UNIVERSITI PUTRA MALAYSIA

DEVELOPMENT OF COMPUTATIONAL METHOD TO DETERMINE MAXIMUM VOID RATIO OF SAND-SOIL PARTICLES USING GRAVITATIONAL SPHERE PACKING METHOD

MOHAMMAD MAHDI ROOZBAHANI

DEVELOPMENT OF COMPUTATIONAL METHOD TO DETERMINE MAXIMUM VOID RATIO OF SAND-SOIL PARTICLES USING GRAVITATIONAL SPHERE PACKING METHOD

By

MOHAMMAD MAHDI ROOZBAHANI

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

October 2012

DEDICATIONS

I would like to dedicate this project to all those who have helped me to complete it.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

DEVELOPMENT OF COMPUTATIONAL METHOD TO DETERMINE MAXIMUM VOID RATIO OF SAND-SOIL PARTICLES USING GRAVITATIONAL SPHERE PACKING METHOD

By

MOHAMMMAD MAHDI ROOZBAHANI

October 2012

Chairman: Professor Bujang Bin Kim Huat, PhD

Faculty: Engineering

Abstract: These days, there has been an increasing interest in virtual reality simulation of the geotechnical field. More recently, literature has emerged that offers linear experimental Equation which positively correlates the maximum and minimum void ratio of sandy-soil particles. However, far too little attention has been paid to virtually simulate one of this parameter in order to rapidly compute these main parameters of the relative density without any need to do experimental works. Gravitational sphere packing which arranges non-overlapping spheres within a confined space has powerful capability to simulate the interaction of large number of particles. The goal of this research is to employ gravitational sphere packing to compute maximum void ratio of sandy-soil particles with particle size distribution as input data.

In this study, pre-processing, processing, and post-processing codes of the op-

timized gravitational sphere packing were developed in MATLAB software. In pre-processing stage, the initial position of the spheres was determined in (x-y) plane by applying different random number distributions. At the stage of processing, three manners of rolling, collision, and stability conditions were considered to pack particles for each incoming sphere within the cylindrical container. The post-process code computes porosity of packed particles through the cylindrical container for both with and without boundary effects. The simulated model was validated by ASTM D 4254 standard method for measuring the maximum void ratio in metal cylindrical container, in which soil particles with different types of sorting are employed for comparison.

Results showed that maximum void ratio of dry sandy-soil particles was computed by considering wall-sphere interaction condition in gravitational sphere packing. However, deactivation of boundary effects led to calculate the porosity of the unconsolidated wet sandy-soil particles. Both above mentioned results had close values in agreement with the experimental works. The study also revealed that log-normally particle size distribution provides the lowest value of the porosity. In addition, particles had high potential to be packed in square container with the lowest value of the porosity.

All in all, the new presented method of optimized gravitational sphere packing rapidly simulates arrangement of the soil particles in dilute suspension. The rapid code implementation of this method can compute the main parameters of the relative density with high accuracy which was verified by ASTM D 4254 standard test.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

PEMBANGUNAN SUATU KAEDAH PENGIRAAN BAGI MENENTUKAN LOMPANG MAKSIMUM PARTIKEL-PARTIKEL TANAH BERPASIR DENGAN MENGGUNAKAN KAEDAH PEMBUNGKUSAN SFERA GRAVITI

Oleh

MOHAMMAD MAHDI ROOZBAHANI

Oktober 2012

Pengerusi: Professor Bujang Bin Kim Huat, PhD

Fakulti: Kejuruteraan

Pada masa kini, minat terhadap simulasi realiti maya dalam bidang geoteknik semakin meningkat. Baru baru ini, kesusasteraan menawarkan persaman eksperimen linear yang mana menghubungkaitkan nisbah lompang maksimum dan minimum partikel tanah pepejal secara positif. Walaubagaimanapun, perhatian yang amat sedikit telah diberikan kepada kaedah simulasi maya untuk mengira salah satu parameter tersebut bagi secara pantas mengira parameter-parameter penting bagi ketumpatan relatif tanpa menjalankan sebarang eksperimen. Kaedah pembungkusan sfera graviti yang menyusun sfera-sfera yang tidak bertindih di dalam ruang yang terbatas mempunyai kebolehan untuk menjalankan simulasi bagi sebilangan besar partikel-partikel. Matlamat penyelidikan ini adalah untuk menggunakan pembungkusan sfera graviti untuk mengira nisbah lompang maksimum bagi tanah zarah berpasir dengan menggunakan taburan saiz partikel sebagai data input.

Di dalam penyelidikan ini, pre-pemprosesan, pemprosesan serta pemprosesan pasca

bagi pembungkusan sfera graviti yang mengoptimumkan telah diperolehi dengan menggunakan perisian MATLAB. Pada peringkat pre-pemprosesan, kedudukan awal sfera-sfera di plane (x-y) telah ditentukan dengan menggunakan nombor taburan rawak yang berbeza. Pada peringkat pemprosesan, tiga kaedah jatuh, perlanggaran dan keadaan stabil telah diambilkira bagi membungkus setiap sfera yang masuk ke dalam silinder. Kod yang digunakan pada peringkat pemprosesan pasca pula mengira keporosan partikel-partikel yang dibungkus melalui silinder bagi keadaan kesan dengan sempadan serta keadaan tanpa kesan sempadan. Model simulasi telah disahkan oleh piawai ASTM D 4254 yang digunakan bagi mengukur nisbah lompang maksimum dalam bekas silinder , di mana partikel-partikel tanah di dalam susunan yang berbeza telah dibandingkan.

Keputusan menunjukkan nisbah lompang maksimum bagi partikel tanah zarah berpasir yang kering, telah diukur dengan mengambil kira interaksi dinding sfera di dalam pembungkusan sfera graviti. Walaubagaimanapun, penyahaktifan kesan sempadan telah membawa kepada pengiraan keporosan partikel tanah zarah berpasir yang basah dan tidak tepu. Kedua-dua keputusan yang dibincangkan, mempunyai nilai-nilai yang hampir sama dengan keputusan yang diperolehi melalui eksperimen. Penyelidikan ini juga mendedahkan taburan saiz partikel log-biasa mempunyai nilai keporosan paling rendah. Partikel-partikel tersebut juga mempuyai potensi yang tinggi untuk dibungkus di dalam bekas segiempat sama dengan nilai keporosan yang paling rendah.

Rumusannya, kaedah baru yang mengoptimumkan pembungkusan sfera graviti ini mensimulasikan dengan pantas kedudukan partikel-partikel tanah di dalam penggantungan yang cair. Kod pantas bagi kaedah ini boleh digunakan bagi mengira parameter-parameter penting bagi ketumpatan relatif dengan ketepatan yang tinggi yang telah disahkan oleh Ujian Piawai ASTM D 4254.

ACKNOWLEDGEMENTS

My utmost gratitude goes to God. I would like to express a special thanks to my academic advisor Professor Bujang Bin Kim Huat. It was a well rewarding experience to have been under his supervision. I deeply appreciate his patient instructions and continuous support he has given me during my study and research. I am also graceful to my supervisory committee member, Dr. Haslinda Nahazanan. I warmly thank Dr. Afshin Asadi for his generous contributions and effective advices to my research. I indeed appreciate Professor Hasanah Mohd Ghazali for her kind supports during the difficult moments of my research.

On a personal note, I am thankful of Professor Lori Graham-Brady for her kind, supportive, and valuable helps during my research.

I would especially like to thank my wife, Elnaz Younessian, for her continuing support and patience. I owe my loving thanks to my parents Hoeseinreza and Fatemeh Roozbahani.

I certify that a Thesis Examination Committee has met on 11 October 2012 to conduct the final examination of Mohammad Mahdi Roozbahani on his thesis entitled "Development of Computational Method to Determine Maximum Void Ratio of Sand-soil Particles using Gravitational Sphere Packing Method" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Degree of Master of Science.

Members of the Thesis Examination Committee were as follows:

Ahmad Rodzi Mahmud, PhD
Associate Professor
Faculty of Engineering
Universiti Putra Malaysia
(Chairperson)

Abang Abdullah Abang Ali, PhD
Professor
Faculty of Engineering
Universiti Putra Malaysia
(Internal Examiner)

Zainuddin Md. Yusoff, PhD
Senior Lecturer
Faculty of Engineering
Universiti Putra Malaysia
(Internal Examiner)

Lori Graham-Brady, PhD
Professor
Faculty of Engineering
Johns Hopkins University
Country USA
(External Examiner)

SEOW HENG FONG, PhD
Professor and Deputy Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of **Master of Science**. The members of the Supervisory Committee were as follows:

Bujang Bin Kim Huat, PhD
Professor
Faculty of Engineering
Universiti Putra Malaysia
(Chairperson)

Haslinda Nahazanan, PhD
Senior Lecturer
Faculty of Engineering
Universiti Putra Malaysia
(Member)

BUJANG BIN KIM HUAT, PhD
Professor and Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:

DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

MOHAMMAD MAHDI ROOZBAHANI

Date:

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

V_V	Volume of voids
V_S	Volume of solids
V_T	Total volume
ϕ	Porosity
D_r	Relative density
e_{max}	Maximum void ratio based on a very loose state of the particles
e_{min}	Minimum void ratio based on a very dense state of the particles
e	In situ void ratio
P	Pressure
γ	Surface tension of the liquid
θ	Contact angle of the liquid
d	Diameter of the capillary
V_B	Bulk volume of the sample
V_a	Volume of the vessel containing the sample
V_b	Volume of the evacuated vessel
P_2	Initial pressure
P_1	Final pressure
N	Coordination Number
C_1, C_2, C_3	Pre-settled sphere centers
C_S	Coordinate of the center of the incoming sphere
r_1, r_2, r_3	Radiuses of pre-settled sphere
r_S	Radius of incoming sphere
C_{1s}	Distance of the two touched spheres
h	Height of spherical cap
a	Radius of base circle

σ	Variance of data
μ	The mean of data
r	Sphere radius
R	Container radius
b	Smallest distance of the axis of the cylinder to the center of the sphere
V_i	Volume of the intersection when $b < R$
V_c	Volume of the intersection when $b > R$
A	Largest root
B	Second largest root
C	Smallest root
$K(k)$	First complete elliptic integral
$E(k)$	Second complete elliptic integral
$\Pi(k, -\alpha^2)$	Third complete elliptic integral
z	The sine of the amplitude
α	The modular angle
k	The parameter
w_i	Residual of the ith data produced by regression smoothing method
r_i	Robust weight function
MAD	Median($\|r_i\|$)
ρ_L	Density of the liquid (Mg/m_3)
m_1	Mass of bottle(in g)
m_2	Mass of bottle and dry soil (in g)
m_3	Mass of bottle, soil and liquid (in g)
m_4	Mass of bottle when full of liquid only (in g)
G_s	Specific gravity

CHAPTER 1

INTRODUCTION

1.1 Motivation of the Research

These days, simulations of the experimental works in the virtual environment have been dramatically increased in the geotechnical field because of the rapid data access and observing inner variations of the soil particles finely. To this end, many methods have been extended such as sphere packing techniques (Tory et al., 1973). Such these methods can simulate laboratory works in geotechnical field. Sphere packing is an arrangement of non-overlapping spheres within a container. This method is widely employed in two different structures which are statistical geometric and numerical. The former one is used in models with a small number of the elements, whereas the latter one is applied for a large number of elements.

The random loose sphere packing was exercised in this research with the statistical geometric procedure which is founded by Tory and Church (Tory et al., 1973). This method has been updated to be utilized in different fields such as powder and nano-technology, chemical, and geological engineering.

The content of this research is concerned with loosely packing of the sandy-soil particles with gravitational sphere packing method. However, different applications of this method have been used to compute an ideal system with low value of the porosity. Thus, a sever optimization is needed to get close this method with reality conditions in the experimental works of the geotechnical field.

Maximum void ratio, e_{max}, of the sandy-soil particles is defined by this method. e_{max} is one of the main parameters of the relative density, D_r, based on the Equation 1.2. Void ratio, e, is the ratio of the volume of voids to the volume of solids according to Equation 1.1. More recently, literature has emerged that offers linear experimental Equation which positively correlates the maximum and minimum void ratio, e_{min}, of sandy-soil particles (Yilmaz and Mollamahmutoglu, 2009; Cubernovski and Ishihara, 2002). However, far too little attention has been paid to virtually simulate e_{max} in order to rapidly compute these main parameters of the relative density (e_{max} and e_{min}) without any need to do experimental works. Therefore, this project comes to this end to simulate one of these parameters, e_{max}, with particle size distribution as input data in order to rapid data access to these parameters in a virtual environment.

$$e = \frac{V_V}{V_S} = \frac{V_V}{V_T - V_V} = \frac{\phi}{1-\phi}; \phi = \frac{V_V}{V_T} \tag{1.1}$$

where;

$V_V =$ volume of voids

$V_S =$ volume of solids

$V_T =$ total volume

$\phi =$ porosity

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}} \tag{1.2}$$

where;

$e_{max} =$ maximum void ratio in correspondence with a very loose state of the particles

$e_{min} =$ minimum void ratio in correspondence with a very dense state of the particles

$e =$ in situ void ratio

In the gravitational or loosely sphere packing, very little research has been reported on the geotechnical behavior of the elements. This research is going to relate this method to the geotechnical field.

1.2 Research Objectives

Two main objectives of current research are categorized as follows:

- To find minimum porosity of the unconsolidated sandy soils after mechanical rearrangement of the sand grains.
- To compute *maximum void ratio* of dry sandy-soil particles based on the statistical geometric approach of the gravitational or loosely sphere packing method.

It should be noted that above objectives are achieved according to the preliminary objectives mentioned below which are directly related to the fundamental concepts of the random loose sphere packing (gravitational sphere packing) developing this method for different applications such as geotechnical field. For example, better perception to choose an efficient container or random numbers for the post-processing computations in porosity.

- To investigate the different porosity values of the identical particles in different rectangular containers.
- To find the lowest value of the porosity by examining different distributions of random numbers into loosely packed multi-sized spheres.

1.3 Scope of the Research

The main scope of this research is divided into two parts which are as follows:

Simulation part: Vital necessities of optimization in the sphere packing method would play crucial role to provide logical results in agreement with experimental works. In Chapter 2, the different methods of the sphere packing are briefly introduced, and exclusive literature review of the gravitational sphere packing has also been described in mentioned Chapter.

Sphere packing method has been optimized in the rapid collision detection and container interaction with soil particles which are explained in Chapter 3. K Nearest Neighbors based on the KD-tress method and binary search are applied to find the neighbors of every incoming sphere in the system, and also 3D Apollonius is applied to break 2 orders equations in simple ones so as to rapid making tangent of spheres. Two conventional containers are considered for this method which are rectangular and cylindrical container. Interactions of the spheres with cylindrical container were solved by applying *complete elliptic integrals*. These intersections were resolved by spherical caps for rectangular container.

It should be noted that some outlier data of the porosity values through the height and diameter of the container are mathematically filtered by *Lowes Local Regression Smoothing* procedure explained in Chapters 3 and 4.

In Chapter 4, The results of the simulated model according to the mentioned objectives are verified by last simulation models of the literatures and current experimental work.

Experimental works: Experimental works were done to find the maximum void ratio for sandy-soil particles with different sorting coefficients which are classified from very well sorted to the very poorly sorted. In addition, specific gravity, and sieve analysis are the Lab works which are discussed in Chapters 3 and 4.

BIBLIOGRAPHY

Abreu, C. R. A., Macias-Salinas, R., Tavares, F. W. and Castier, M. 1999. A Monte Carlo simulation of the packing and segregation of spheres in cylinders. *Brazilian Journal of Chemical Engineering* 16 (4): 1–12.

Al-raoush, R., Thompson, K. and Willson, C. S. 2003. Comparison of Network Generation Techniques for Unconsolidated Porous Media. *Techniques* 1700 (June 2002): 1687–1700.

Alonso, M., Satoh, M. and Miyanami, K. 1992. Void-size distribution in two-dimensional random packings of equal-sized disks. *The Canadian Journal of Chemical Engineering* 70 (1): 28–32.

Anishchik, S. and Medvedev, N. 1995. Three-Dimensional Apollonian Packing as a Model for Dense Granular Systems. *Physical Review Letters* 75 (23): 4314–4317.

Asmar, B. N., Langston, P. a. and Matchett, a. J. 2003. Modeling particle settling behavior using the discrete element method. *Advanced Powder Technology* 14 (5): 519–532.

ASTM. D4254, Standard Test Methods for Minimum Index Density and Unit Weight of Soils and Calculation of Relative Density 1.

ASTM. D4404-10, Standard Test Method for Determination of Pore Volume and Pore Volume Distribution of Soil and Rock by Mercury Intrusion Porosimetry.

ASTM. F316-03, Standard Test Methods for Pore Size Characteristics of Membrane Filters by Bubble Point and Mean Flow Pore Test.

Aydilek, a. H. 2006. A semi-analytical methodology for development of woven geotextile filter selection criteria. *Geosynthetics International* 13 (2): 59–72.

Bargiel, M. 2008. Geometrical Properties of Simulated Packings of Spherocylinders. *Science And Technology* 126–135.

Bargiel, M. and Mo, J. 1991. C-language program for the irregular close packing of hard spheres. *Computer Physics Communications* 64.

Bargiel, M. and Tory, E. M. 1993. Packing fraction and measures of disorder of ultradense irregular packings of equal spheres. I. Nearly ordered packing. *Advanced Powder Technology* 4 (2): 79–101.

Beard, D. C. and Weyl, P. K. 1973. Influence of Texture on Porosity and Permeability of Unconsolidated Sand. *The American Association of Petroleum Geologists Bulletin* 57: 349–369.

Belheine, N., Plassiard, J.-P., Donzé, F.-V., Darve, F. and Seridi, a. 2009. Numerical simulation of drained triaxial test using 3D discrete element modeling. *Computers and Geotechnics* 36 (1-2): 320–331.

Bhandari, A. and Han, J. 2010. Investigation of geotextilesoil interaction under a cyclic vertical load using the discrete element method. *Geotextiles and Geomembranes* 28 (1): 33–43.

BS. 1377:Part 2:1990, Soils for civil engineering purposes. Part 2. Classification tests.

Burkardt, J. and Tech, V. 2010. Computational Geometry Lab : TETRAHEDRONS. *Test* .

Catherine, O., Cui, L. and Bray, J. D. 2004. Three-Dimensional Discrete Element Simulations of Direct Shear Tests. In *2nd International PFC Symposium*.

Chan, S. and Ng, K. 1988. Geometrical characteristics of the pore space in a random packing of equal spheres. *Powder Technology* 54 (2): 147–155.

Chen, P. 2008. *Quantitative Study of Granular Flow , Mixing , and Segregation Using the Discrete Element Method (DEM)*. PhD thesis, Northwestern University.

Cleveland, W. S. 1979. Robust Locally and Smoothing Weighted Regression Scatterplots. *Journal of the American Statistical Association* 74 (368): 829– 836.

Collins, R. E. 1961. *Flow of Fluids through Porous Materials*. Von Nostrand-Reinhold.

Cubernovski, M. and Ishihara, K. 2002. Maximum and minimum void ratio characteristics of sands. *Soil and Foundations Japanese Geotechnical Society* 42: 65–78.

Cundall, P. a. and Hart, R. D. 1992. Numerical Modelling of Discontinua. *Engineering Computations* 9 (2): 101–113.

Cundall, P. a. and Strack, O. D. L. 1979. A discrete numerical model for granular assemblies. *Géotechnique* 29 (1): 47–65.

Dullien, F. A. L. 1992. *Porous Media Fluid Transport and Pore Structure*. 2nd edn. New York: Academic Press. Inc.

Ezrukov, A. L. B., Toyan, D. I. S. and Argie, M. O. B. 2001. Spatial Statistics for Simulated Packing of Spheres. *Image (Rochester, N.Y.)* 203–206.

Fazekas, S. 2007. *Distinct Element Simulations of Granular Materials*. PhD thesis, University of Technology and Economics Budapest, Hungary.

Formisani, B. 1991, Packing and fluidization properties of binary mixtures of spherical particles.

Galakhov, a. and Shevchenko, V. 1992. Analysis of two-dimensional packings. *Ceramics International* 18 (4): 213–221.

Gao, S., Meegoda, J. N. and Hu, L. 2012. Two methods for pore network of porous media. *International Journal for Numerical and Analytical Methods in Geomechanics* .

Gentle, J. E., Chambers, J., Eddy, W., Hardle, W., Sheather, S. and Tierney, L. 2005. *Random Number Generation and Monte Carlo Methods*. Second. edn. New York: springer.

Georgalli, G. and Reuter, M. 2007. A particle packing algorithm for pellet design with a predetermined size distribution. *Powder Technology* 173 (3): 189–199.

German, R. M. and Park, S. J. 2008. *Mathematical relations in particulate materials processing : ceramics, powder metals, cermets, carbides, hard materials, and minerals*. Wiley.

Gong, G. 2008. *DEM simulations of drained and undrained behaviour*. PhD thesis, University of Birmingham.

Goodling, J., Vachon, R., Stelpflug, W., Ying, S. and Khader, M. 1983. Radial porosity distribution in cylindrical beds packed with spheres. *Powder Technology* 35 (1): 23–29.

Gotoh, K. 1978. A random packing structure of equal spheres statistical geometrical analysis of tetrahedral configurations. *Powder Technology* 20 (2): 233–242.

Graff, L. 2010. *Discrete Element Method Simulation of Wear due to the Soil-tool Interaction*. PhD thesis, University of Saskatchewan.

Grznar, J., Booth, D. and Sebastian, P. 1997. A Robust Smoothing Approach to Statistical Process Control. *Journal of Chemical Information and Modeling* 37 (2): 241–248.

Haughey, D. 1966. Local voidage variation in a randomly packed bed of equal-sized spheres. *Chemical Engineering Science* 21 (10): 905–915.

ISO. 14688, Geotechnical investigation and testing – Identification and classification of soil – Part 1: Identification and description.

Jao van de Lagemaat, K. D. B. and Frank, A. J. 2001. Relation between Particle Coordination Number and Porosity in Nanoparticle Films:Implications to Dye-Sensitized Solar Cells. *The Journal of Physical Chemistry* 105 (50).

Jerier, J.-F., Richefeu, V., Imbault, D. and Donzé, F.-V. 2010. Packing spherical discrete elements for large scale simulations. *Computer Methods in Applied Mechanics and Engineering* 199 (25-28): 1668–1676.

Jiang, M., Yu, H. and Harris, D. 2005. A novel discrete model for granular material incorporating rolling resistance. *Computers and Geotechnics* 32 (5): 340–357.

Jodrey, W. S. 1981. Computer Simulation of Equal Spheres of Isotropic , Homogeneous , Dense Random Packing. *Powder Technology* 30: 111 – 118.

Jodrey, W. S. and Tory, E. M. 1979. Simulation of random packing of spheres. *Simulation* 32 (1): 1–12.

Jodrey, W. S. and Tory, E. M. 1985. Computer simulation of close random packing of equal spheres'. *Physical Review A* 32 (4): 2347–2351.

John I. D'Souzae, D. H. N. M. 2008. Mercury Intrusion Porosimetry : A Tool for Pharmaceutical Particle Characterization. *Pharmaceutical Reviews* 6 (2).

Kern, W. F. and Bland, J. R. 1948. *Solid Mensuration with Proofs*. 2nd edn. New York: Wiley.

Kern, W. F. and Bland, J. R. 1994. Radial porosity distribution in cylindrical beds packed with spheres. *Chemical Engineering Science* 49 (6): 863–876.

Korkut, E. N. 2003. *Geotextiles as Biofilm Attachment Baffles for Wastewater Treatment*. PhD thesis, Drexel University.

Kozicki, J. and Tejchman, J. 2008. Modelling of fracture process in concrete using a novel lattice model. *Granular Matter* 10 (5): 377–388.

Kristiansen, K., Wouterse, A. and Philipse, A. 2005. Simulation of random packing of binary sphere mixtures by mechanical contraction. *Physica A: Statistical Mechanics and its Applications* 358 (2-4): 249–262.

Kyrylyuk, A. V., Wouterse, A. and Philipse, A. P. 2009. Random Packings Of Rod-Sphere Mixtures Simulated By Mechanical Contraction. *Physics* 211–214.

Lamarche, F. and Leroy, C. 1990. Evaluation of the volume of intersection of a sphere with a cylinder by elliptic integrals. *Computer Physics Communications* 59: 359–369.

Landry, J. and Grest, G. 2004. Granular packings with moving side walls. *Physical Review E* 69 (3): 1–9.

Lobo-Guerrero, S. and Vallejo, L. 2005. Discrete element method evaluation of granular crushing under direct shear test conditions. *Journal of geotechnical and ...* (October): 1295–1301.

Lochmann, K., Oger, L. and Stoyan, D. 2006. Statistical analysis of random sphere packings with variable radius distribution. *Solid State Sciences* 8 (12): 1397–1413.

Macrae, J. C. and Gray, W. a. 1961. Significance of the properties of materials in the packing of real spherical particles. *British Journal of Applied Physics* 12 (4): 164–172.

Markauskas, D. 2006. Compacting of particles for biaxial compression test by the discrete element method. *Journal of Civil Engineering and Management* XII (2): 153–161.

Mcwhirter, J. D., Crawford, M. E. and Klein, D. E. 1997. Wall Region Porosity Distributions for Packed Beds of Uniform Spheres with Modified and Unmodified Walls. *Transport in Porous Media* 99–118.

Mitchell, J. K. and Soga, K. 2005. *Fundamentals of Soil Behavior*. Third edit edn. New Jersey: John Wiley & Sons, Inc.

Montgomery, D. C. 2003. *Applied Statistics and Probability for Engineers Third Edition*. Third edit edn. John Wiley.

Mueller, G. 2005. Numerically packing spheres in cylinders. *Powder Technology* 159 (2): 105–110.

Mueller, G. E. 1992. Radial void fraction distributions in randomly packed fixed beds of uniformly sized spheres in cylindrical containers. *Powder Technology* 72 (3): 269–275.

Mueller, G. E. 1993. Angular void fraction distributions in randomly packed fixed beds of uniformly sized spheres in cylindrical containers. *Powder Technology* 77 (3): 313–319.

Mueller, G. E. 2010. Radial porosity in packed beds of spheres. *Powder Technology* 203 (3): 626–633.

Nelder, B. J. A. and Meadf, R. 1964. A simplex method for function minimization .

Nolan, G. and Kavanagh, E. 1994. The size distribution of interstices in random packings of spheres 78: 231–238.

Nolan, G. and Kavanagh, P. 1992. Computer simulation of random packing of hard spheres. *Powder Technology* 72 (2): 149–155.

Nolan, G. and Kavanagh, P. 1993. Computer simulation of random packings of spheres with log-normaldistributions. *Powder Technology* 76 (3): 309–316.

OSullivan, C. 2011. Particle-Based Discrete Element Modeling: A Geomechanics Perspective. *International Journal of Geomechanics* (1985).

Panchanathan, V. 1973. of grains . General parameter evaluation. *Powder Technology* 8: 19–26.

Quickenden, T. 1974. Random packing in two dimensions and the structure of monolayers. *Journal of Colloid and Interface Science* 48 (3): 382–393.

Ransing, R., Gethin, D., Khoei, a., Mosbah, P. and Lewis, R. 2000. Powder compaction modelling via the discrete and finite element method. *Materials & Design* 21 (4): 263–269.

Reboul, N., Vincens, E. and Cambou, B. 2008. A statistical analysis of void size distribution in a simulated narrowly graded packing of spheres. *Granular Matter* 10 (6): 457–468.

Rubinstein, R. Y. and Kroese, D. P. 2008. *Simul and the Monte Carlo Method*. 2nd edn., , vol. 64. Wiley.

Scheidegger, A. E. 1974. *The Physics of Flow through Porous Media*. 3rd edn. Univ. of Toronto Press.

Shi, Y. and Zhang, Y. 2008. Simulation of random packing of spherical particles with different size distributions. *Applied Physics A* 92 (3): 621–626.

Sjah, J. and Vincens, E. 2012. Determination of the constriction size distribution of granular filters by filtration tests. *Distribution* .

Sobolev, K. and Amirjanov, A. 2010. Application of genetic algorithm for modeling of dense packing of concrete aggregates. *Construction and Building Materials* 24 (8): 1449–1455.

Stoyan, Y. G. 1983. Mathematical methods for geometric design. *Proceedings of International Conference PROLAMAT 82* 67–86.

Stoyan, Y. G. and Yaskov, G. N. 2009. Packing identical spheres into a cylinder. *International Transactions in Operational Research* 17 (1): 51–70.

Suzuki, M., Shinmura, T., Iimura, K. and Hirota, M. 2008. Study of the Wall Effect on Particle Packing Structure Using X-ray Micro Computed Tomography. *Adv Powder Technol* 19 (2): 183–195.

Tavarez, F. A. 2005. *Discrete Element Method for Modeling Solid and Particulate Materials*. PhD thesis, Universityof Wisconsin-Madison.

Thornton, C. 1991. Impact of elastic spheres with and without adhesion. *Powder Technology* 65 (1-3): 153–166.

To, L. T. and Stachurski, Z. H. 2004. Random close packing of spheres in a round cell. *Journal of Non-Crystalline Solids* 333 (2): 161–171.

Tory, E. M., Church, B. H., Tam, M. K. and Ratner, M. 1973. Simulated random packing of equal spheres. *The Canadian Journal of Chemical Engineering* 51 (4): 484–493.

Washburn, E. W. 1921. Note on a Method of Determining the Distribution of Pore Sizes in a Porous Material. *Proc Natl Acad Sci U S A* 7(4) (1): 115–116.

Widuliski, u., Kozicki, J. and Tejchman, J. 2009. Numerical simulations of triaxial test with sand using DEM. *Arch. Hydro-Eng. Environ. Mech* 56 (3): 149–171.

Williams, S. and Philipse, a. 2003. Random packings of spheres and spherocylinders simulated by mechanical contraction. *Physical Review E* 67 (5): 1–9.

Yang, A., Miller, C. T. and Turcoliver, L. D. 1996. Simulation of correlated and uncorrelated packing of random size spheres. *Physical Review E: Statistical Physics, Plasmas, Fluids, and Related Interdisciplinary Topics* 53 (2): 1516–1524.

Yao, M. 2001. *Three-Dimensional Discrete Element Method Analysis of Cohesive Soil*. PhD thesis, Johns Hopkins University.

Yilmaz, Y. and Mollamahmutoglu, M. 2009. Characterization of Liquefaction Susceptibility of Sands by Means of Extreme Void Ratios and/or Void Ratio Range. *Journal of Geotechnical and Geoenvironmental Engineering* 135 (12): 1986.

Zhang, W., Thompson, K. E., Reed, A. H. and Beenken, L. 2006. equilateral cylindrical particles. *Chemical Engineering Science* 61: 8060–8074.