BLOCK BACKWARD DIFFERENTIATION FORMULA FOR SOLVING ORDINARY AND ALGEBRAIC DIFFERENTIAL EQUATIONS

By

NAGHMEH ABASI

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

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DEDICATIONS

To

My lovely parents

and

My adorable sisters, Neda and Nikoo

and

My beloved brother, Mohammad Hasan
Abstract of thesis presented to the Senate of Universiti Putra Malaysia in
fulfilment of the requirement for the degree of Doctor of Philosophy

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January 2014

Chair: Dato’ Mohamed Bin Suleiman, PhD
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This research focuses on solving semi-explicit index-1 Differential Algebraic Equations (DAEs) which is a special case of Differential Algebraic Equations (DAEs). Block Backward Differentiation Formula (BDF) methods of constant and variable step sizes are considered to produce more than one solutions per step for the DAEs concurrently. A formula of the 2-point with off-step points using block BDF method of constant step size for solving stiff ODEs is developed. The stability analysis shows that the method is A-stable. The method has competitive results in comparison with the existing block BDF method in terms of accuracy and time. The 2-point, 3-point and 2-point with off-step points block backward differentiation formulae of constant step size are extended for solving semi-explicit index-1 Differential Algebraic Equations (DAEs). Newton’s iteration is used for the implementation of the methods. It is seen that the block BDF methods applied are more suitable than the existing BDF method in terms of accuracy and the time is competitive. In addition, a 3-point block backward differentiation formula using variable step size for solving stiff Ordinary Differential Equations (ODEs) is formulated. The strategy applied for selecting the step size and the stability regions are described. The accuracy of the developed method is seen to be better than the existing variable step block BDF. Solving semi-explicit index-1 DAEs using 2-point and 3-point block backward differentiation formula of variable step size are also considered. The strategies involved in the choosing and controlling the step size of both methods are described. The codes developed indicate that the methods have outperformed the existing method in reducing the error while the time is competitive. The numerical results indicate that the block BDF methods of constant and variable step size for solving semi-explicit index-1 DAEs have better accuracy and efficiency in comparison with the existing constant and variable step BDF methods.
Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

KAEDAH BLOK FORMULASI BEZA KE BELAKANG UNTUK MENYELESAIKAN PERSAMAAN PEMBEZAAN BIASA DAN ALJABAR

Oleh

NAGHMEH ABASI

Januari 2014

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I certify that a Thesis Examination Committee has met on 10 January 2014 to conduct the final examination of Naghmeh Abasi on her thesis entitled "Block Backward Differentiation Formula for Solving Ordinary and Algebraic Differential Equations" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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