UNIVERSITI PUTRA MALAYSIA

BLOCK BACKWARD DIFFERENTIATION FORMULA FOR SOLVING ORDINARY AND ALGEBRAIC DIFFERENTIAL EQUATIONS

NAGHMEH ABASI

IPM 2014 1

BLOCK BACKWARD DIFFERENTIATION FORMULA FOR SOLVING ORDINARY AND ALGEBRAIC DIFFERENTIAL EQUATIONS

By

NAGHMEH ABASI

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

January 2014

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DEDICATIONS

To

My lovely parents

and

My adorable sisters, Neda and Nikoo

and

My beloved brother, Mohammad Hasan

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

BLOCK BACKWARD DIFFERENTIATION FORMULA FOR SOLVING ORDINARY AND ALGEBRAIC DIFFERENTIAL EQUATIONS

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January 2014

Chair: Dato' Mohamed Bin Suleiman, PhD
Faculty: Institute for Mathematical Research (INSPEM)

This research focuses on solving semi-explicit index-1 Differential Algebraic Equations (DAEs) which is a special case of Differential Algebraic Equations (DAEs). Block Backward Differentiation Formula (BDF) methods of constant and variable step sizes are considered to produce more than one solutions per step for the DAEs concurrently. A formula of the 2-point with off-step points using block BDF method of constant step size for solving stiff ODEs is developed. The stability analysis shows that the method is A-stable. The method has competitive results in comparison with the existing block BDF method in terms of accuracy and time. The 2-point, 3-point and 2-point with off-step points block backward differentiation formulae of constant step size are extended for solving semi-explicit index-1 Differential Algebraic Equations (DAEs). Newton's iteration is used for the implementation of the methods. It is seen that the block BDF methods applied are more suitable than the existing BDF method in terms of accuracy and the time is competitive. In addition, a 3-point block backward differentiation formula using variable step size for solving stiff Ordinary Differential Equations (ODEs) is formulated. The strategy applied for selecting the step size and the stability regions are described. The accuracy of the developed method is seen to be better than the existing variable step block BDF. Solving semi-explicit index-1 DAEs using 2-point and 3-point block backward differentiation formula of variable step size are also considered. The strategies involved in the choosing and controlling the step size of both methods are described. The codes developed indicate that the methods have outperformed the existing method in reducing the error while the time is competitive. The numerical results indicate that the block BDF methods of constant and variable step size for solving semi-explicit index-1 DAEs have better accuracy and efficiency in comparison with the existing constant and variable step BDF methods.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putran Malaysia
sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

KAEDAH BLOK FORMULASI BEZA KE BELAKANG UNTUK MENYELESAIKAN PERSAMAAN PEMBEZAAN BIASA DAN ALJABAR

Oleh

NAGHMEH ABASI

Januari 2014

Pengerusi: Dato' Mohamed Bin Suleiman, Ph.D.
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Kajian ini tertumpu kepada penyelesaian persamaan pembezaan aljabar (PPA) semi tak tersirat indeks-1 yang merupakan kes khas PPA . Kaedah blok formulasi beza ke belakang (FBB) dengan saiz langkah tetap dan saiz langkah boleh ubah dipertimbangkan untuk menghasilkan lebih daripada satu penyelesaian serentak dalam setiap langkah. Satu formula 2-titik FBB dengan saiz langkah tetap dengan titik-titik luar langkah untuk menyelesaikan persamaan pembezaan biasa (PPB) kaku dibangunkan. Analisis kestabilan menunjukkan kaedah ini adalah A stabil. Kaedah ini mempunyai keputusan yang kompetitif dibandingkan dengan blok FBB yang sedia ada dari segi kejituan dan masa. Blok FBB 2-titik, 3-titik dan 2-titik luar langkah dengan saiz langkah tetap di perluaskan untuk menyelesaikan PPA semi tak-tersirat indexs-1. Lelaran Newton digunakan bagi pelaksanaan kaedah tersebut. Dilihat bahawa penggunaan kaedah ini adalah lebih sesuai berbanding kaedah FBB yang sedia ada serta kompetitif dari segi kejituan dan masa. Blok FBB 3-titik yang menggunakan saiz langkah boleh ubah dirumus untuk menyelesaikan PBB kaku. Strategi yang digunakan untuk memilih saiz langkah dan rantau kestabilan dihuraikan. Kejituan kaedah yang dibangunkan dilihat lebih baik daripada langkah boleh ubah blok FBB yang sedia ada. Penyelesaian PPA semi taktersirat indeks-1 yang menggunakan blok FBB 2-titik dan 3-titik dengan saiz langkah boleh ubah juga dipertimbangkan. Strategi-strategi yang terlibat untuk memilih dan mengawal saiz langkah dalam kedua-dua kaedah dihuraikan. Kod-kod yang dibangunkan menunjukkan kaedah yang dibangunkan mengatasi kaedah yang sedia ada dalam mengurangkan ralat masa pengiraan adalah kompetitif. Keputusan berangka menunjukkan bahawa kaedah blok FBB yang diperluaskan kepada saiz langkah tetap dan boleh ubah mempunyai kejituan dan kecekapan yang lebih baik berbanding dengan kaedah FBB saiz langkah tetap dan boleh ubah apabila menyelesaikan PPA semi tak tersirat indeks-1.

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I certify that a Thesis Examination Committee has met on 10 January 2014 to conduct the final examination of Naghmeh Abasi on her thesis entitled "Block Backward Differentiation Formula for Solving Ordinary and Algebraic Differential Equations" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy.

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