



UNIVERSITI PUTRA MALAYSIA

BEHAVIOUR OF EXTERNALLY PRESTRESSED BEAMS

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**BEHAVIOUR
OF
EXTERNALLY PRESTRESSED BEAMS**

By

Chan Kim Liang

**A Project Report submitted in partial fulfillment of the requirements for
the Degree of Master of Science
in the Faculty of Engineering
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PROJECT APPROVAL SHEET

This project report attached hereby, entitled “**Behaviour of Externally Prestressed Beams**” and submitted by **Chan Kim Liang** in partial fulfillment of the requirement for the degree of Master of Science (Structural Engineering & Construction) is hereby attached.

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“Great works are performed not by strength, but by perseverance”

– Samuel Johnson (1709-1984)

ABSTRACT

Externally prestressed beams are defined as structural concrete beams with external prestressing tendons placed outside the concrete section.

The behaviour of externally prestressed beams was studied by evaluating the flexural response of the beam based on modified bond reduction coefficients to account for second order effects and using a structural software to model the overall behaviour of the beam with incremental loading.

Theoretical calculations based on modified bond reduction coefficients were found to be acceptable within the test results. The predicted results from computer modeling showed that the structural behaviour of externally prestressed beam can be satisfactorily predicted from initial zero load to the ultimate loading stage.

TABLE OF CONTENTS

TITLE	PAGE
Project Approval Sheet	i
Permission To Make Photocopies	ii
Acknowledgement	iii
Abstract	iv
Table of Contents	v
List Of Figures	viii
List Of Tables	x
List Of Appendices	xi
Notation	xiii
 CHAPTER 1 INTRODUCTION	
1.1 General	2
1.2 Historical Development of Prestressing	3
1.3 External Prestressing	5
1.4 Scope and Objectives	8
1.5 Structure of Project Report	9
 CHAPTER 2 LITERATURE REVIEW	
2.1 General	11
2.2 Analysis Based On Bond Reduction Coefficients	11
2.3 Analysis By Modeling	17
2.4 Other Areas Of Research	27

TITLE	PAGE
CHAPTER 3 METHODOLOGY AND THEORETICAL CONSIDERATIONS	
3.1 General	28
3.2 Analysis By Bond Reduction Coefficients	30
3.2.1 Bond Coefficients	30
3.2.2 Ultimate Limit State	35
3.2.3 Deflections	39
3.3 Experimental Programme	40
3.3.1 Beam Fabrication	42
3.3.2 Instrumentation	43
3.3.3 Testing Procedure	44
3.4 Analysis By Computer Modeling	45
CHAPTER 4 EXPERIMENTAL RESULTS AND THEORETICAL ANALYSIS	
4.1 Prestressing Of External Tendons	47
4.2 Experimental Result Of RC 2 & RC3	47
4.3 Prestressing Stresses And Losses	47
4.3.1 The Total Losses	47
4.3.2 The Friction Loss	48
4.4 Load-Deflection Response	55
4.5 Load – Change Of Stress Curve	62
4.6 Cracking Load & Cracking Pattern	72
4.7 Ultimate Tendon Stress	74
4.8 Ultimate Moment of Resistance	75
4.9 Type Of Failure	76

TITLE	PAGE
CHAPTER 5 CONCLUSIONS AND RECOMMENDATION	
5.1 Conclusions	78
5.2 Recommendation	79
Bibliography	80



LIST OF FIGURES

	PAGE
Figure 1.1 Prestressing principle in linear and circular prestressing	4
Figure 2.1 Dowel action in internal tendon and unrestrained movement of external tendon	16
Figure 2.2 Modeling of externally prestressed concrete beam	25
Figure 2.3 Stress-Strain Relationships	25
Figure 2.4 Force equilibrium at a deviator	26
Figure 3.1 Load–Deflection Curve of Externally Prestressed Beam	29
Figure 3.2 Type of loading and configuration of external tendons and deviators	33
Figure 3.3 Bending moment diagrams and strain distributions of concrete at tendon level for beams with different span-depth ratio	34
Figure 3.4 Strain and Stress distributions at critical section of an externally prestressed beam for ultimate flexure limit state	38
Figure 3.5 Flowchart for the analysis of externally prestressed beams at ultimate limit state	38
Figure 3.6 Configuration of test beams	41
Figure 3.7 Typical Setup Of Testing Procedure	44
Figure 3.8 Prototype Beam Model	46
Figure 4.1 Friction Loss Vs Load RC 2 – Experiment	50
Figure 4.2 Friction Loss Vs Load RC 3 – Experiment	50
Figure 4.3 Friction Loss Vs Load RC 1 – Computer Modelling	51
Figure 4.4 Friction Loss Vs Load RC 2 – Computer Modelling	52

Figure 4.5	Friction Loss Vs Load RC 3 – Computer Modelling	53
Figure 4.6	Friction Loss Vs Load RC 4 – Computer Modelling	54
Figure 4.7	Load – Deflection Curve RC 2	57
Figure 4.8	Load – Deflection Curve RC 3	57
Figure 4.9	Load – Deflection Curve RC 1 - Computer Modelling	58
Figure 4.10	Load – Deflection Curve RC 2 – Computer Modelling	59
Figure 4.11	Load – Deflection Curve RC 3 – Computer Modelling	60
Figure 4.12	Load – Deflection Curve RC 4 – Computer Modelling	61
Figure 4.13	Load – Δf_{ps} Curve RC 2 (Position 1 & 2) – Experiment	64
Figure 4.14	Load – Δf_{ps} Curve RC 2 (Position 3 & 4) – Experiment	64
Figure 4.15	Load – Δf_{ps} Curve RC 3 (Position 1 & 2)- Experiment	65
Figure 4.16	Load – Δf_{ps} Curve RC 3 (Position 3& 4) – Experiment	65
Figure 4.17	Load – Δf_{ps} Curve RC 3 (Position 5 & 6) – Experiment	66
Figure 4.18	Load – Tendon Stress RC 2 – Experiment	67
Figure 4.19	Load – Tendon Stress RC 3 – Experiment	67
Figure 4.20	Load – Δf_{ps} Curve RC 1 – Computer Modelling	68
Figure 4.21	Load – Δf_{ps} Curve RC 2 – Computer Modelling	69
Figure 4.22	Load – Δf_{ps} Curve RC 3 – Computer Modelling	70
Figure 4.23	Load – Δf_{ps} Curve RC 4 – Computer Modelling	71
Figure 4.24	Cracking Pattern in RC 2	73
Figure 4.25	Cracking Pattern in RC3	73
Figure 4.26	Flexural –Shear Failure in RC 2 & RC 3	77

LIST OF TABLES

	PAGE
Table 4.1 Losses After Prestressing	48
Table 4.2 Cracking Load	72
Table 4.3 Ultimate Tendon Stress	75
Table 4.4 Ultimate Moment of Resistance	76



LIST OF APPENDICES

	PAGE
APPENDIX A	
CONCRETE MIX DESIGN	A1
CUBE TEST RESULTS	A2
APPENDIX B	
CALCULATION OF NEUTRAL AXIS	B1
APPENDIX C	
CALCULATION OF SECOND MOMENT OF INERTIA	C1
APPENDIX D	
CALCULATION OF PRESTRESS FORCE	D1
APPENDIX E	
CHECK SECTION FOR OVER-REINFORCED SECTION	E1
APPENDIX F	
COMPUTATION OF CRACKING MOMENT USING BS 8110	F1
APPENDIX G	
COMPUTATION OF ULTIMATE MOMENT OF RESISTANCE USING BS 8110	G1
APPENDIX H	
COMPUTATION OF DEFLECTION AND CAMBER USING BS8110	H1
APPENDIX I	
CALCULATION OF ULTIMATE MOMENT OF RESISTANCE BY STRAIN COMPATIBILITY	I1

APPENDIX J

SUMMARY OF SECTION PROPERTIES

J1

APPENDIX K

COMPUTATION OF ULTIMATE MOMENT OF RESISTANCE
USING BOND REDUCTION COEFFICIENT (NG C.K.)

K1

APPENDIX L

Iteration Process to Calculate Ultimate Moment of Resistance
For RC 1 & 2

L1

Iteration Process to Calculate Ultimate Moment of Resistance
For RC 3

L3

Iteration Process to Calculate Ultimate Moment of Resistance
For RC 4

L5

APPENDIX M

COMPUTATIONAL OF ULTIMATE STRESS USING
BOND COEFFICIENTS BY VARIOUS RESEARCHES

M1

APPENDIX N

DETAILS RESULTS OF EXPERIMENTAL
TEST OF RC 2 & RC 3

N1

APPENDIX O

DETAILS RESULTS OF COMPUTER MODELING
USING SAP 2000

O1

APPENDIX P

PHOTOGRAPHS

P1

NOTATION

Notation is generally in accordance with BS 8119, and the principal symbols are listed below. Other symbols are defined in the text or shown in the diagram where necessary.

A_{ps} = area prestress of external tendons

A_s = area of internal tensile reinforcement

A_s' = area of internal compressive reinforcement

c = neutral axis depth at critical section under consideration

d_{ps0} = initial tendon depth

d_s = effective depth of internal tensile reinforcement

d_s' = effective depth of internal compressive reinforcement

e_u = tendon eccentricity

e_0 = initial tendon eccentricity

ε_{cu} = strain in the top of concrete fibre at ultimate (= 0.003) according to ACI Code

E_c = modulus of elasticity of the concrete

E_{ps} = modulus of elasticity of external tendons

f_{ps} = Ultimate stress of prestressing tendons

Δf_{ps} = stress increase at ultimate flexure limit state

f_{pe} = effective prestress of external tendon

f_{py} = yield strength of prestressing external tendon

f_y = yield strength of internal tensile reinforcement

f_y' = yield strength of internal compressive reinforcement

f'_c = concrete cylinder strength

F_i = tensile forces in the cable segments (i) at the deviator (i)

F_{i+1}	= tensile forces in the cable segments $(i+1)$ at the deviator (i) .
h	= overall beam height
I_e	= effective moment of inertia
I_{tr}	=moment of inertia based on the transformed section
I_{cr}	=moment of inertia based on the cracked section
k_c	= depth of centroid of the concrete in compression zone
L	= effective beam span
L_d	= distance from the beam support to deviator point
L_s	= distance from the beam support to loading point
M	=friction coefficient at the deviator
M_u	=ultimate moment of resistance
M_{cr}	=cracking moment of resistance
S_d	= distance between two deviators placed symmetrically with respect to the centre line of the beam.
ρ_p	= prestressing ratio
Ω_u	=bond reduction coefficient at ultimate limit state
Φ_u	= ultimate curvature
Δ_{total}	=maximum total deflection due to prestressing force and external applied load

CHAPTER 1

INTRODUCTION

1.1 General

In reinforced concrete structure, loads which are carried in a member by bending are effectively transferred through it as compressive and tensile forces. Since concrete has low tensile strength, steel reinforcement must be provided in all structural concrete members subject to such bending forces to control tensile cracking and, ultimately, to prevent failure.

Normally steel reinforcements are placed within the tension zones of concrete members to carry the internal tensile forces across flexural cracks to the supports. Here the applied moment is resisted by the compression of the uncracked portion of the concrete section and tension in the reinforcing bars.

However the presence of reinforcement does not prevent the development of tensile cracks in members but only limits the magnitude of the strain in the bars to prevent excessive large cracks.

One major problem with the presence of cracks in ordinary reinforced concrete is the corrosion of the reinforcement when exposed to water and chemical contaminants. A further effect of cracking to ordinary reinforced members is the substantial loss in stiffness which occurs after cracking. The second moment of area of the cracked section, I_c , is far less than the second

moment of area before cracking, I_u . This would cause a large increase in the deformation of the member.

Since corrosion is generally only a problem for structures in aggressive exterior environments (bridges, marine structures, etc.) and is not critical in the majority of buildings, an alternative form of reinforced concrete that was found suitable is prestressed concrete. In prestressed concrete, compressive stresses are introduced into a member to reduce the tensile stresses which result from bending due to the applied loads. The compressive stresses are generated in a member by tensioned steel anchored at the ends of the members and/or bonded to the concrete.

1.2 Historical Development of Prestressing

Prestressed concrete is a new concept which dates back to 1872, when *P. H. Jackson*, an engineer from California, patented a prestressing system that used a tie rod to construct beams or arches from individual blocks. [See [Figure 1.1](#)] In 1888, *C. W. Doehring* of Germany obtained a patent for prestressing slabs with metal wires. However this new concept of prestressing was not successful due the loss of the prestress with time.

The breakthrough for prestressing came when *R. E. Dill* of Alexandria, Nebraska, recognized the effect of the shrinkage and creep (transverse material flow) of concrete on the loss of prestress. He subsequently developed the idea that successive post-tensioning of unbonded rods would

compensate for the time-dependent loss of stress in the rods due to the decrease in the length of the member because of creep and shrinkage.

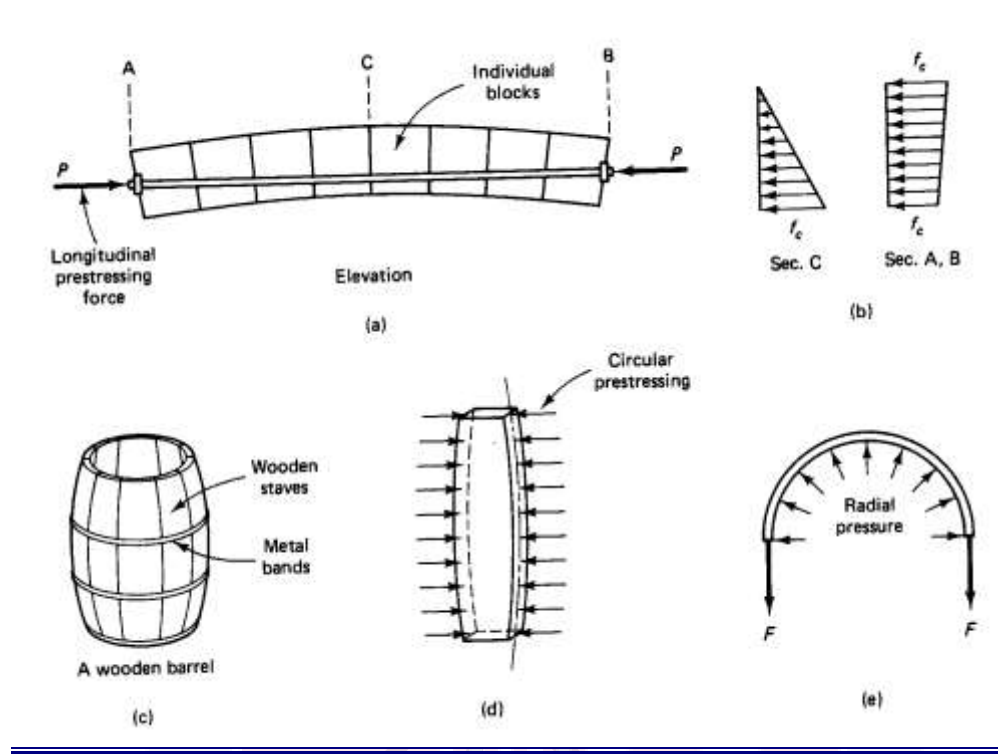


Figure 1.1 Prestressing principle in linear and circular prestressing [*Prestressed Concrete-A Fundamental Approach* by Edward G. Nawy pp.5-18]

Then in the early 1920s, *W. H. Hewett* of Minneapolis developed the principles of circular prestressing which would stressed horizontal reinforcement around walls of concrete tanks to prevent cracking and subsequently achieving watertightness. Thereafter, prestressing of tanks and pipes developed at an accelerated pace in the United States, with thousands of tanks of water, liquid, and gas storage built and much mileage of prestressed pressure pipe laid in the two to three decades that followed.

Linear prestressing continued to develop in Europe and in France through *Eugene Freyssinet*, who proposed in 1926 through 1928 methods to overcome prestress losses through the use of high-strength and high-ductility steels. In 1940, he introduced the now well-known and well-accepted Freyssinet system comprising the conical wedge anchor for 12-wire tendons.

G. Magnel of Ghent, Belgium, and *Y. Guyon* of Paris extensively developed and used the concept of prestressing for the design and construction of numerous bridges destroyed by World War II in western and central Europe. The Magnel system also used wedges to anchor the prestressing wires. They differed from the original Freyssinet wedges in that they were flat in shape, accommodating the prestressing of two wires at a time.

Between 1930s and 1960s, *P. W. Abeles* of England introduced and developed the concept of partial prestressing. *F. Leonhardt* of Germany, *V. Mikhailov* of Russia, and *T. Y. Lin* of the United States also contributed a great deal to the design of prestressed concrete. These twentieth-century developments have led to the extensive use of prestressing throughout the world especially in the United States.

1.3 External Prestressing

External prestressing is a prestressing system in which the concrete structural members are prestressed longitudinally using tendons located completely outside the concrete section to provide flexural resistance. The

flexural response of an externally prestressed beam has been commonly equated to that of a beam with internal unbonded tendons.

It is used not only in new construction works but is also an efficient method for strengthening existing structures. External prestressing is an efficient method in the construction of new bridges especially segmental bridge box girders and in the rehabilitation and strengthening of existing concrete bridge structures.

Some of the advantages of external prestressing system include:-

- More economical construction
- Easier tendon layout, placement and consolidation of concrete
- Better corrosion protection as compared to a conventional tendon system.

The use of external prestressing system to strengthen existing concrete bridges is conceptually different from that used in the traditional design and construction of new concrete bridges.

In the design of new concrete bridges, external tendons are the primary reinforcements and therefore the analysis and design could be achieved using methods similar to unbonded post-tensioned construction.

When used for strengthening, external tendons represent only a part of the total flexural reinforcement. The remaining reinforcement could be in the

form of steel reinforcement, internal steel prestressing or a combination of both, depending of the type of concrete structure to be rehabilitated.

A study to examine experimentally the benefits of external prestressing in strengthening of concrete flexural members was carried out by *Harajli, M.H.* in 1993 with the objective to evaluate its effect on both service load behaviour and nominal flexural resistance.

The experiment included three different structural systems:-

- reinforced concrete,
- prestressed concrete and
- partially prestressed concrete

together with the two profiles of external prestressing namely, straight horizontal and single point draped (deviator at midspan) external tendon.

Results showed that providing external prestressing steel by a relatively moderate amount resulted in the increase in the nominal flexural resistance by up to 146%. This was achieved without significant reduction in ductility or ultimate flexural deformation of member.

In this investigation by *Harajli, M.H.*, external prestressing using various steel profile for different levels of prestressing, was shown to have

- reduce cracks widths or close cracks completely
- reduce the service load deflections induced under cyclic fatigue loading

- stiffer service-load deflection response and therefore a reduction in live load deflection.

External prestressing using a straight horizontal profile was relatively less effective in increasing the flexural response as compared to a deviated profile but the service load-deflection response was similar for these two types of profiles.

The secondary advantage of external prestressing is to prolong the fatigue life of concrete flexural members subjected to repetitive type loading. This is because external prestressing reduces the mean stress level and stress ranges in the internal tension reinforcement which is of primary importance to the fatigue life of concrete flexural members.

1.4 Scope and Objectives

In the study of the behaviour of externally prestressed beams, two factors which makes the analysis complicated are:-

- Stress increase in the external tendons is dependent on the overall deformation of the member
- Eccentricity variations of external tendons under load, commonly referred as second-order effects.

Therefore since the elongation of the external tendons is influenced by the total deformation of the structural member rather than solely on the section under consideration, most of the proposed methods of analysis for the

behaviour of externally prestressed beams involve complicated numerical analysis that are only possible only with the use of computers.

The scope and objectives of this study are as follows:-

- a) To compare the various empirical methods proposed by various researchers to analyse an externally prestressed beam.
- b) To investigate the structural behaviour and performance of an externally prestressed beam experimentally.
- c) To model the behaviour of externally prestressed beams using an existing structural software.

To achieve the stated objectives, theoretical investigations into previous analytical studies will be reviewed. Experimental studies on beams with parameters that include various deviator spacing will be carried out. Finite element method (FEM) using an existing structural software (SAP 2000 -8 Non Linear) to model and analyse the experimental test beams. The structural response of these different beam models will be found in terms of stresses, maximum deflection and ultimate resistances loads. These results can be compared with existing test results.

1.5 Structure of Project Report

This thesis is divided into five chapters. The introduction in Chapter 1 reviews the “birth” of prestressing and its development over the centuries. A brief description of the role of external prestressing along with scope and objective of this study was described.

A survey of all recent literature concerning external prestressing was presented in Chapter 2. The methodology of analyzing external prestressed beams using empirical formulas in the codes with bond reduction coefficients will be reviewed together with discussion on existing computer modeling techniques.

In Chapter 3, the theoretical considerations cover the flexural behavior of external prestressing and a detailed review of the theories involved in using analysis by bond coefficients. The methodology of experimental procedures that will be carried out will be described in detail followed by a review of computer modeling by FEM that will be used to compare with results from theoretical analysis and experiment studies.

Experimental and theoretical analysis results are presented and discussed in Chapter 4. A comparison with the computer analysis results will also be discussed.

Finally, Chapter 5 will review the conclusions drawn from this study with recommendations for further studies in the future.

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