

## **A method of estimating the p-adic sizes of common zeros of partial derivative polynomials associated with a complete cubic form**

### **ABSTRACT**

Let  $x = (x_1, x_2, \dots, x_n)$  be a vector in the space  $Q^n$  with  $Q$  field of rational numbers and  $q$  be a positive integer,  $f$  a polynomial in  $x$  with coefficient in  $Q$ . The exponential sum associated with  $f$  is defined as  $S(f; q) = \sum_{x \pmod q} \exp(2\pi i f(x)/q)$ , where the sum is taken over a complete set of residues modulo  $q$ . The value of  $S(f; q)$  depends on the estimate of cardinality  $|V|$ , the number of elements contained in the set  $V = \{x \pmod q \mid f_x \equiv 0 \pmod q\}$  where  $f_x$  is the partial derivative of  $f$  with respect to  $x$ . To determine the cardinality of  $V$ , the p-adic sizes of common zeros of the partial derivative polynomials need to be obtained. In this paper, we estimate the p-adic sizes of common zeros of partial derivative polynomials of  $f(x, y)$  in  $Q_p[x, y]$  with a complete cubic form by using Newton polyhedron technique. The polynomial is of the form  $f(x, y) = ax^3 + bx^2y + cxy^2 + dy^3 + 32ax^2 + bxy + 12cy^2 + sx + ty + k$ .

**Keyword:** Exponential sums; Cardinality; P-adic sizes; Newton polyhedron