer for example injection or suction in industry where the buoyancy force arises from thermal diffusion caused by the temperature gradient, such as polymer fibre coating or the coating of wires.

Due to the growing use of the boundary layer in stretching or shrinking sheet in manufacturing and processing industries, we attempt to understand the friction and heat transfer characteristics itself. This is very important for the engineers and researchers to understand and be familiar with the flow behaviour and properties of such fluids. The nature of the flow and heat transfer of stretching or shrinking cylinder near stagnation point is use to predict the flow behaviour in the equipment process. Lastly, the kinematics of both stretching or shrinking and the simultaneous heating and cooling during such processes has a decisive influence on the quality of the final products.

1.6 Outline of the Thesis

Overall this thesis is divided into seven chapters. Chapter 1 contains the introduction, research background, problem statements, objectives and scope of the study, significance of study and lastly the outline of the thesis. Chapter 2 consists of the literature review regarding the problems stated in the thesis and the numerical method that had been used to solve the problems numerically. This thesis is based on Ishak (2009)'s where we are using the similarity transformations of this problem since we are dealing with the cylindrical case. Therefore, the general formulation for Ishak (2009) was shown in detail in Chapter 3. Meanwhile, Chapter 4 until 7 will be discussed further about the four problems that had been done in this study.

In Chapter 4, we studied the stagnation point flow and mass transfer with chemical reaction past a stretching or shrinking cylinder. Chapter 5 discusses on the stagnation point flow over a stretching or shrinking cylinder with prescribed surface heat flux. In Chapter 6, we investigate the stagnation point flow and heat transfer over an exponentially stretching or shrinking cylinder while in Chapter 7 we extend the work in Chapter 6 to the case where the cylinder is permeable, i.e. when there is suction or injection (blowing) through the cylinder surface.

Lastly, Chapter 8 contains the summary of the study as well as possible further research that can be carried out.

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