



**UNIVERSITI PUTRA MALAYSIA**

***NONLINEAR THREE DIMENSIONAL FINITE ELEMENTS FOR  
COMPOSITE CONCRETE BEAMS***

**OMID KOHNEHPOOSHI**

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**By**

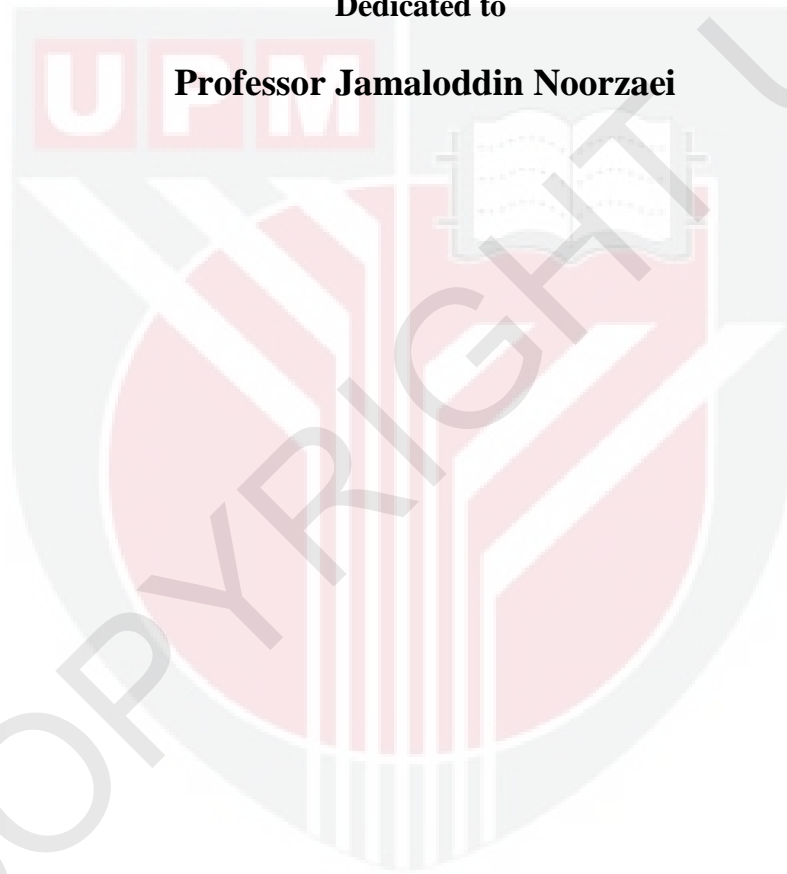
**OMID KOHNEHPOOSHI**

**Thesis Submitted to the School of Graduate Studies, University Putra Malaysia,  
in Fulfilment of the Requirements for the Degree of Doctor of Philosophy**

**April 2012**

**Dedicated to**

**Professor Jamaloddin Noorzaei**



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment  
of the requirement for the degree of Doctor of Philosophy

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COMPOSITE CONCRETE BEAMS**

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**April 2012**

**Chairman: Professor Jamaloddin Noorzaei, PhD**

**Faculty : Engineering**

The use of reinforced concrete (RC) for major structural systems has increased in the recent years. Many structures such as bridges, nuclear containment systems and high rise buildings require sophisticated analysis and design procedures. RC girders for bridges have been widely used in the construction of many interstate systems and can be found in the bridge inventory of most transportation agencies.

Numerical modelling of RC slab-girder bridges is used for realistic analysis of bridge superstructure. However, in the analysis of composite structures, interface structural modelling of various-type elements such as beam, truss, plate, membrane and solid element should be taken into account in order to achieve accuracy. This can be carried out by introducing suitable interface elements and taking interface translation and rotational degrees of freedom into consideration.

The development and application of the interface element to model the interface behaviour between concrete and FRP are still very limited. Some of the available literature reviewed has shown that shell element has been extensively used to model

FRP materials. However, little information is available regarding the use of plate bending element for FRP materials, and hence, there is a need to formulate an interface element which is compatible with the bending characteristics of this material. Therefore in this study, new interface elements have been developed, while modified constitutive law have been applied and new computational algorithm is utilised. The new elements are the Truss-link element to model the interaction between concrete and reinforcement bars, the interface element between two plate bending elements and the interface element to represent the interfacial behaviour between FRP, steel and concrete.

With the tremendous advancement in the computer technologies, three dimensional (3D) finite element (FE) models have been used to study the behaviour of reinforced concrete beams, with and without externally bonded fibre reinforced polymer (FRP) plates. The current investigation focused on the development of effective and suitable modelling of reinforced concrete beam with and without strengthening. The modelling includes physical and constitutive models, from which the nonlinear finite-element (FE) codes were developed with pre- and post-processing. The programme was written using FORTRAN language and it works under FORTRAN-95 Power Station.

The accuracy and efficiency of the finite element programmes were achieved by analyzing several examples from the literature. For instance, the finite element discretisation of different numerical examples was used for the verification purposes and also computational efficiency. The application of the 3D FE code was further enhanced by carrying out the numerical analysis of the RC beams post-tensioned in

the critical shear region, analysis of single lap joint, three dimensional finite element analysis of FRP strengthened RC beams, as well as analysis of the anchored reinforcing bars under monotonic pull-out load and the 3D nonlinear finite element analysis of girder bridge.

During these applications, the distributions of deflection, slip of bars inside the concrete, normal and shear stresses of concrete, axial and shear stresses in the FRP plates, normal and shear stresses of single lap joint and slip of bars due to pull-out test were monitored. Acceptable distributions of slip, deflection, shear and axial stresses in the FRP plate have also been found. These results show that the new elements, which include Truss-linkage element and interface elements, are effective and appropriate to be used for structural component modelling.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**PEMBANGUNAN MODEL UNSUR TERHINGGA TIGA DIMENSI UNTUK  
JAMBATAN KONKRIT TETULANG**

Oleh

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Penggunaan konkrit tetulang (RC) sebagai satu bahan binaan untuk sistem struktur utama telah meningkat sejak kebelakangan ini. Kebanyakan struktur seperti jambatan, sistem kurungan nuklear, dan bangunan tinggi memerlukan analisis dan prosedur reka bentuk yang canggih. Galang RC telah digunakan dengan meluas dalam kebanyakan sistem pembinaan jambatan antara negeri dan boleh didapati dalam inventori jambatan dikebanyakan agensi pengangkutan.

Model berangka bagi jambatan galang-papak RC boleh digunakan untuk analisis superstruktur jambatan yang realistik. Dalam analisis struktur komposit, hubung kait antara sifat pelbagai jenis struktur seperti rasuk, kekuda, plat, membran dan unsur padu seharusnya diambil kira dengan teliti untuk mencapai keputusan yang tepat. Ini dapat dicapai dengan memperkenalkan unsur-unsur antara yang sesuai dan mengambilkira terjemahan pengantaraan dan darjah kebebasan berputar.

Pembangunan dan aplikasi unsur antara untuk model perlakuan antara konkrit dan penggunaan tetulang serat polimer (FRP) adalah terhad. Kajian sorotan menunjukkan

unsur kelompong digunakan dengan banyaknya bagi model unsur bahan FRP. Walaubagaimanapun hanya sedikit maklumat menggunakan unsur lenturan plat bagi unsur bahan FRP yang diperolehi, oleh kerana itu adalah sangat perlu untuk diformulasikan unsur antara yang bersesuaian dengan sifat lenturan bahan ini. Oleh yang demikian, dalam kajian ini, unsur antara baru telah dimajukan, pengubahsuaian model juzukan telah dicadangan dan algoritma pengiraan baru telah digunakan. Unsur-unsur baru termasuk unsur rangkaian-kekuda untuk memodel interaksi antara konkrit dan tetulang, unsur pengantaraan antara dua plat lenturan dan unsur pengantara untuk mewakili perlakuan antara FRP, tetulang dan konkrit.

Dengan kemajuan yang sangat cepat dan banyak dalam teknologi komputer, unsur terhingga tak linear (FE) model tiga dimensi (3D) dibangunkan untuk menyelidik sifat rasuk konkrit dengan dan tanpa peneguhan luaran menggunakan plat tetulang serat polimer (FRP). Penyelidikan ini memfokus kepada pembangunan model yang efektif dan sesuai bagi rasuk konkrit bertetulang dengan dan tanpa penguat. Model tersebut mengandungi model fizikal dan juzukan, yang dengannya kod unsur terhingga telah dibangunkan lengkap dengan pra dan pasca pemprosesan. Kod telah ditulis menggunakan bahasa FORTRAN di dalam FORTRAN-95 Power Station.

Ketepatan dan kecekapan kod unsur terhingga telah dicapai dengan menganalisis beberapa contoh tanda aras yang didapati dari kajian terdahulu. Pendiskretan unsur terhingga bagi contoh berangka yang berbeza digunakan untuk tujuan pengesahan dan juga kecekapan pengiraan. Penggunaan kod 3D FE telah terus dipertingkatkan dengan melaksanakan analisis berangka bagi rasuk RC pasca tegangan dalam rantau ricih yang genting, analisis sambungan tindih tunggal, analisis unsur terhingga tiga



dimensi (3D) bagi rasuk RC yang diperkuatkan dengan FRP, analisis bar tetulang tertambat yang dibebankan secara monotonic dan analisis tiga (3D) dimensi unsur terhingga tak linear bagi galang jambatan.

Ketika penggunaan ini, pengagihan pesongan, gelinciran tetulang di dalam konkrit, normal dan tekanan ricih ke atas konkrit, tekanan paksi dan tekanan ricih dalam plat-plat FRP, tekanan normal dan tekanan ricih ke atas sambungan tindih tunggal, gelinciran tetulang daripada ujian sisipan telah diawasi, apabila berkaitan. Didapati pengagihan gelinciran, pesongan, tekanan ricih dan tekanan paksi dalam plat FRP yang ditemui adalah munasabah. Keputusan ini menunjukkan unsur baru termasuk unsur-unsur rangkaian-kekuda dan unsur-unsur pengantaraan adalah berkesan dan bersesuaian untuk digunakan bagi model komponen struktur.

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I certify that an Examination Committee has met on 6<sup>th</sup> April 2012 to conduct the final examination of **Omid Kohnehpooshi** on his **Doctor of Philosophy** thesis entitled “Three Dimensional finite Element Analysis For Composite Concrete beams” in accordance with University Pertanian Malaysia (Higher Degree) Act 1980 and University Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

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## DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or other institutions.

**OMID KOHNEHPOOSHI**

Date: 6 April 2012

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