



**UNIVERSITI PUTRA MALAYSIA**

***ONE-DIMENSIONAL HIGH-ORDER COMPACT METHOD  
FOR SOLVING EULER'S EQUATIONS***

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**FK 2010 5**

**ONE-DIMENSIONAL HIGH-ORDER COMPACT METHOD FOR  
SOLVING EULER'S EQUATIONS**

By

**MAHMUD ABD HAKIM BIN MOHAMAD**

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia,  
in Fulfilment of the Requirements for the Degree of Master of Science**

**September 2010**

Dedicated

to

My beloved wife Hafsa,

Our daughters Sumaiyah & Najihah

All My Parents,

Hj. Mohamad and Hjh. Jawahil, Ir. Hj. Mohamed Makki and Hjh. Siti Rohani

For all their love, support, motivation and understanding

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

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**Chairman: Professor Ir. ShahNor Basri, PhD, F. A. Sc.**

**Faculty : Engineering**

In the field of computational fluid dynamics, many numerical algorithms have been developed to simulate inviscid, compressible flows problems. Among those most famous and relevant are based on flux vector splitting and Godunov-type schemes. Previously, this system was developed through computational studies by Mawlood (2004). However the new test cases for compressible flows, the shock tube problems namely the receding flow and shock waves were not investigated before by Mawlood (2004). Thus, the objective of this study is to develop a high-order compact (HOC) finite difference solver for one-dimensional Euler equation. Before developing the solver, a detailed investigation was conducted to assess the performance of the basic third-order compact central discretization schemes. Spatial discretization of the Euler equation is based on flux-vector splitting. From this observation, discretization of the convective flux terms of the Euler equation is based on a hybrid flux-vector splitting, known as the advection upstream splitting method (AUSM) scheme which combines the accuracy of flux-difference splitting and the robustness of flux-vector splitting. The AUSM scheme is based on the third-order compact scheme to the approximate finite difference equation was completely

analyzed consequently. In one-dimensional problem for the first order schemes, an explicit method is adopted by using time integration method. In addition to that, development and modification of source code for the one-dimensional flow is validated with four test cases namely, unsteady shock tube, quasi-one-dimensional supersonic-subsonic nozzle flow, receding flow and shock waves in shock tubes. From these results, it was also carried out to ensure that the definition of Riemann problem can be identified. Further analysis had also been done in comparing the characteristic of AUSM scheme against experimental results, obtained from previous works and also comparative analysis with computational results generated by van Leer, KFVS and AUSMPW schemes. Furthermore, there is a remarkable improvement with the extension of the AUSM scheme from first-order to third-order accuracy in terms of shocks, contact discontinuities and rarefaction waves.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia  
sebagai memenuhi keperluan untuk ijazah Master Sains

**KAEDAH PADAT PERINGKAT TINGGI SATU DIMENSI UNTUK  
MENYELESAIKAN PERSAMAAN EULER**

Oleh

**MAHMUD ABD HAKIM BIN MOHAMAD**

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Terdapat banyak algoritma berangka dalam bidang dinamik bendalir pengkomputeran, yang dihasilkan untuk simulasi aliran likat termampat. Di antara skim-skim yang paling terkenal dan relevan adalah berdasarkan pada jenis skim “Flux Vector Splitting” dan skim “Godunov”. Sebelumnya, sistem ini dibangunkan melalui kajian pengkomputeran oleh Mawlood (2004). Namun ujian kes baru untuk aliran termampat, masalah tiub kejutan iaitu aliran surut dan gelombang kejutan tidak diselidiki terlebih dahulu oleh Mawlood (2004). Dengan demikian, objektif kajian ini ialah untuk membangunkan penyelesaian perbezaan terhingga peringkat tinggi padat (HOC) bagi persamaan Euler satu dimensi. Satu penyiasatan rapi telah dijalankan bagi menilai keupayaan skim pendiskretan padat peringkat tiga asas pusat sebelum penyelesaian dibina. Pendiskretan ruang untuk persamaan Euler adalah berasaskan pemisahan fluks-vector. Daripada pemerhatian ini, istilah pendiskretan fluks berolak persamaan Euler adalah berasaskan pemecahan vector-fluks hybrid yang dikenali sebagai skim kaedah pemecahan ‘advection’ hulu yang menggabungkan ketepatan pemecahan perbezaan fluks dan kekuatan pemecahan vektor-fluks. Skim AUSM yang berdasarkan skim padat peringkat ketiga merupakan

suatu penghampiran kepada persamaan perbezaan telah lengkap dianalisa secara berperingkat. Dalam menyelesaikan masalah aliran satu dimensi untuk kaedah pengkamiran masa, satu kaedah tak tersirat digunakan pada skim yang mempunyai kejutan tertib pertama. Selain itu, pembangunan dan pengubahsuaian terhadap kod sumber bagi aliran satu dimensi disahkan dengan empat kes ujian iaitu, tiub kejutan tidak mantap, aliran muncung supersonik-subsonik mirip satu matra, aliran surut dan gelombang kejutan dalam tibu kejutan. Daripada hasil kajian ini, ia juga dilakukan untuk memastikan bahawa definisi masalah “Riemann” boleh dikenalpasti. Analisis selanjutnya juga telah dilakukan dalam membandingkan ciri-ciri skim AUSM terhadap keputusan eksperimen, yang diperolehi daripada kerja-kerja sebelumnya dan juga analisis perbandingan dengan hasil pengkomputeran yang dihasilkan oleh skim-skim van Leer, KFVS dan AUSMPW. Selanjutnya, terdapat penambahbaikan terhadap skim AUSM daripada ketepatan peringkat pertama kepada ketiga melibatkan kejutan, perhubungan keadaan yang terputus dan penyebab gelombang.

## ACKNOWLEDGEMENT

First and foremost I thank Allah the Almighty for the blessings and opportunities that He has provided for me to accomplish this study.

I would like to express my sincere thanks to my advisor and committee chairman, Professor Dr. Ir. ShahNor Basri, who was a constant source of advice, encouragement and support throughout this study. I am deeply indebted and most grateful to Dr. Abdul Aziz Jaafar, the supervisory committee member, who guided this work during his service at Universiti Putra Malaysia. I wish to express my thanks to the other member of my supervisory committee, Associate Professor Dr.-Ing. Ir. Renuganth Varatharajoo for his helpful discussions and advice. I wish to express my thanks to the other members of my research study especially to Associate Professor Dr Ahmad Samsuri Mokhtar, Dr Mahmood Khalid Mawlood and also Dr Bambang Basuno for his helpful discussions and advice. I highly appreciate their time and willingness to serve in my supervisory committee.

I wish to extend my thanks to the staff of the Department of Aerospace Engineering for their friendly dealing and moral support. I also thank technicians of the computer lab, my friends and colleagues for sharing their knowledge and encouragement.

Thanks must also go to my family members, my wife, and lovely daughters for their patience, understanding and encouragement as well as the wonderful environment they provided for the successful completion of this work.



I certify that a Thesis Examination committee has met on 2 September 2010 to conduct the final examination of Mahmod Abd Hakim bin Mohamad on his thesis entitled “One-Dimensional High -Order Compact Method for Solving Euler’s Equations” in accordance with the Universities and University College Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The committee recommends that the student be awarded the Master of Science.

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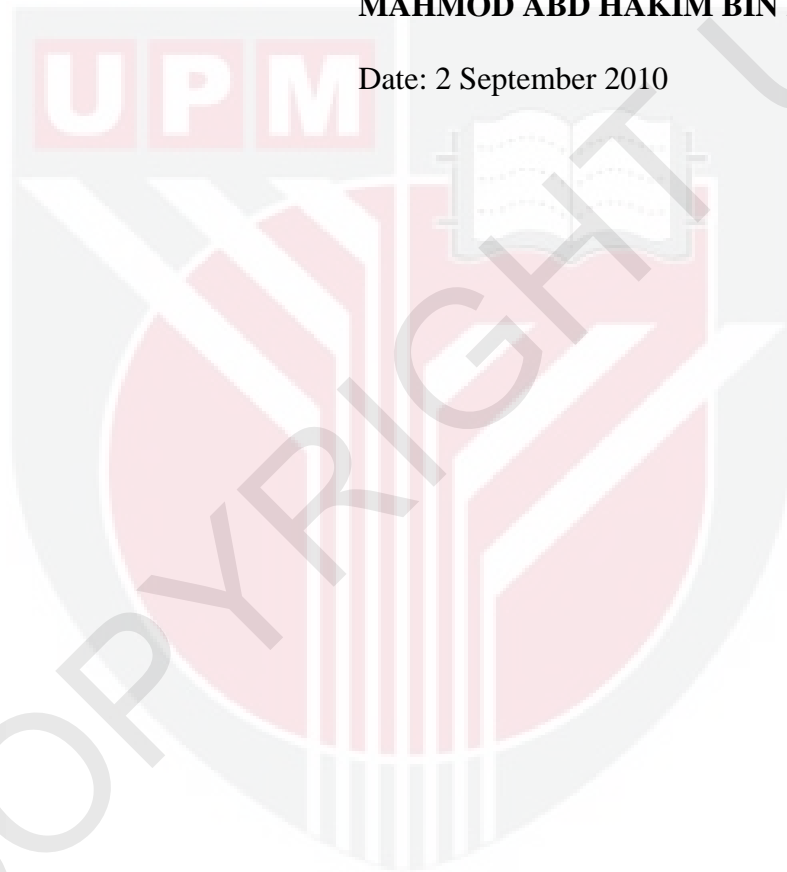
## DECLARATION

I declare that the thesis is my original work except for quotation and citations, which have been duly acknowledged. I also declare that it has not been previously and is not concurrently, submitted for any degree at Universiti Putra Malaysia or other institutions.

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**MAHMUD ABD HAKIM BIN MOHAMAD**

Date: 2 September 2010



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