UNIVERSITI PUTRA MALAYSIA

BEHAVIOUR OF EXTERNALLY PRESTRESSED BEAMS

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BEHAVIOUR
OF
EXTERNALLY PRESTRESSED BEAMS

By
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A Project Report submitted in partial fulfillment of the requirements for
the Degree of Master of Science
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PROJECT APPROVAL SHEET

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Finally, many thanks to my wife, MaryAnn and my children, Andrew and Stefanie for their patience and without their support and encouragement this report would not have been possible.

“Great works are performed not by strength, but by perseverance”

– Samuel Johnson (1709-1984)
ABSTRACT
Externally prestressed beams are defined as structural concrete beams with external prestressing tendons placed outside the concrete section.

The behaviour of externally prestressed beams was studied by evaluating the flexural response of the beam based on modified bond reduction coefficients to account for second order effects and using a structural software to model the overall behaviour of the beam with incremental loading.

Theoretical calculations based on modified bond reduction coefficients were found to be acceptable within the test results. The predicted results from computer modeling showed that the structural behaviour of externally prestressed beam can be satisfactorily predicted from initial zero load to the ultimate loading stage.
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Notation is generally in accordance with BS 8119, and the principal symbols are listed below. Other symbols are defined in the text or shown in the diagram where necessary.

\( A_{ps} \) = area prestress of external tendons

\( A_s \) = area of internal tensile reinforcement

\( A_s' \) = area of internal compressive reinforcement

\( c \) = neutral axis depth at critical section under consideration

\( d_{ps0} \) = initial tendon depth

\( d_s \) = effective depth of internal tensile reinforcement

\( d_s' \) = effective depth of internal compressive reinforcement

\( e_u \) = tendon eccentricity

\( e_0 \) = initial tendon eccentricity

\( \varepsilon_{cu} \) = strain in the top of concrete fibre at ultimate (= 0.003) according to ACI Code

\( E_c \) = modulus of elasticity of the concrete

\( E_{ps} \) = modulus of elasticity of external tendons

\( f_{ps} \) = Ultimate stress of prestressing tendons

\( \Delta f_{ps} \) = stress increase at ultimate flexure limit state

\( f_{pe} \) = effective prestress of external tendon

\( f_{py} \) = yield strength of prestressing external tendon

\( f_y \) = yield strength of internal tensile reinforcement

\( f_y' \) = yield strength of internal compressive reinforcement

\( f'c \) = concrete cylinder strength

\( F_i \) = tensile forces in the cable segments \((i)\) at the deviator \((i)\)
\( F_{i+1} = \) tensile forces in the cable segments \((i+1)\) at the deviator \((i)\).

\( h = \) overall beam height

\( I_e = \) effective moment of inertia

\( I_t = \) moment of inertia based on the transformed section

\( I_{cr} = \) moment of inertia based on the cracked section

\( k_c = \) depth of centroid of the concrete in compression zone

\( L = \) effective beam span

\( L_d = \) distance from the beam support to deviator point

\( L_s = \) distance from the beam support to loading point

\( M = \) friction coefficient at the deviator

\( M_u = \) ultimate moment of resistance

\( M_{cr} = \) cracking moment of resistance

\( S_d = \) distance between two deviators placed symmetrically with respect to the centre line of the beam.

\( \rho_p = \) prestressing ratio

\( \Omega_u = \) bond reduction coefficient at ultimate limit state

\( \Phi_u = \) ultimate curvature

\( \Delta_{total} = \) maximum total deflection due to prestressing force and external applied load
CHAPTER 1
INTRODUCTION

1.1 General

In reinforced concrete structure, loads which are carried in a member by bending are effectively transferred through it as compressive and tensile forces. Since concrete has low tensile strength, steel reinforcement must be provided in all structural concrete members subject to such bending forces to control tensile cracking and, ultimately, to prevent failure.

Normally steel reinforcements are placed within the tension zones of concrete members to carry the internal tensile forces across flexural cracks to the supports. Here the applied moment is resisted by the compression of the uncracked portion of the concrete section and tension in the reinforcing bars.

However the presence of reinforcement does not prevent the development of tensile cracks in members but only limits the magnitude of the strain in the bars to prevent excessive large cracks.

One major problem with the presence of cracks in ordinary reinforced concrete is the corrosion of the reinforcement when exposed to water and chemical contaminants. A further effect of cracking to ordinary reinforced members is the substantial loss in stiffness which occurs after cracking. The second moment of area of the cracked section, $I_c$, is far less than the second
moment of area before cracking, $I_m$. This would cause a large increase in the deformation of the member.

Since corrosion is generally only a problem for structures in aggressive exterior environments (bridges, marine structures, etc.) and is not critical in the majority of buildings, an alternative form of reinforced concrete that was found suitable is prestressed concrete. In prestressed concrete, compressive stresses are introduced into a member to reduce the tensile stresses which result from bending due to the applied loads. The compressive stresses are generated in a member by tensioned steel anchored at the ends of the members and/or bonded to the concrete.

1.2 Historical Development of Prestressing

Prestressed concrete is a new concept which dates back to 1872, when P. H. Jackson, an engineer from California, patented a prestressing system that used a tie rod to construct beams or arches from individual blocks. [See Figure 1.1] In 1888, C. W. Doehring of Germany obtained a patent for prestressing slabs with metal wires. However this new concept of prestressing was not successful due the loss of the prestress with time.

The breakthrough for prestressing came when R. E. Dill of Alexandria, Nebraska, recognized the effect of the shrinkage and creep (transverse material flow) of concrete on the loss of prestress. He subsequently developed the idea that successive post-tensioning of unbonded rods would
compensate for the time-dependent loss of stress in the rods due to the decrease in the length of the member because of creep and shrinkage.

Figure 1.1 Prestressing principle in linear and circular prestressing [Prestressed Concrete-A Fundamental Approach by Edward G. Nawy pp.5-18]

Then in the early 1920s, W. H. Hewett of Minneapolis developed the principles of circular prestressing which would stressed horizontal reinforcement around walls of concrete tanks to prevent cracking and subsequently achieving watertightness. Thereafter, prestressing of tanks and pipes developed at an accelerated pace in the United States, with thousands of tanks of water, liquid, and gas storage built and much mileage of prestressed pressure pipe laid in the two to three decades that followed.
Linear prestressing continued to develop in Europe and in France through Eugene Freyssinet, who proposed in 1926 through 1928 methods to overcome prestress losses through the use of high-strength and high-ductility steels. In 1940, he introduced the now well-known and well-accepted Freyssinet system comprising the conical wedge anchor for 12-wire tendons.

G. Magnel of Ghent, Belgium, and Y. Guyon of Paris extensively developed and used the concept of prestressing for the design and construction of numerous bridges destroyed by World War II in western and central Europe. The Magnel system also used wedges to anchor the prestressing wires. They differed from the original Freyssinet wedges in that they were flat in shape, accommodating the prestressing of two wires at a time.

Between 1930s and 1960s, P. W. Abeles of England introduced and developed the concept of partial prestressing. F. Leonhardt of Germany, V. Mikhailov of Russia, and T. Y. Lin of the United States also contributed a great deal to the design of prestressed concrete. These twentieth-century developments have led to the extensive use of prestressing throughout the world especially in the United States.

1.3 External Prestressing

External prestressing is a prestressing system in which the concrete structural members are prestressed longitudinally using tendons located completely outside the concrete section to provide flexural resistance. The
The flexural response of an externally prestressed beam has been commonly equated to that of a beam with internal unbonded tendons.

It is used not only in new construction works but is also an efficient method for strengthening existing structures. External prestressing is an efficient method in the construction of new bridges especially segmental bridge box girders and in the rehabilitation and strengthening of existing concrete bridge structures.

Some of the advantages of external prestressing system include:

- More economical construction
- Easier tendon layout, placement and consolidation of concrete
- Better corrosion protection as compared to a conventional tendon system.

The use of external prestressing system to strengthen existing concrete bridges is conceptually different from that used in the traditional design and construction of new concrete bridges.

In the design of new concrete bridges, external tendons are the primary reinforcements and therefore the analysis and design could be achieved using methods similar to unbonded post-tensioned construction.

When used for strengthening, external tendons represent only a part of the total flexural reinforcement. The remaining reinforcement could be in the
form of steel reinforcement, internal steel prestressing or a combination of both, depending of the type of concrete structure to be rehabilitated.

A study to examine experimentally the benefits of external prestressing in strengthening of concrete flexural members was carried out by Harajli, M.H. in 1993 with the objective to evaluate its effect on both service load behaviour and nominal flexural resistance.

The experiment included three different structural systems:-
- reinforced concrete,
- prestressed concrete and
- partially prestressed concrete

together with the two profiles of external prestressing namely, straight horizontal and single point draped (deviator at midspan) external tendon.

Results showed that providing external prestressing steel by a relatively moderate amount resulted in the increase in the nominal flexural resistance by up to 146%. This was achieved without significant reduction in ductility or ultimate flexural deformation of member.

In this investigation by Harajli, M.H., external prestressing using various steel profile for different levels of prestressing, was shown to have
- reduce cracks widths or close cracks completely
- reduce the service load deflections induced under cyclic fatigue loading
stiffer service-load deflection response and therefore a reduction in live load deflection.

External prestressing using a straight horizontal profile was relatively less effective in increasing the flexural response as compared to a deviated profile but the service load-deflection response was similar for these two types of profiles.

The secondary advantage of external prestressing is to prolong the fatigue life of concrete flexural members subjected to repetitive type loading. This is because external prestressing reduces the mean stress level and stress ranges in the internal tension reinforcement which is of primary importance to the fatigue life of concrete flexural members.

1.4 Scope and Objectives

In the study of the behaviour of externally prestressed beams, two factors which makes the analysis complicated are:

- Stress increase in the external tendons is dependent on the overall deformation of the member
- Eccentricity variations of external tendons under load, commonly referred as second-order effects.

Therefore since the elongation of the external tendons is influenced by the total deformation of the structural member rather than solely on the section under consideration, most of the proposed methods of analysis for the
behaviour of externally prestressed beams involve complicated numerical analysis that are only possible only with the use of computers.

The scope and objectives of this study are as follows:-

a) To compare the various empirical methods proposed by various researchers to analyse an externally prestressed beam.

b) To investigate the structural behaviour and performance of an externally prestressed beam experimentally.

c) To model the behaviour of externally prestressed beams using an existing structural software.

To achieve the stated objectives, theoretical investigations into previous analytical studies will be reviewed. Experimental studies on beams with parameters that include various deviator spacing will be carried out. Finite element method (FEM) using an existing structural software (SAP 2000 -8 Non Linear) to model and analyse the experimental test beams. The structural response of these different beam models will be found in terms of stresses, maximum deflection and ultimate resistances loads. These results can be compared with existing test results.

1.5 Structure of Project Report
This thesis is divided into five chapters. The introduction in Chapter 1 reviews the “birth” of prestressing and its development over the centuries. A brief description of the role of external prestressing along with scope and objective of this study was described.
A survey of all recent literature concerning external prestressing was presented in Chapter 2. The methodology of analyzing external prestressed beams using empirical formulas in the codes with bond reduction coefficients will reviewed together with discussion on existing computer modeling techniques.

In Chapter 3, the theoretical considerations covers the flexural behavior of external prestressing and a details review of the theories involved in using analysis by bond coefficients. The methodology of experimental procedures that will be carried out will be described in detail followed by a review of computer modeling by FEM that will be used to compare with results from theoretical analysis and experiment studies.

Experimental and theoretical analysis results are presented and discussed in Chapter 4. A comparison with the computer analysis results will also be discussed.

Finally, Chapter 5 will review the conclusions drawn from this study with recommendations for further studies in the future.
BIBLIOGRAPHY


