



**UNIVERSITI PUTRA MALAYSIA**

**IMPROVING BUCKLING AND POST-BUCKLING OF SHAPE MEMORY  
ALLOY LAMINATED COMPOSITE PLATES SUBJECTED TO MECHANICAL  
AND THERMAL LOADING USING FINITE ELEMENT METHOD**

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**By**

**ZAINUDIN BIN A.RASID**

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of  
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**Chairman: Assoc. Prof. Rizal Zahari, PhD**

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This research work focuses on simulation work involving development of finite element formulations and its finite element based software validated against experimental results reported in the literature to subsequently facilitate parametric studies. Nitinol shape memory alloy with its well-known property of the shape memory effect is used to improve post-buckling of laminated composite plates subjected to mechanical, thermal and thermo-mechanical loadings. Two finite element formulations for the post-buckling of composite plates with embedded shape memory alloy, namely the total strain and the incremental strain formulations are used. Both formulations are derived based on the first order shear deformation theory while the strength of material approach is used to include the effect of recovery stress in the constitutive equation. Thermal loading can be uniform or non-uniform throughout the width and thickness of the composite plates. The properties and recovery stress of the nitinol are either determined by solving the Brinson's model or taken from experimental data of others. The formulations were solved using the Newton-Raphson's method and source codes were developed for this purpose. Parametric studies were conducted theoretically to investigate the effects of the shape memory alloy on the post-buckling behaviour of composite plates with regard to several composite related and shape

memory alloy related parameters. The addition of shape memory alloy wires within layers of composite plates has resulted in the significant improvement in the composite critical loads. In the case of simply supported boundary condition, the increase of the critical load can be up to 70% for the shape memory alloy layer thickness equal to one fourth of the total thickness of other layers. The post-buckling paths of the composite plates subjected to mechanical, thermal and thermo-mechanical loadings are stable and substantially improved after the addition of the shape memory alloy. For the four types of configurations under studied here, the improvement of the active strain energy tuning method is at the highest in the case of the symmetric angle-ply plate where bifurcation for this plate occurs at the ratio of the load over critical load of  $P/P_{cr} = 3$ . It is interesting also to see that while the best mechanical post-buckling paths occur if the shape memory alloy layer is located in the middle of the plate, the location of the shape memory alloy layers has no effect on the thermal post-buckling paths. In the case of the tent-like temperature distribution, the non-uniform temperature distribution where the ratio of the temperature of the uniform temperature rise part to the temperature gradient,  $T_1/T_0 = 1$  has allowed the post-buckling response to occur earlier compared to the case of  $T_0 = 0$  loading. At the same time for both cases of the active property tuning and the active strain energy tuning, the post-buckling paths are improved with the increase of the ratio  $T_1/T_0$ . Furthermore, the thermal post-buckling paths that are degraded initially due to the compressive loading are shown to jump upward significantly with the addition of the shape memory alloy. At the end, this research has shown that the developed model and the source codes are able not only to show the significant improvement made by the shape memory alloy on the post-buckling behaviour of composite plates subjected to mechanical, thermal and thermo-mechanical loadings but also to demonstrate the post-buckling behaviour of the shape memory alloy composite plates subjected to several parameter changes.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**PENAMBAHBAIKAN KEPADA LENGKOKAN AND PASCALENGKOKAN BAGI PLAT KOMPOSIT TERLAMINAT DENGAN ALOI MEMORI BENTUK YANG DIKENAKAN BEBANAN MEKANIK DAN HABA MENGGUNAKAN KAEDAH UNSUR TERHINGGA**

Oleh

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Dalam kajian ini, aloi memori bentuk nitinol dengan kelakuan kesan memori bentuknya yang terkenal itu telah digunakan untuk memperbaiki kelakuan pascalengkokan bagi plat komposit berlapis yang dikenakan bebanan mekanik, haba dan haba mekanik. Dua perumusan unsur terhingga untuk kelakuan pascalengkokan bagi plat komposit dengan aloi memori bentuk terbenam iaitu formulasi terikan jumlah dan formulasi terikan tokokan telah digunakan. Kedua-dua model telah diterbitkan berasaskan teori ubah bentuk ricih tertib pertama di mana pendekatan kekuatan bahan telah digunakan bagi memasukkan kesan tegasan pemulihan ke dalam persamaan jujuk bagi plat komposit. Bebanan haba adalah termasuklah yang seragam atau tidak seragam di sepanjang keluasan dan ketebalan plat. Kelakuan dan tegasan pemulihan bagi aloi memori bentuk ditentukan dengan menyelesaikan model Brinson ataupun dengan menggunakan data-data ujikaji pengkaji lain. Kedua-dua model telah diselesaikan menggunakan kaedah Newton-Raphson dan kod sumber telah dibangunkan untuk tujuan ini. Kajian berparameter telah dilakukan secara teori dalam menyiasat kesan aloi memori bentuk ke atas kelakuan pascalengkokan bagi plat komposit berlapis merujuk kepada beberapa kelakuan yang berkaitan dengan komposit dan kelakuan yang berkaitan dengan aloi memori bentuk. Penambahan aloi

memori bentuk di antara lapisan plat komposit telah menghasilkan penambahbaikan yang bererti kepada beban kritikal. Untuk kes keadaan sempadan plat disokong mudah, penambahan beban kritikal sebanyak 70% terhasil bila ketebalan lapisan aloi memori bentuk adalah satu perempat ketebalan lapisan yang berbaki. Laluan pascalengkokan bagi plat komposit adalah didapati stabil dengan penambahbaikan yang bererti bagi semua kes-kes bebanan mekanik, haba dan mekanik haba setelah aloi memori bentuk dibenam. Bagi empat jenis tatarajah yang dikaji di sini, penambahbaikan adalah tertinggi dalam kes plat lapis-serong simetri di mana dwi-kewujudan berlaku pada nisbah beban kepada beban kritikal,  $P/P_{cr} = 3$ . Adalah menarik untuk menyaksikan laluan pascalengkokan bagi kes bebanan haba tidak dipengaruhi oleh kedudukan lapisan aloi memori bentuk sedangkan bagi kes bebanan mekanik, laluan yang terbaik adalah bila lapisan aloi memori bentuk berada di tengah-tengah plat. Bagi kes bebanan taburan suhu seperti khemah, taburan suhu yang tak seragam di mana nisbah suhu bahagian tak seragam kepada suhu bahagian tetap,  $T_1/T_0 = 1$  telah membenarkan tindakbalas pascalengkokan berlaku lebih awal berbanding kes  $T_0 = 0$ . Pada masa yang sama, bagi kedua-dua kes penalaan sifat aktif dan penalaan tenaga terikan aktif, laluan pascalengkokan bertambah baik dengan penambahan nisbah suhu  $T_1/T_0$ . Tambahan pula, laluan pascalengkokan yang telah berkurang akibat bebanan mekanik yang telah dikenakan lebih awal telah didapati mengalami anjakan yang besar ke arah kedudukan lebih baik dengan penambahan aloi memori bentuk. Akhir sekali, kajian ini telah menunjukkan bahwa model-model yang telah dibangunkan bersama kod sumbernya telah bukan sahaja menzahirkan penambahbaikan yang bererti yang dibuat oleh aloi memori bentuk terhadap kelakuan pascalengkokan plat komposit berlapis yang dikenakan beban mekanik, haba dan haba-mekanik bahkan juga memperlihatkan kelakuan pascalengkokan plat terhadap beberapa perubahan parameter.

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I certify that a Thesis Examination Committee has met on **the 19<sup>th</sup> November 2012** to conduct the final examination of Zainudin bin A. Rasid on his thesis entitled **“Improving Buckling and Post-Buckling of Shape Memory Alloy Laminated Composite Plates Subjected to Mechanical and Thermal Loading Using Finite Element Method”** in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U. (A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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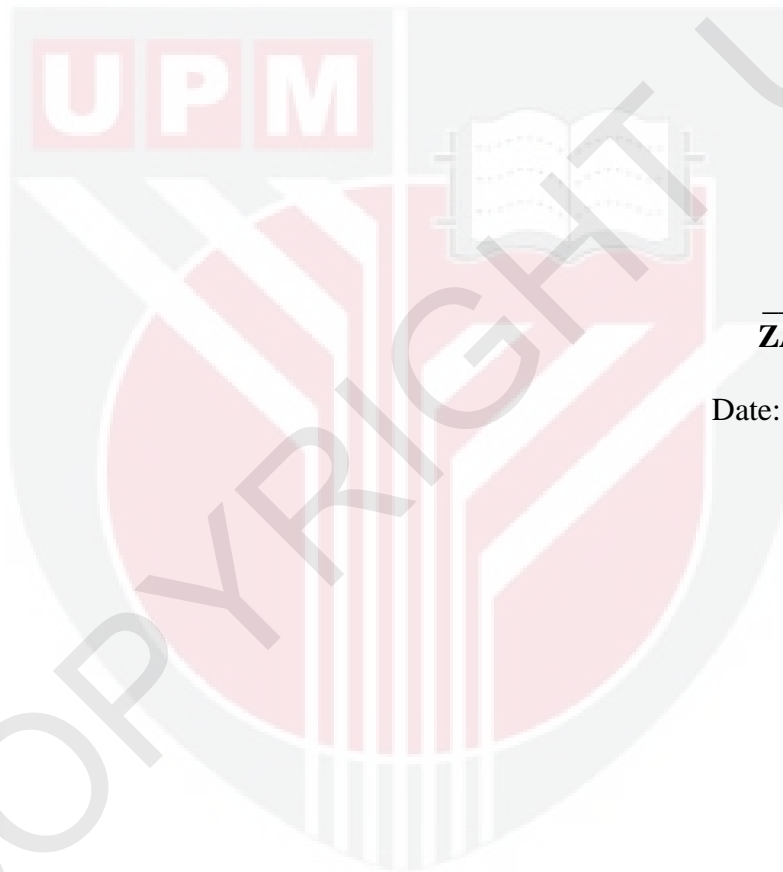
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## DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.



**ZAINUDIN A. RASID**

Date: 19<sup>th</sup> November, 2012

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