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FINITE ELEMENT ANALYSIS FOR CRASHWORTHINESS OF AUTOMOTIVE SIDE DOOR REINFORCEMENT

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FINITE ELEMENT ANALYSIS FOR CRASHWORTHINESS OF AUTOMOTIVE SIDE DOOR REINFORCEMENT

By

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FINITE ELEMENT ANALYSIS FOR CRASHWORTHINESS OF AUTOMOTIVE SIDE DOOR REINFORCEMENT
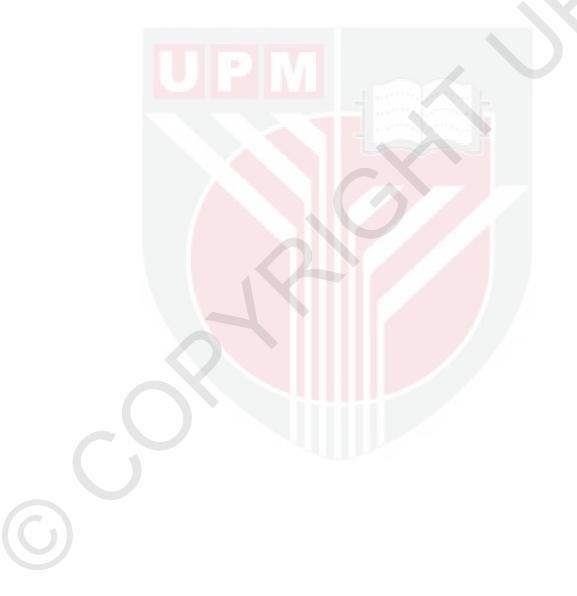
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Chairman: Associate Professor Rizal Bin Zahari, PhD
Faculty: Engineering

Side impact collision is caused 40 percent serious injury during the car accident. In side impact collision, one of the most important concerns for vehicle engineers is allocating sufficient protection to automotive occupants. Although the crash zones which absorbing the energy and decreasing the magnitude of side impact force are less compared with the rear and frontal collision. However, extending the physical space between occupant and door are caused increasing the weight of vehicle. Consequently, the weight reduction of vehicle plays a crucial role to improve fuel efficiency and reducing emission gas. This factor is caused that the thin-wall structures are considered widely in vehicle industry. Side door square beam (SDSB) is one of the important parts in vehicle which plays the important role in absorbing energy of impact force and decreasing deflection. The main objective of this research is to present a practical and systematic methodology for improving the crashworthiness design of vehicle as well as obtaining the optimal design point. This research includes 2 main parts. In the first main part, the importance of side impact collision and the crashworthiness factors (SEA, PL) are investigated for side door square beam (SDSB) subjected to side impact. In the second main part, the proposed methodology is based on respond surface functions including the constrained single objective, weight and geometrical average methods. These optimization methods are employed to get the optimal design point.In this research the crashworthiness improvement and weight reduction of an SDSB are two problems which are considered. Reducing the impact force (PL) and deflection as well as increasing the specific energy absorption (SEA) are three targets which are investigated in this research. The material alloys of aluminium and steel as a reference are assigned to simple structure of SDSB to investigate the maximum SEA. It's observed that aluminium 2011 has the maximum SEA. In addition, the effect of increasing thickness of simple structure made of aluminium 2011 is compared with reinforced structure with the same material to investigate the less deflection and maximum SEA. It's observed that reinforced structure has the less deflection and maximum SEA compared with the simple structure. All the aforementioned comparing steps are performed by LS-DYNA software which is particularly used for finite element crash analysis. A multi-objective optimization is applied to find optimal point design to maintain maximum SEA and minimum PL for reinforced structure. Response surface methodology (RSM) is considered for design optimization in terms

of single optimization and multi-objective optimization. Single –objective optimization is applied to maximize SEA while the impact force is constrained and then minimized impact force while SEA is constrained. In the second part, multi-objective optimization which include the weighted average method and geometrical average method are presented to find the optimal point of structure design. The optimization methods are solved by MATLAB software. The result shows aluminum 2011 is the light weight material to absorb more energy compared with the steel. On the other hand reinforced structure made aluminum 2011 has the less deflection. The achievement structure is optimized to find optimal design point including maximum SEA and minimum PL. The optimal design point of thicknesses is obtained 0.7 mm for the selected reinforced structure and 0.27 mm for the rib. The PL and SEA of this optimal design point is 83171 Kn and   N.mm/ton respectively.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk Ijazah Master Sains

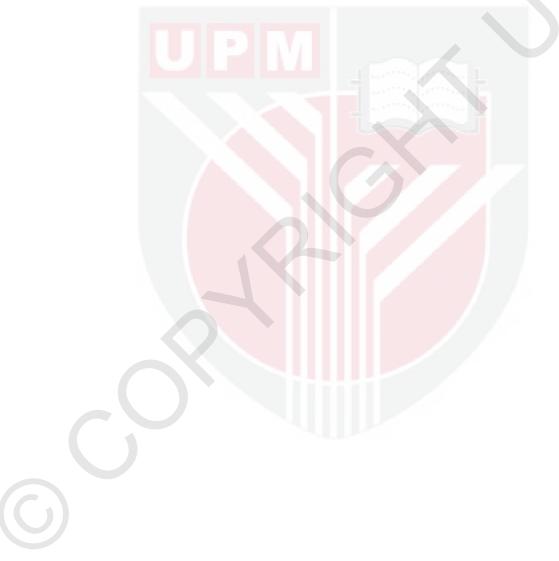# FINITE ELEMENT ANALYSIS FOR CRASHWORTHNISS OF AUTOMOTIVE SIDE DOOR REINFORCEMENT

Oleh

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Pengerusi:Professor Madya Rizal Bin Zahari, PhD
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Sehingga kini, perlanggaran akibat hentaman sisi telah menyumbang kepada 40 peratus daripada kecederaan serius sewaktu kemalangan kereta. Di dalam perlanggaran hentaman sisi, salah satu daripada perkara penting kepada jurutera automotif ialah bagaimana untuk menyediakan perlindungan secukupnya kepada penumpang kereta. Seperti yang diketahui, zon remukkan yang menyerap tenaga pada ketika hentaman sisi berlaku adalah lebih kecil berbanding dengan zon remukkan di bahagian hadapan atau belakang kereta. Bagaimanapun, sekiranya ruang sisi kereta di antara penumpang dan pintu kereta diluaskan ianya akan hanya menambahkan berat kenderaan. Oleh yang demikian, pengurangan berat kenderaan memain peranan bagi meningkatkan kecekepan bahan api serta mengurangkan pemancaran gas. Faktor tersebut telah menyebabkan penggunaan struktur rasuk berdinding nipis digunakan dengan meluasnya sebagai perlindungan tambahan terhadap hentaman bahagian sisi kenderaan. Rasuk keratan segiempat sama (SDSB) merupakan salah satu struktur terpenting yang sering digunakan pada kenderaan bagi menyerap tenaga akibat daya hentaman di samping mengurangkan pesongan. Di dalam kajian ini, peningkatan kebolehremukkan dan pengurangan berat pada SDSB merupakan masaalah yang dipertimbangankan. Pengurangan daya hentaman (PL) dan pesongan serta penambahan penyerapan tenaga tertentu (SEA) merupakan tiga parameter yang diterokai di dalam kajian ini. Dua jenis bahan alloy iaitu keluli dan aluminium dijadikan sebagai bahan struktur mudah SDSB bagi menyiasat SEA maksima. Hasil pemerhatian telah menunjukkan bahawa aluminium 2011 mempunyai SEA yang maksima jika dibandingkan dengan keluli. Sebagai kajian tambahan, kesan penambahan ketebalan struktur mudah diperbuat oleh aluminium 2011 dibandingkan dengan struktur terkukuh bagi menyiasat pesongan dan SAE maksima. Daripada pemerhatian, didapati struktur terkukuh mempunyai pesongan paling kurang dan SAE tertinggi jika dibandingkan dengan struktur mudah tersebut. Kesemua langkah-langkah tersebut dijalankan dengan menggunakan perisian analisis unsur terhingga LS-DYNA yang sering digunakan secara terperinci untuk anlisis remukkan. Seterusnya, pengoptimuman pelbagai-objektif diaplikasikan bagi mencari titik rekabentuk optimum bagi mengekalkan SAE maksima dan pengurangkan PL bagi struktur terkukuh. Kaedah permukaan tindakbalas (RSM) dipertimbangkan bagi pengoptimuman rekabentuk dari segi pengoptimuman objektif tunggal dan
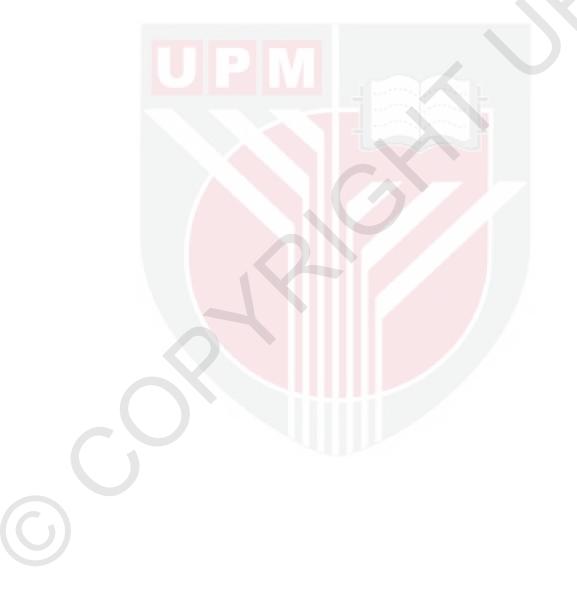
pengoptimuman pelbagai-objektif. Pengoptimuman objektif tunggal digunakan bagi memaksimakan SEA sementara daya hentaman dikekang. Seterusnya, pengoptimuman dijalankan di mana SAE pula dikekang sementara daya hentaman dijadikan minima. Pada bahagian kedua, pengoptimuman pelbagai-objektif termasuk kaedah purata wajaran dan kaedah purata geometri dibentangkan bagi mendapatkan titik optimal rekabentuk struktur. Kaedah pengoptimuman diselesaikan oleh perisian MATLAB. Keputusan menunjukkan bahawa aluminium 2011 adalah lebih ringan dan mempunyai SEA lebih tinggi berbanding dengan keluli. Sebaliknya, struktur terkukuh aluminium 2011 mempunyai pesongan paling minima. Struktur pencapaian dioptimumkan bagi mendapatkan titik rekabentuk optimum termasuk SEA maksima serta PL minima. Titik rekabentuk optimum ketebalan ialah 0.7mm bagi struktur terkukuh yang terpilih manakala 0.27mm untuk struktur rusuk. Titik rekabentuk optimum bagi PL dan SEA ialah masing-masing 83171 kN dan 1.3002×10^{9} N.mm/kg.

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"In the name of Allah, Most gracious, Most merciful"

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LIST OF ABBREVIATIONS

SDSB   Side Door Square Bean

PL   Peak Load

SEA   Specific Energy Absorption

FE   Finite Element models

RBF   Radial Basis Function

RSM   Response Surface Method

CAE   Computer Aided Engineering

CHAPTER 1

INTRODUCTION

1.1 Background

Side impact becomes a serious problem for protecting occupant during the crash due to a little physical space between the occupant and door. By comparison between the other type of accident, its observed that vehicle during the side impact can absorb five times less energy before injury caused to occupants comparing with frontal accident (Fildes et al., 1991). Meanwhile most of researches have been considered the frontal impact. Bumper is the main part of vehicle which they analyzed (Marzbanrad et al., 2009). However, side door body in side impact required more attention due to less crash zone (Ghadianlou & Abdullah, 2013).

These days vehicle safety has become a significant factor in vehicle sales. Safer vehicle has become a goal that each company attempt to accomplish it. According to Khalil and Du Bois (2004) , in modern vehicle design, safer is the first concern. In the past, Vehicle Company spent a lot of money for experimental test to analysis the result. These test are time consuming and so expensive. By applying finite element methods these problems are solved.

Thin wall structure is used widely in Vehicle Company due to their lightweight material. They have a good capacity to absorb energy and resistance to avoid folding and bending during the crash. In this study rectangular beams with simple and reinforced structure are investigated in side impact to analysis the amount of their absorbing energy and deflection. Applying an appropriate thin-walled structure will caused decreasing the amount of peak load. Consequently, the occupant safety will improve. In addition, using these light materials leads to improving fuel efficiency of vehicle.

An optimization method is a numerical approach to find an optimum solution. An approximation method is the approach to collect data of complex design and create approximate models. Hence approximate model is considered to predict crash behaviour. One of the approximated methods is the response surface method (RSM) which is considered mainly in crash responses. Using this method for automotive component bring us crashworthiness improvement. In impact behaviour of vehicle, minimizing the peak load (PL) and maximizing the specific energy absorption (SEA) are two targets which required obtaining simultaneously. In this research work finding the optimal design point is achieved by using RSM optimization methods. This point must meet the maximum SEA and minimum PL.

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1.2 Problem Statement

Improving vehicle fuel efficiently is a significant factor in automotive industry. Decreasing 10 % of vehicle weight leads to save more than five percent of fuel (Zhang et al., 2008). Many researches are analyzed crash but without considering the effect of energy absorption capacity through impact collision. According to statistic, it's reported that 35 % of fatalities are occurred by side impact (Dong et al., 2007). Vehicle safety has become a significant factor in vehicle sales. Safer vehicle has become a goal that each company attempt to accomplish it.

The computational cost of nonlinear finite element method is enormously expensive. A type of automotive structure includes many parts with complex geometry and different materials. The whole collision occurs during approximately 100(mls) and each part go through each other and stresses surpass material elastics. Considering this, each modeling and analysis requires huge calculation. This means, the optimization methods must applied instead of finite element approach.

An acceptable design must meet all aforementioned requirements considering improving fuel efficiently and safer design. A side square door beam plays an important role to absorb the energy of impact force. Consequently, reduction the weight of beam is a significant factor to reduce the fuel consumption. The second problem is improving crashworthiness of the square beam using optimization methods, needs to decrease impact force and increase the energy absorption with respect to limitation of deflection.

1.3 Research Objectives

This research study is present to obtain an optimized reinforced structure of SSDB. A finite element method is employed as a modelling to obtain the PL and SEA. On the other hand single objective and multi-objective optimization tools are used as a means for getting the optimal design point of structure. These objectives of current work are:

1- To investigate the effects of material and modifying structure configuration of SSBD subject to side impact

2- To determine the optimal design point of modifying structure geometry subjected to side impact by applying optimization methods.

Based on previous research studies mentioned in chapter 2 modifying the structure of beam and changing the material are caused improving SEA and reducing of impact force. Consequently, finding the light weight material is caused improving fuel efficiency. Thus, there are research hypotheses which are analyzed to prove aforementioned goals.

1.4   Research Scope

As it is mentioned, the target of this research is to improve the crashworthiness of SSDB. To achieve this goal, the simple rectangular structure of SSDB is selected for analyzing the impact behaviour. The materials which are used for SSDB are alloy aluminium 3105-HB, 2011 and alloy steel AISI1006. These materials are chosen due to using widely in automotive industry. Analyzing the impact behaviour of the modifying structure of SSDB is another factor to find less deflection. Evaluations of impact test are performed by LS_DYNA 3.1 Beta software. Then the optimization methods are applied to obtain the optimal design point. In this research study, the results are optimized by MATLAB software.

1.5   Thesis Organization

This thesis consists of five chapters. First chapter consists of background, problem statement and thesis organization. In the second chapter, literature review is done that it related to the present study. Next chapter is studied on the nonlinear finite element and optimization methodology of square beam. The results of finite element and optimization methods are presented in chapter four. Finally, the last chapter discussed the recommendations and conclusions.

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