SERIAL AND PARALLEL SYSTEMS IN LONGITUDINAL LIFETIMA DATA

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Introductions

Lifetime data arise from either a failure due to a single cause/risk alone or a failure in the presence of other causes (Cox and Oakes, 1984). The latter phenomenon is known as competing risks. A useful analogy of a failure of an individual from one of several competing risks is that of a failure in a series system due to the malfunctioning of one of the components. However, in a certain situation failure does not actually occur when a particular component fails. This is analogous to the survival of a parallel system consisting of several components. Analysis of life data (Kalbfeisch) and Prentice, 1980) from life testing, reliability test and clinical trials normally use model based on the serial system. There are cases where the parallel system best described the data concerned.

Materials and Methods

A computerised data archives were prepared for quick and efficient access and retrieval. Parametric assumptions on the underlying distributions of lifetime were first examined in detail with main emphasis when the risks were assumed independent. The case when the risks were dependent was then pursued. Various models supporting the mixture system were investigated through simulation studies and through true life data. The validity of the model was then examined and statistical dignostics were carried out. Simulation studies were done in generating data points of desired properties through special computer programme and through some appropriate simulation packages. The statistical properties inherent to the model were investigated. Attempt would be made to develop some new diagnostic methods to assess the validity of the model and to assess the influence of cases that matters. As customary of lifetime data the concomitant variables or covariates play an important statistical role. The effect of these covariates associated with an individual on the failure time was incorporated in the model by expressing the parameters of the distribution as a log-linear function of these covariates. For parallel models with m unit system derive from random variables that were identically distributed and follows the exponential then the density of

$T = \max(X_1, X_2, \ldots, X_m)$ is given by

$$h(t, \lambda) = \frac{m}{\lambda}\exp\left(-\frac{t}{\lambda}\right)\left(1 - \exp\left(-\frac{t}{\lambda}\right)\right)^{(m-1)}$$

Let $\lambda = \exp(\beta Z)$, where $Z = (1, z_1, z_2, \ldots, z_p)'$ is a vector of covariates associated with the individual, and $\beta = (\beta_0, \beta_1, \ldots, \beta_p)'$ is a vector of regression paremeters, then transforming $u = \log(t)$ yields a density

$$g(u, Z) = m\exp(u - \beta Z)\exp[-\exp(u - \beta Z)]\{1 - \exp[-\exp(u - \beta Z)]\}^{(m-1)}$$

which defines a regression model given by $u_i = \beta Z_i + \varepsilon_i$ where each error term ε_i is distributed with density

$$f(\varepsilon) = m\exp(\varepsilon)\exp[-\exp(\varepsilon)]\{1 - \exp[-\exp(\varepsilon)]\}^{(m-1)}$$

The observed life time is $y_i = \min(u_i, c_i)$, $i = 1, 2, \ldots, n$ where c_i are censoring values together with the indicator variable

$$\delta_i = \begin{cases} 1 ; & u_i \le c_i \\ 0 ; & u_i > c_i \end{cases}$$

The Maximum Likelihood Estimation (MLE), (Hamada and Tse, 1988) for the lifetime data with censoring begins with the formation of the likelihood function. With $w_i = y_i - \beta Z_i$ the likelihood is thus:

$$l(\beta) = \sum_{i=1}^{n}\{\delta_i \log(g(w_i)) + (1 - \delta_i)\log(1 - G(w_i))\}$$

where $G(w_i)$ is the distribution function of W.

Results and Discussion

The estimated parameters were obtained by the methods of iterative least square (Schmee and Hahn, 1979). Appropriate models were then established for various real data and compared to that from simulations. The performance of Wald, Rao and the likelihood ratio statistics concerning the regression parameters were investigated. They appear to be in agreement in so far if the sample size was large enough, that is, the tests were asymptotically equivalent and optimal. However the likelihood ratio performs better in small finite sample, and it appears to be more applicable to model under study The study also shows that the likelihood ratio is superior than the other two statistics for testing hypotheses of the regression parameter. It has also good power performance.

Conclusions

Many lifetime data can be modelled statistically as an m-unit parallel system with error components from the exponentially family. Initially the standard exponential distribution is assumed, and the covariates associated with an individual were incorporated through the min of the distribution as linear combination which enables us to use the iterative least square method to estimate the parameters. We were able to identify the superiority of some of the test statistics for testing hypotheses of the regression parameter.

References

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