UNIVERSITI PUTRA MALAYSIA

NUMERICAL SOLUTIONS OF HYPERSINGULAR INTEGRALS AND INTEGRAL EQUATIONS OF THE FIRST KIND

SUZAN JABBAR OBAIYS

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By

SUZAN JABBAR OBAIYS

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

October 2013

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DEDICATIONS

Specially Dedicated to

Mum and Dad,

My Husband, Ahmad Fahad,

And,

My lovely sons Anas, Anwar and Sanad who have always stood by me and dealt with all of my absence from many family occasions with a smile.

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Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

NUMERICAL SOLUTIONS OF HYPERSINGULAR INTEGRALS AND INTEGRAL EQUATIONS OF THE FIRST KIND

By

SUZAN JABBAR OBAIYS

October 2013

Chairman: Associate Professor Zainidin Eshkuvatov, Ph.D.

Faculty: Institute For Mathematical Research

In this thesis, two problems are considered:

i) An automatic quadrature scheme is presented for the evaluation of hypersingular integral of the form

$$Q_i(f,x) = \int_{-1}^{1} \frac{w_i(t)f(t)}{(t-x)^2}dt, \quad x \in [-1,1],\ i = 0,1,2, \tag{1}$$

where $w_0(x) = 1,\ w_1(x) = \sqrt{1-x^2},\ w_2(x) = \frac{1}{\sqrt{1-x^2}}$ are the weights, and the function f imperatives to have certain smoothness or continuity properties.

ii) We also described the approximate solutions of hypersingular integral equations of the form

$$\int_{-1}^{1} Q(t)\Big[\frac{K(t,x)}{(t-x)^2} + L(t,x)\Big]dt = f(x), \quad x \in (-1,1), \tag{2}$$

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where $K(t,x)$ and $L(t,x)$ are regular square-integrable functions of t and x, and $K(x,x) \neq 0$. The density function $Q(t)$ satisfies the Hölder-continuous first derivative, means that $Q(t) \in C^{1,\alpha}[-1,1]$. The real function f is approximated by the orthogonal Chebyshev polynomials of the first and second kinds $T_n(x)$ and $U_n(x)$ respectively.

For the first problem in (1), an automatic quadrature scheme (AQS) for hypersingular integrals is derived. The numerical results show that the Chebyshev polynomials give a very good approximation by choosing the appropriate weight function. Particular attention is paid to the error estimate of the numerical solutions of Eq. (1). The error rate is calculated by Chebyshev norm for the class of functions $C^{N+2,\alpha}[-1,1]$, which is defined as

$$\parallel e_N \parallel_c = \max_{-1 \leq a \leq t \leq b \leq 1} |f(t) - P_N(t)|. \tag{3}$$

For the second problem in (2), we first consider the characteristic hypersingular integral equation of the form

$$\frac{1}{\pi} \,=\!\!\!\!\!\!\int_{-1}^{1} \frac{\phi(t)dt}{(t-x)^2} = f(x), \qquad |x| < 1, \tag{4}$$

where $K(t,x) = 1$ and $L(t,x) = 0$. By applying the Galerkin method, Eq. (4) can be reduced to a system of linear algebraic equations. The exactness of the numerical solutions of Eq. (4), when the density function $\phi(t)$ is a polynomial of degree 3, is proved.

While for the case of $K(t,x) = 1$ and $L(t,x) \neq 0$, an efficient expansion method for approximating the solution of Eq. (2) is presented.

MATLAB codes are developed to obtain the numerical results for all proposed problems. The numerical examples assert the theoretical results

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Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

NUMERICAL SOLUTIONS OF HYPERSINGULAR INTEGRALS AND INTEGRAL EQUATIONS OF THE FIRST KIND

Oleh

SUZAN JABBAR OBAIYS

October 2013

Pengerusi: Professor Madya Zainidin K. Eshkuvatov, Ph.D.

Fakulti: Institute Penyelidikan Matematik

Dalam tesis ini, dua permasalahan dipertimbangkan:

i) Skim Quadrature Automatik dipersembahkan untuk menilai kamiran hipersingular dalam bentuk:

$$Q_i(f,x) = \int_{-1}^{1} \frac{w_i(t)f(t)}{(t-x)^2} dt, \quad x \in [-1,1],\ i = 0,1,2, \tag{1}$$

dengan, $w_0(x) = 1$, $w_1(x) = \sqrt{1-x^2}$, $w_2(x) = \frac{1}{\sqrt{1-x^2}}$ adalah pemberat, dan fungsi f adalah tertakluk kepada ciri kemulusan dan keselanjaran.

ii) Kami juga menggambarkan penyelesaian hampir bagi kamiran hipersingular dalam bentuk

$$\int_{-1}^{1} Q(t)\Big[\frac{K(t,x)}{(t-x)^2} + L(t,x)\Big] dt = f(x), \quad x \in (-1,1), \tag{2}$$

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dengan $K(t,x)$ dan $L(t,x)$ adalah fungsi regular dalam t dan x yang boleh kamir kuasa dua dan $K(x,x) \neq 0$. Fungsi ketumpatan $Q(t)$ mematuhi terbitan pertama keselanjaran Hölder $Q(t) \in C^{1,\alpha}[-1,1]$.

Fungsi nyata f dihampirkan dengan polinomial Chebyshev orthogonal jenis pertama dan kedua $T_n(x)$ dan $U_n(x)$.

Untuk masalah pertama dalam (1), skim quadrature automatik (AQS) untuk kamiran hipersingular diterbitkan. Keputusan berangka menunjukkan polinomial Chebyshev memberikan penghampiran paling baik dengan pilihan fungsi pemberat yang sesuai.

Tumpuan khusus diberikan kepada anggaran ralat penyelesaian berangka bagi persamaan (1). Kadar ralat telah dikira dengan norma Chebyshev untuk fungsi kelas $C^{N+2,\alpha}[-1,1]$, yang didefinasikan sebagai

$$\| e_N \|_c = \max_{-1 \le a \le t \le b \le 1} |f(t) - P_N(t)|. \tag{3}$$

Untuk permasalahan dalam (2), kami pertimbangkan persamaan kamiran hipersingular cirian dalam bentuk

$$\frac{1}{\pi} \,=\!\!\!\!\!\!\int_{-1}^{1} \frac{\phi(t)dt}{(t-x)^2} = f(x), \qquad |x| < 1, \tag{4}$$

dengan $K(t,x) = 1$ dan $L(t,x) = 0$. Dengan mengaplikasi kaedah Galerkin, persamaan Eq. (4) boleh ditulis sebagai sistem persamaan linear aljabar. Ketepatan penyelesaian berangka bagi Eq. (4) apabila fungsi ketumpatan $\phi(t)$ adalah polinomial darjah ketiga, dibuktikan.

Manakala bagi kes $K(t,x) = 1$ dan $L(t,x) \neq 0$, kaedah pengembangan efisien untuk penghampiran kepada penyelesaian bagi Eq. (2) dipersembahkan.

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Kod MATLAB dibangunkan untuk mendapatkan penyelesaian berangka bagi semua masalah yang dicadangkan. Contoh berangka mengesahkan keputusan teoritikal.

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I certify that a Thesis Examination Committee has met on 2 September 2013 to conduct the final examination of **Suzan Jabbar Obaiys** on her thesis entitled **Numerical Solutions Of Hypersingular Integrals And Integral Equations Of The First Kind** in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the **Doctor of Philosophy**.

Members of the Thesis Examination Committee were as follows:

Fudziah Ismail, PhD
Professor
Faculty of Science
Universiti Putra Malaysia
(Chairperson)

Mohd Rizam Abu Bakar, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Internal Examiner)

Ali Hassan Mohamed Murid, PhD
Associate Professor
Faculty of Science
Universiti Teknologi Malaysia
(Internal Examiner)

Takemitsu Hasegawa, Ph.D.
Professor
Department of Information Science
Faculty of Engineering
University of Fukui
Japan
(External Examiner)

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NORITAH OMAR, Ph.D.
Associate Professor and Deputy Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy.

The members of the Supervisory Committee were as follows:

Zainidin K. Eshkuvatov, Ph.D.
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Chairperson)

Nik Mohd Asri Nik Long, Ph.D.
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Member)

Zanariah Abdul Majid, Ph.D.
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Member)

BUJANG BIN KIM HUAT, Ph.D.
Professor and Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

SUZAN JABBAR OBAIYS

Date: 2 October 2013

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