UNIVERSITI PUTRA MALAYSIA

INTERACTION BETWEEN INCLINED AND CURVED CRACKS PROBLEMS IN PLANE ELASTICITY

MOHD RADZI ARIDI

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INTERACTION BETWEEN INCLINED AND CURVED CRACKS PROBLEMS IN PLANE ELASTICITY

By

MOHD RADZI ARIDI

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Master of Science

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INTERACTION BETWEEN INCLINED AND CURVED CRACKS PROBLEMS IN PLANE ELASTICITY

By

MOHD RADZI ARIDI

June 2014

Chair: Nik Mohd Asri Nik Long, PhD
Institute: Institute for Mathematical Research

In this thesis, the interaction between inclined and curved cracks problem in plane elasticity is formulated into the hypersingular integral equations using the complex variable function method. Then, using the curved length coordinate method, the cracks are mapped into a straight line which require less collocation points, hence give faster convergence.

In order to solve the equations numerically, the quadrature rules are applied and we obtained the unknown coefficients with $M+1$ collocation points. The obtained unknown coefficients will later be used in calculating the stress intensity factors. Firstly, we investigated the problems between straight and curved cracks problem in plane elasticity. The results give good agreements with the existence results. Then, we extended the problem for the interaction between inclined and curved cracks problem in plane elasitcity. For numerical examples, four types of loading modes have been presented : Mode I, Mode II, Mode III and Mix Mode.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk Sarjana Sains.

HUBUNGAN ANTARA MASALAH REKAHAN CONDONG DAN MELENGKUNG PADA SATAH ELASTIK

Oleh

MOHD RADZI ARIDI

Jun 2014

Pengerusi: Nik Mohd Asri Nik Long, PhD
Institut : Institut Penyelidikan Matematik

Dalam tesis ini, hubungan antara rekahan menaik dan melengkung pada satah elastik diformulakan kepada persamaan pengamiran hipersingular dengan menggunakan kaedah fungsi pembolehubah kompleks. Kemudian, dengan menggunakan kaedah koordinat panjang lengkung, retakan dipetakan pada satu garis lurus yang hanya memerlukan titik kolokasi yang sedikit, seterusnya menghasilkan penumpuan yang cepat.

Untuk menyelesaikan persamaan secara berangka, kaedah kuadratur digunakan untuk memperoleh pekali yang tidak diketahui dengan menggunakan sebanyak $M + 1$ titik kolokasi. Pekali yang terhasil digunakan untuk mengira faktor keamatan tekanan. Sebagai permulaan, hubungan rekahan lurus dan melengkung pada satah elastik dikaji. Keputusan yang baik dicapai berdasarkan kajian sebelum ini. Kemudian, kajian diteruskan untuk hubungan antara rekahan menaik dan melengkung pada satah elastik. Sebagai contoh penyelesaian berangka, empat jenis mod muatan diambil kira: Mod I, Mod II, Mod III and Mod Campuran.

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I certify that a Thesis Examination Committee has met on **Date of Viva voce** to conduct the final examination of **Mohd Radzi Aridi** on his thesis entitled "**Interaction Between Inclined and Curved Cracks Problems in Plane Elasticity**" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the **Master of Science in Mathematics**.

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Associate Professor
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Professor
School of Mathematics
Universiti Sains Malaysia
Malaysia
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NORITAH OMAR, PhD
Associate Professor and Deputy Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

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Associate Professor
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Universiti Putra Malaysia
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LIST OF ABBREVIATIONS

$LEFM$	linear elastic fracture mechanics
$EPFM$	elastic plastic fracture mechanics
$SIFs$	stress intensity factors
WS	weakly singular
$S1$	singular type 1
$S2$	singular type 2
HS	hypersingular
$R1A$	Fredholm type 1
$R1B$	Fredholm type 2
$R2$	Fredholm type 3
K	Stress intensity factor parameter
K_1	Stress intensity factor for Mode I
K_2	Stress intensity factor for Mode II
K_3	Stress intensity factor for Mode III
N	Normal expression
T	Tangential expression
$N + iT$	Right hand term expression
z	Dislocation point

CHAPTER 1

INTRODUCTION

1.1 Preliminaries

Fracture mechanics is one of the engineering field of mechanics concerned with the study of crack propagation in materials. It also known as solid mechanic that deals with the mechanical behaviour of cracked bodies. Using methods of analytical solid mechanics, one can calculate the driving force on a crack. Those of the experimental solid mechanics characterize the resistance of materials to fracture.

Predicting the fatigue life of cracked components is one of the most important tasks in engineering of fracture mechanics. The fracture mechanics plays as an important tool in improving the mechanical performance of mechanical structures. Based on the theories of elasticity and plasticity, it applies the stress and strain to the materials in order to predict the mechanical failure of bodies. Fracture mechanics can be divided into two main categories, Linear Elastic Fracture Mechanics (LEFM) and Elastic Plastic Fracture Mechanics (EPFM).

LEFM deals with the basic theory of fracture with sharp cracks in elastic bodies. By assuming the material is isotropic and linear elastic, the stress field near the crack tip can be evaluated using the theory of elasticity. The crack will grow when the stresses near the crack tip exceed the material fracture toughness. However, LEFM is limited for small-scale yielding only, when the inelastic deformation is small compared to the size of the crack. In order to overcome this limitation, EPFM will be used if large zones of plastic deformation develop before the crack grows. Under EPFM, by assuming the material is isotropic and elastic-plastic, the strain energy fields or opening displacement near the crack tips can be evaluated. The crack will grow when the energy or opening exceeds the critical value.

Development of fracture mechanics understanding based on linear elasticity can be found from the pioneer work by Inglis (1913), Griffith (1920), Westgaard (1939) and Irwin (1957). Inglis (1913) done a first major step in the direction of quantification of the effects of crack-like defects. He observed the stress analysis of an elliptical hole in an infinite linear elastic plate loaded at its outer boundaries and modeled the crack-like discontinuity by making the minor axis very much less than the major. However, his solution poses difficulty for the limit of a perfectly sharp crack. In other words, the stresses approach infinity at the crack tip. In order to overcome the problem, Griffith (1920) extended Inglis's solution and employed an energy balance approach rather than focusing on the crack tip stresses directly. He observed that when a crack has grown into a solid, a region of material adjacent to the free surface is unloaded and its strain energy released.

Westgaard (1939) derived the asymptotic solution for a stationary crack loaded dynamically using the complex stress functions. His method provides a powerful technique for solving the infinite linear elastic plane containing a crack or array of cracks. Based on Griffith's work, Irwin (1957) developed the energy release rate into a more useful form for engineering problems. Using Westgaard's approach, he described the stresses and displacements near the crack tip by a single parameter. This crack tip characterizing parameter later become known as the stress intensity factors. As a result, many researchers paid attention on evaluating the stress intensity factors and computed data of the stress intensity factors have been mainly used in evaluating the safety of components. In relation to the stress intensity factors, a set of rules can be obtained for predicting the fatigue life of the cracked components.

1.2 Analysis of stress

Assume that a volume V of arbitrary shape and the forces acting on the infinite small volume element dV have the form $\overrightarrow{\Phi}dV$ where $\overrightarrow{\Phi}$ is some finite vector for any point (x, y, z). A body force, acting on a volume element dV may be represented by a vector $\overrightarrow{\Phi}dV$. It applies to some point of the element dV and must be understood in the sense that the resultant force vector $\overrightarrow{\Psi}$ acting on any finite volume V of the body. The resultant force may be represented by a triple integral (Muskhelishvili, 1957)

$$\overrightarrow{\Psi} = \iiint_V \overrightarrow{\Phi}dV = \iiint_V \overrightarrow{\Phi}dxdydz \tag{1.1}$$

Similarly, the resultant moments of these forces about the axes $0_x, 0_y, 0_z$ of an orthogonal, are given by

$$\begin{aligned} M_x &= \iiint_V (y\sigma_{zn} - z\sigma_{yn})dxdydz \\ M_y &= \iiint_V (z\sigma_{xn} - x\sigma_{zn})dxdydz \\ M_z &= \iiint_V (x\sigma_{yn} - y\sigma_{xn})dxdydz \end{aligned} \tag{1.2}$$

where $\sigma_{xn}, \sigma_{yn}, \sigma_{zn}$ are the components of the vector $\overrightarrow{\Phi}$. The components of the stress vector acting on the plane normal to 0_x are denoted by $\sigma_{xx}, \sigma_{xy}, \sigma_{xz}$ where σ_{xx} is the normal stress component acting on this plane while σ_{xy}, σ_{xz} are the tangential or shear stress components. Similarly, $\sigma_{yx}, \sigma_{yy}, \sigma_{yz}$ and $\sigma_{zx}, \sigma_{zy}, \sigma_{zz}$ are the stress components acting on the plane normal to 0_y and 0_z respectively. Noted that the subscript notation for a stress component σ_{ab} represent the stress on the plane a along b direction.

1.3 Stress intensity factors

From the viewpoint of fracture analysis, the stress intensity factors is the coefficient obtainable from the singular stress field in the front of a crack. The behavior of the singular stress field was actually observed by Muskhelishvili (1957) for the

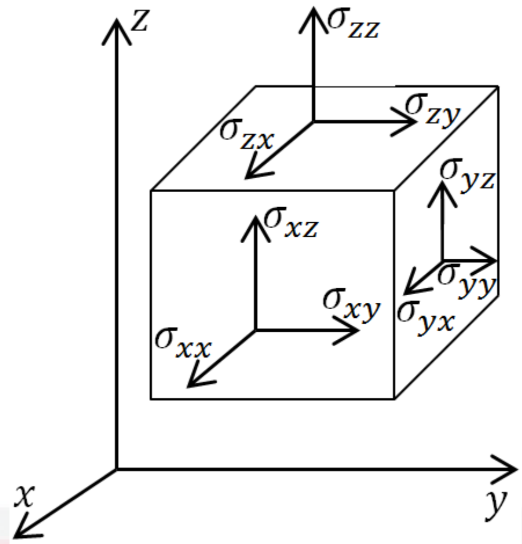

Figure 1.1: The stress components (Muskhelishvili, 1957).

collinear crack case. His pioneering book contains the fundamental equations of the mechanics of elastic bodies and general formula for elementary applications.

In predicting the stress behaviour at the crack tip, the stress intensity factor, K is used in fracture mechanics. Since the pioneer work of Irwin (1957), the stress intensity factors is a major achievement in the theoretical foundation of LEFM. In other words, it is usually used to a homogeneous and linear elastic material which give a small scale yielding at the crack tip. Under LEFM, the stress distribution, σ_{ij} near the crack tip in polar coordinates (r, θ) with origin at the crack tip is given by (Tada et al., 2000)

$$\sigma_{ij}(r,\theta) = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) \tag{1.3}$$

where K is the stress intensity factor and f_{ij} is a dimensionless quantity that depends on the load and geometry.

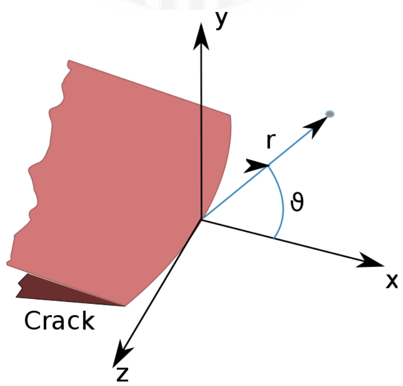

Figure 1.2: Polar coordinate at the crack tip (Tada et al., 2000).

Irwin (1957) proposed three modes of fracture based on the relative movement of the faces of the crack. From Figure 1.3, the three load types are categorized as mode I, mode II and mode III. Mode I is the normal or tensile mode where the crack surfaces move directly apart while mode II is the shear or sliding mode where the crack surfaces slide over one another in a direction perpendicular to the leading edge of the crack. Mode III is the tearing or antiplane shear mode where the crack surfaces move relative to one another and parallel to the leading edge of the crack.

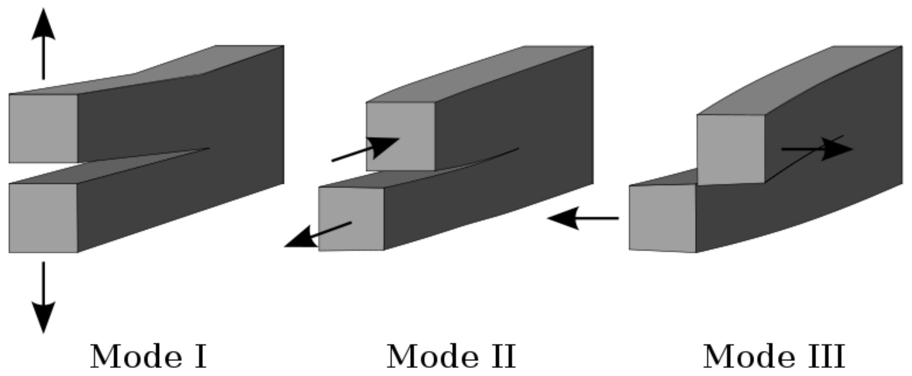

Figure 1.3: The mode I, mode II and mode III crack loading (Rooke and Cartwright, 1976).

The stress intensity factor for mode I is presented as K_1 and applied to the crack opening mode while K_2 and K_3 represented the stress intensity factors for mode II (shear mode) and mode III (tearing mode) respectively. These factors are defined as

$$\begin{aligned} K_1 &= \lim_{r\to 0} \sqrt{2\pi r}\sigma_{yy}(r,0) \\ K_2 &= \lim_{r\to 0} \sqrt{2\pi r}\sigma_{yx}(r,0) \\ K_3 &= \lim_{r\to 0} \sqrt{2\pi r}\sigma_{yz}(r,0) \end{aligned} \tag{1.4}$$

1.4 Problem statements

In this thesis, we investigate the crack problems between straight or inclined with curved cracks in plane elasticity. Based on work by Chen (2003) and using the superposition techniques described by Kachanov (1987), the multiple cracks problems can be solved numerically.

The research questions of these problems are:

1. how to formulate the multiple cracks problem between straight or inclined with curved crack?
2. how to obtain the hypersingular integral equations for the above mentioned problems?
3. how to map the multiple cracks into a real axis?

4. how to construct the quadrature rule to obtain the unknown coefficients?

1.5 Objectives

Based on the identified problem, the objectives of this investigation are:

1. to formulate the mathematical model for the interaction between straight or inclined crack with curved crack.
2. to obtain the hypersingular integral equations for the above mentioned problems.
3. to map the multiple cracks into a real axis by using the curved length coordinate.
4. to develop the numerical scheme for solving the hypersingular integral equation appears in these problems.

1.6 Scope of study

The scope of this research will be mainly focused on formulation of the hypersingular integral equation for two different crack problems. Four types of loading modes will be considered in this research which are Mode I, Mode II, Mode III and Mix Mode to represent the numerical results.

1.7 Outline of thesis

This thesis covers six chapters with the following contents:

Chapter 1 gives a brief introduction of this study in viewpoint of fracture analysis. Some keywords, for example LEFM, EPFM and SIFs are introduced. The research questions and the objectives for this research are also included in this chapter. Chapter 2 focuses on the previous work done by many researchers. This chapter reviews the method for solving cracks problems such as singular integral equations, Fredholm integral equation, hypersingular integral equation, finite element method and boundary element method. Chapter 3 will cover the methodology for solving the crack problem. A compact survey for plane elasticity crack problems is also carried out. The concept of the complex variable function method is emphasized. The hypersingular integral equations and superposition principle are introduced. The right hand term for the equation is also discussed in this chapter. Chapter 4 discusses the interaction between straight and curved cracks in plane elasticity. In this chapter, the formulation of the problem, curved length coordinate method, quadrature rule and stress intensity factor are presented. The interaction between inclined and curved cracks in plane elastivity is discussed in chapter 5. The method approach in this chapter follows as Chapter 4 with different position between the two cracks. Lastly, Chapter 6 contains the summary of the study and the suggestion for future research.

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