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Review Article

A Review on Robot Motion Planning Approaches

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ABSTRACT

The ability of a robot to plan its own motion seems pivotal to its autonomy, and that is why the motion planning has become part and parcel of modern intelligent robotics. In this paper, about 100 research are reviewed and briefly described to identify and classify the amount of the existing work for each motion planning approach. Meanwhile, around 200 research were used to determine the percentage of the application of each approach. The paper includes comparative tables and charts showing the application frequency of each approach in the last 30 years. Finally, some open areas and challenging topics are presented based on the reviewed papers.

Keywords: Autonomy, intelligent robotics, robot motion planning

INTRODUCTION

A fundamental need in robotics is to have the algorithms that convert high level specifications of tasks from human into low-level descriptions movements. The terms motion planning and trajectory planning are often used for these kinds of problems. A classic version of motion planning problem, which is sometimes referred to as the piano mover's problem, is defined as follows:

Let be a robot system having k degree of freedom and free to move within two or three dimensional domain v which is bounded by various obstacles whose geometry is known to the system. The motion planning problem for β is, given the initial and desired final placements of the system β, to determine whether a continuous motion from the initial placement to the final one exists during which β avoids collision with the known obstacles, and if so, to plan such a motion (Halperin, 1994).

In this pure formulation of the problem, the only interest is on the geometric aspects of the motion and ignores many issues, such as acceleration, speed, uncertainty or incompleteness in the geometric data, control strategies for executing the motion, etc.

The basic issues and steps in any motion planning formulation are: Computation of Configuration, Object Representation, Approaches to Motion planning, Search Methods, and Local Optimization of motion (Hwang & Narendra, 1992).

From this early piano mover's problem, motion planning has evolved to address a huge number of variations on the problem, allowing applications in areas such as animation of digital characters, surgical planning, automatic verification of factory layouts, mapping of unexplored environments, navigation of changing environments, assembly sequencing, and drug design. New applications

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bring new considerations that must be addressed in the design of motion planning algorithms. In this review, after surveying about 200 papers in the field, the amount of existing works were collected and classified for future analysis.

In the following sections, each group of approaches are firstly introduced, and this is followed by an analysis of the amount of usages of each approach through different time sections. Finally, based on the reviewed researches in the field of motion planning, the open areas and future challenges in this field are discussed.

MATERIAL AND METHODS

A considerable amount of research is available in the field of robot motion planning approaches. The discipline was launched in mid 60's, but it was not until the work of Lozano and Wesley (1979) on spatial planning that motion planning drew most researchers' attention. The current developed methods are variations of a few general approaches, such as Bug Algorithms, Roadmap, Cell Decomposition, Potential Fields, Sampling-based motion planning, Kalman filtering, Heuristic Approaches and, Mathematical programming. These methods are not necessarily mutually exclusive, and their combination is often used in developing a motion planner (Masehin & Amin Naseri, 2004; Dongbin & Jianqiang, 2006). After surveying a total of 198 papers in the field (from 1980 to 2010), the amount of existing works for each approach was identified and classified. In total, ninety seven papers were used to briefly describe each approach. The following sections introduce each of these approaches and mention the most important works of each one.

Bug Algorithm

Even a simple planner can present interesting and difficult issues. The Bug1 and Bug2 algorithms (Lumelsky & Stepanov, 1987) are among the earliest and simplest sensor-based planners with provable guarantees. These algorithms assume the robot is a point operating in the plane with a contact sensor or a zero range sensor to detect obstacles. When the robot has a finite range (non-zero range) sensor, the Tangent Bug algorithm (Kamon *et al*., 1996) is a Bug derivative that can then use that sensor information to find shorter paths to the goal. The Bug and Bug-like algorithms are straightforward to implement. Moreover, a simple analysis shows that their success is guaranteed, when possible. These algorithms require two behaviours; namely, Motion to the Goal and Boundary Following. It has been proven that the path length in Bug 1 and Bug 2 has the following condition:

$$L_{Bug1} \leq d(q_{start}, q_{goal}) + 1.5\sum_{i=1}^{n} p_i$$

$$L_{Bug2} \leq d(q_{start}, q_{goal}) + 0.5\sum_{i=1}^{n} n_i p_i$$

Where, $d(q_{\text{start}}, q_{\text{goal}})$ is the Euclidian distance between the start and goal, p_i is the perimeter of the i_{th} obstacle, n is the number of obstacles, and n_i is the number of the intersection between M-line and the i_{th} obstacle.

A Performance Comparison of Bug Navigation Algorithms is provided in James and Bräunl (2007).

Potential Fields

The Potential Fields concept shown in *Fig. 1* (Choset *et al*., 2005) was first introduced by Oussama Khatib (1986). A potential function is a differentiable real-valued function $U : R^m \rightarrow R$. The value

of a potential function can be viewed as energy and hence, the gradient of the potential is force. The gradient is a vector which points in the direction that locally maximally increases U.

$$\nabla U(q) = DU(q)^T = \left[\frac{\partial U}{(\partial q_1)(q)}, \cdots, \frac{\partial U}{(\partial q_m)(q)}\right]^T$$

A robot in the potential field method is treated as a point that is represented in the configuration space as a particle under the influence of an artificial potential field U whose local variations reflect the structure of the free space.

The potential function can be defined over free space as the sum of an Attractive potential pulling the robot towards the goal configuration and a Repulsive potential pushing the robot away from the obstacles (Latombe, 1991).

$$U(q) = U_{att}(q) + U_{rep}(q)$$

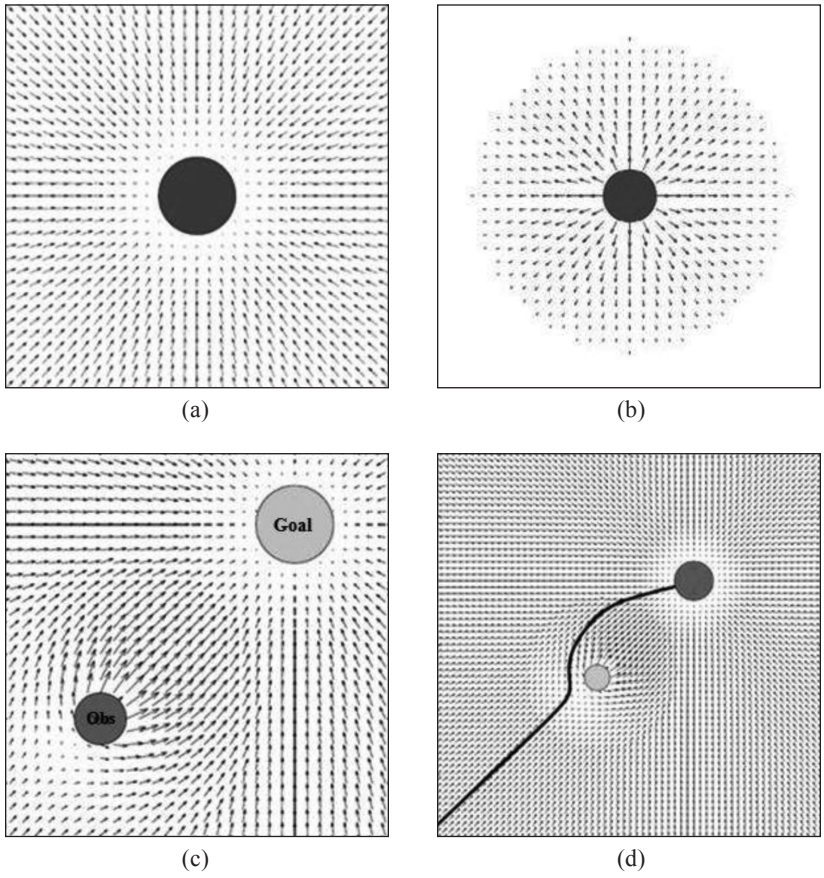

Fig. 1: Potential Fields, (a) Attractive Potential field near the goal, (b) Repulsive potential field near the obstacles, (c) Interaction between attractive and repulsive potential fields, and (d) the final path.

The attractive ($U_1 att\,(q)$) and repulsive ($U_{\text{rep}}(q)$) potential functions can be formulized as follows:

$$U_{att}(q) = \frac{1}{2}\zeta d^2(q, q_{goal})$$

$$(U_{rep}(q)) = \begin{cases} \frac{1}{2}\eta\left(\frac{1}{D(q)} - \frac{1}{Q^*}\right)^2, & D(q) \le Q^* \\ 0 \qquad\qquad\qquad\quad , & D(q) \le Q^* \end{cases}$$

Where ζ is a parameter used to scale the effect of the attractive potential, the $Q^* \in R$ factor allows the robot to ignore obstacles sufficiently far away from it and η can be viewed as a gain on the repulsive gradient.

Due to its low computational costs, the potential Fields method remains as a major path planning approach, especially when a high degree of freedom is involved (Hwang & Narendra, 1992).

Roadmaps

In the roadmap approach, the free Configuration space (C_{free}), i.e. the set of feasible motions, is retracted, reduced to, or mapped onto a network of one-dimensional lines. This particular approach is also called the retraction, skeleton, or highway approach. The search for a solution is limited to the network, and motion planning becomes a graph-searching problem. In this approach, motion planning is done in three steps; first, the robot is moved from its starting configuration to a point on the roadmap, using a canonical method; second, the robot is moved from goal configuration to a point on the roadmap likewise; and third, the two points on the roadmap are connected using lines in the roadmap. The roadmap must represent all topologically distinct feasible paths in C-space (Hwang & Narendra, 1992). Otherwise, the motion planning algorithm is not complete. The well-known roadmaps are Visibility graph (Asano *et al*., 1985), Voronoi diagram (Osamu, 1989), Silhouette (Canny, 1988), Cell Decomposition (Keil & Sack, 1985), and the Subgoal Network (Faverjon & Toumassoud, 1987) (*Fig. 2*).

Sampling–Based Motion Planning

The Probabilistic Roadmap planner (PRM) (Kavraki *et al*., 1996) demonstrated the tremendous potential of the sampling-based methods. PRM fully exploits the fact that it is cheap to check whether or not a single robot configuration is in Q_{free}. PRM creates a roadmap in Q_{free}. It uses rather coarse sampling to obtain the nodes of the roadmap and very fine sampling to obtain the roadmap edges, which are the free paths between node configurations. After the roadmap has been generated, planning queries can be answered by connecting the user-defined initial and goal configurations to the roadmap and using the roadmap to solve the path-planning problem at hand. Initially, node sampling in PRM was done using a uniform random distribution. This planner is called basic PRM. It was observed that random sampling worked very well for a wide variety of problems (Owermars & Svestka, 1995) and ensured the probabilistic completeness of the planner (Kavaraki *et al*., 1998). However, it was also observed by Kavaraki (1995) that random sampling is only a baseline sampling for PRM and many other sampling schemes are useful and also bound to be efficient for many planning problems as the analysis of the planner revealed. Today, these sampling schemes range from importance sampling in areas that during the course of calculations are found difficult to explore, to deterministic sampling such as quasirandom sampling and sampling on a grid.

PRM was conceived as a multiple-query planner. When PRM is used to answer a single query, some modifications are made: the initial and goal configurations are added to the roadmap nodes

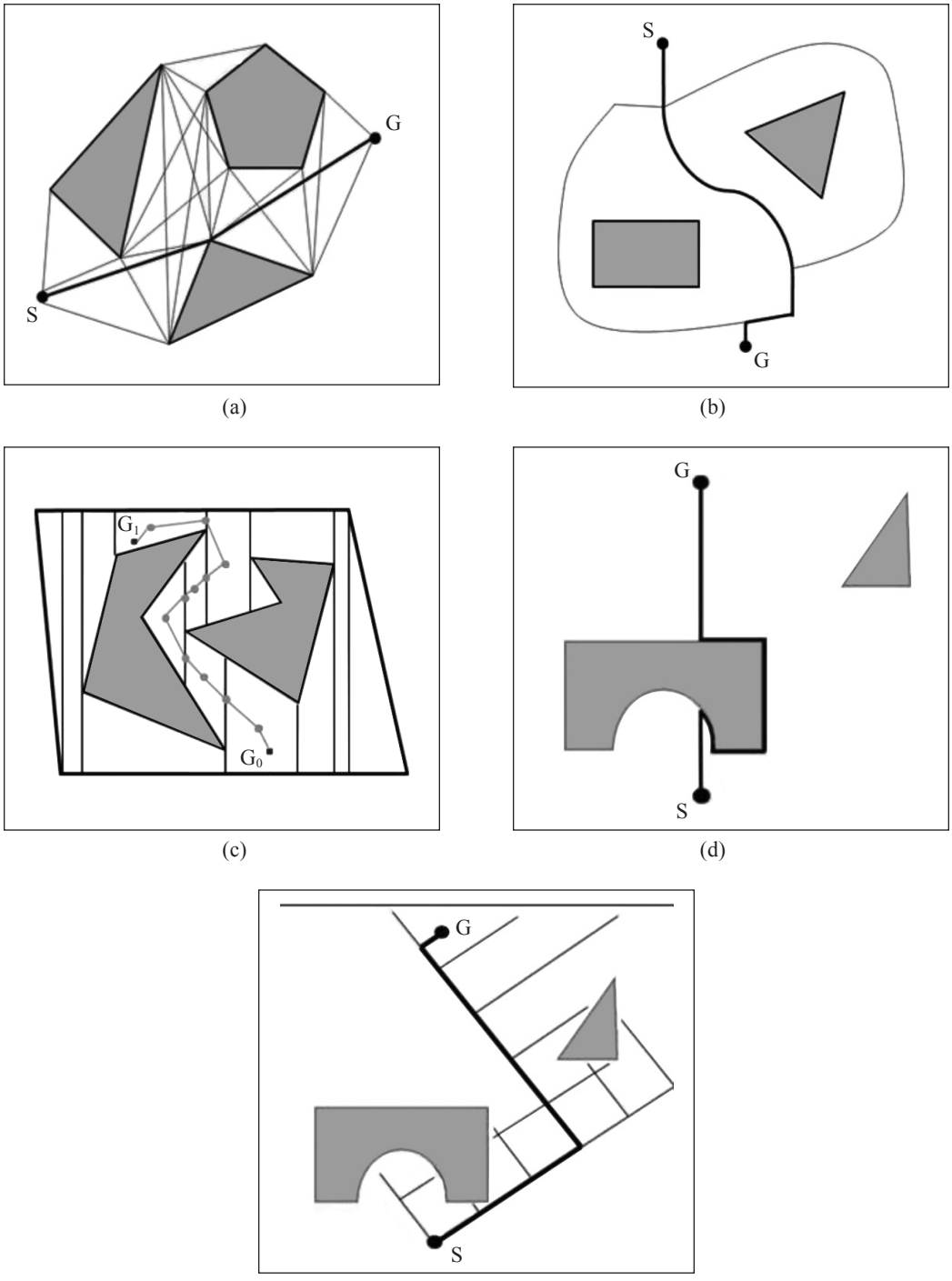

Fig. 2: Roadmaps; (a) Visibility graph, (b) Voronoi diagram, (c) Cell decomposition, (d) Silhouette, (e) Subgoal networks

and the construction of the roadmap is done incrementally and stopped when the query at hand can be answered. However, PRM may not be the fastest planner to be used for single queries. There are some other sampling-based planners that are particularly effective for single-query planning, including the Expansive-Spaces Tree planner (EST) (Hsu, 2000) and the Rapidly-exploring Random Tree planner (RRT) (Kuffner & LaValle, 2000). These planners exhibit excellent experimental performance.

A combination of the above methods is also possible and desirable in many cases. The Sampling-Based Roadmap of Trees (SRT) planner (Bekris *et al*., 2003) constructs a PRM-style roadmap of single-query-planner trees. It has been observed that for very difficult path planning problems, single-query planners need to construct large trees in order to find a solution. In some cases, the cost of constructing a large tree may be higher than the cost of constructing a roadmap of with SRT. This illustrates the distinction between the multiple-query and single-query planning, and its importance.

Despite their simplicity, which is exemplified in the basic PRM planner, the sampling-based planners are capable of dealing with robots with many degrees of freedom and with many different constraints. Among other, the sampling-based planners can take into account kinematic and dynamic constraints (Hsu *et al*., 2002), closed-loop kinematics (Han & Amato 2001), stability constraints (Kuffner *et al*., 2001), reconfigurable robots (Fitch *et al*., 2003), energy constraints (Lamiraux & Kavaraki 2001), contact constraints (Ji & Xiao, 2001), visibility constraints (Danner & Kavaraki 2000) and others. Clearly, some planners are better at dealing with specific types of constraints than the others. For example, EST and RRT planners are particularly useful for problems that involve kinematic and dynamic constraints.

Meanwhile, PRM, EST, RRT, SRT, and their variants have changed the way path planning is performed for high-dimensional robots. They have also paved the way for the development of planners for problems beyond basic path planning (Choset *et al*., 2005).

Kalman Filtering

Heretofore, the planner has been assumed to have the access to either an exact geometric description of its environment or a suite of sensors (e.g. sonar) that provide perfect information about the environment. In this part, cases for which the robot's knowledge of the world derives from measurements provided by imperfect, noisy sensors are first to be considered. The Kalman filter is one of the most useful estimation tools available today. In other words, the Kalman filtering provides a recursive method of estimating the state of a dynamical system in the presence of noise (Maybeck, 1990). A key feature of the Kalman filter is that it simultaneously maintains the estimates of both the state vector and the estimate error covariance matrix (P) which are equivalent to saying that the output of a Kalman filter is a Gaussian probability density function (*PDF*) with the mean and covariance P. In the context of localization, the Kalman filter output is then a distribution of likely robot positions instead of a single position estimate. As such, the Kalman filter is a specific example of a more general technique known as the probabilistic estimation techniques (Choset *et al*., 2005).

Mathematical Programming

The mathematical programming approach represents the requirement of obstacle avoidance with a set of inequalities on the configuration parameters. Motion planning is then formulated as a mathematical optimization problem that finds a curve between the start and goal configurations minimizing a certain scalar quantity. Since such an optimization is non-linear and has many inequality constraints, a numerical method is used to find the optimal solution. Reiswijk *et al*. (1992)

implemented a method of planning geometrical trajectories for two cooperating robots in an assembly cell. A trajectory of robot joints in joint-interpolated space is partitioned into separate convex or concave sub-paths by Sinha and Ho (1992). In this way, they abstract the trajectory in such a way that it becomes smoother for piecewise trajectories, and the information amount required to describe a path reduces drastically. Delaplace *et al*. (1992) implemented vision-sensor-based information to a real world tricycle robot. A closed-loop control system is used to maintain the distance of the robot and a straight or curved trajectory. Papanikolopoulos and Khoshla (1992) suggested a method to solve the robotic visual tracking problem by combining linear quadratic Gaussian control technique with the optical flow technique of vision-based motion tracking. Meanwhile, Sinha and Benmounah (1992) computed a safe path from the start to goal points by applying the trigonometric calculations of angles, along with the steering wheel of the robot which must be orientated. Reiswijk *et al*. (1992) proposed a new approach to optimize pre-calculated trajectories in joint space. They focused on a simultaneous integration of time-optimal curve, and thus benefited from a parallel execution which reduced computational time. An initial path which possibly has collision with obstacles is iteratively improved by performing a dynamic programming search in a sub-manifold of the C-space containing the current path in (Barraquand & Ferbach, 1993). Gifford and Murphy (1996) applied dynamic programming by triangulating the workspace and finding the shortest paths through vertex-nodes. Habibi *et al*. (2007) presented a novel algorithm for path planning of point robots in 2D known environment using binary integer programming.

Heuristic Approaches

The aforementioned conventional approaches suffer from many drawbacks, such as high time complexity in high dimensions, and trapped in local minima, which make them inefficient in practice. On the other hand, it is proven that the path planning problem is NP-complete (Canny, 1988). Therefore, probabilistic algorithms have been developed, including Probabilistic Roadmaps and Rapidly-exploring Random Trees, with high-speed implementation as their major advantages. Other related approaches in motion planning are Level set and Linguistic Geometry. To fix the local minima problem, many heuristic and Meta-heuristic algorithms are used in motion planning, such as the combination of the Simulated Annealing technique and Potential Fields. Other related approaches include Neural Network (Zhu & Yang, 2006), Genetic Algorithms (Quingfu *et al*., 2007), Simulated Annealing (Manousakis *et al*., 2005), Ant Colony Optimization (Mohamad *et al*., 2006), Particle Swarm Optimization (Saska *et al*., 2006), Stigmergy (Cazangi *et al*., 2006), Wavelet Theory (Pai & Reissel, 1998), Tabu Search (Masehian & Amin Naseri, 2004) and Fuzzy Logic (Lee & Wu, 2003). Heuristic algorithms do not guarantee to find a solution, but if they do, are likely to do so much faster than deterministic methods.

RESULTS AND DISCUSSIONS

A total of ninety seven papers were surveyed in this research, covering a sufficient depth of works in the robot motion planning field for the time span of 1980 to 2010. One hundred and ninety eight papers were considered so as to provide different comparisons among these approaches. At the same time, the researchers also tried to bring together major applications of conventional and heuristic techniques in the literature and to come to conclusions about the nature and the course of research in motion planning discipline. The motion planning algorithms were started by some simple methods, and after a while, some more complex methods were also developed. After that, the heuristic methods were developed to increase the effectiveness and efficiency of the solutions. As illustrated in *Fig. 3* (considering a total of 198 papers), the application of the heuristic methods

was increased due to their success in coping with the problems of combinatorial explosion and local minima. As shown in *Fig. 3*, the 80th decade was the climax period for the conventional methods and, the appearance period for the heuristic methods. Meanwhile, the 90th decade was the descent time for the conventional approaches and improvement of the heuristic ones. The last 10 years (2001-2010) were the climax period for the heuristic methods, and the appearance period for the compound and meta-heuristic methods. Some studies which are in this field include those by Masehian and Amin Naseri (2004) and Dongbin and Jianqiang (2006). It seems that in the near future, the application of the heuristic approaches will decline, whereas the application of the compound and meta-heuristic methods will improve in order to achieve some better solutions in shorter time and lesser cost, but with more effectiveness and efficiency.

Tables 1-3 and *Fig. 4-5* represent the percentage of each approach and compare these approaches together. In total, about 25% of the papers are related to conventional approaches and 75% to heuristic approaches. The severe different between the portions of the conventional and heuristic methods indicates the rate of interest to the heuristic methods, according to their ability to decrease the time and error.

Table 1 (considering a total of 198 papers) shows the portions of the surveyed major methods in detail. As given in Table 1, it is indicated that research were mostly done in the fields of fuzzy logic, neural networks and genetic algorithms, whereas fewer studies were conducted in the areas of visibility graph and cell decomposition.

Table 2 and *Fig. 4* (considering a total of 50 papers in the field of Conventional approaches) provide a more detailed analysis on the conventional approaches and their relative application in robot motion planning. As presented in this table, about 32% of the research was done in the field of mathematical programming.

Table 3 and *Fig. 5* (considering a total of 148 papers in the field of heuristic approaches) show a more detailed analysis on the heuristic approaches and their relative application in robot motion

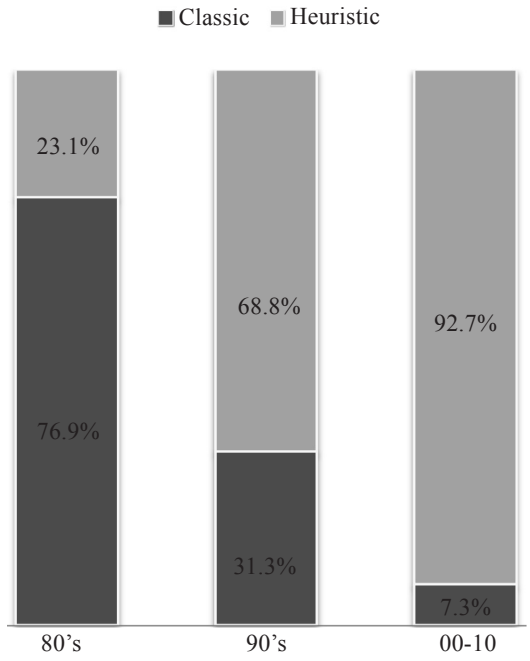

Fig. 3: A Comparison of the conventional and heuristic algorithms

TABLE 1
A Comparison of major motion planning approaches

Approach	10 year periods (from 1980 up to now)			Total
	80's (%)	90's (%)	00-10 (%)	
BUG algorithms	18.18	2.13	2.50	3.92
Visibility graph	9.09	2.13	0.00	1.96
Voronoi diagram	18.18	0.00	2.50	2.94
Silhouette	18.18	2.13	0.00	2.94
Subgoal networks	9.09	2.13	0.00	1.96
Cell decomposition	9.09	2.13	0.00	1.96
Potential fields	9.09	4.26	0.00	2.94
Mathematical programming	0.00	17.02	2.50	8.82
Neural networks	9.09	14.89	12.50	12.75
Genetic algorithm	9.09	12.77	15.00	12.75
Simulated annealing	0.00	6.38	5.00	4.90
Ant colony optimization	0.00	6.38	12.50	7.84
Particle swarm optimization	0.00	4.26	12.50	5.88
Stigmergy	0.00	2.13	5.00	2.94
Wavelets	0.00	4.26	2.50	2.94
Tabu	0.00	4.26	2.50	3.92
Fuzzy	9.09	14.89	27.50	18.63

TABLE 2
Conventional approaches in motion planning

Approach	80's (%)	90's (%)	00-10 (%)	Total (%)
BUG algorithms	20	0.7	33	14.3
Visibility graph	10	0.7	0	7.1
Voronoi diagram	10	0	33	10.7
Silhouette	10	0.7	0	10.7
Subgoal networks	10	0.7	0	7.1
Cell decomposition	10	0.7	0	7.1
Potential fields	10	1.4	0	10.7
Mathematical programming	0	95.1	33	32.1
Total	100	100	100	100

planning. It is important to note that the Fuzzy logic is the most researched approach in the field. The complexities of the modern systems emphasize the type of imprecision rather than randomness. Even in a fully automated system, the critical parametric changes have to be made by a human expert, who usually expresses his control and diagnostic strategies linguistically as a set of heuristic decision rules. Fuzzy logic, as a mathematical tool to handle uncertainties, has been used to model the systems that are hard to define precisely. As a methodology, the fuzzy set theory incorporates imprecision and subjectivity into the model formulation and solution process. Therefore, the fuzzy logic has the most portions among the heuristic methods for robot motion planning.

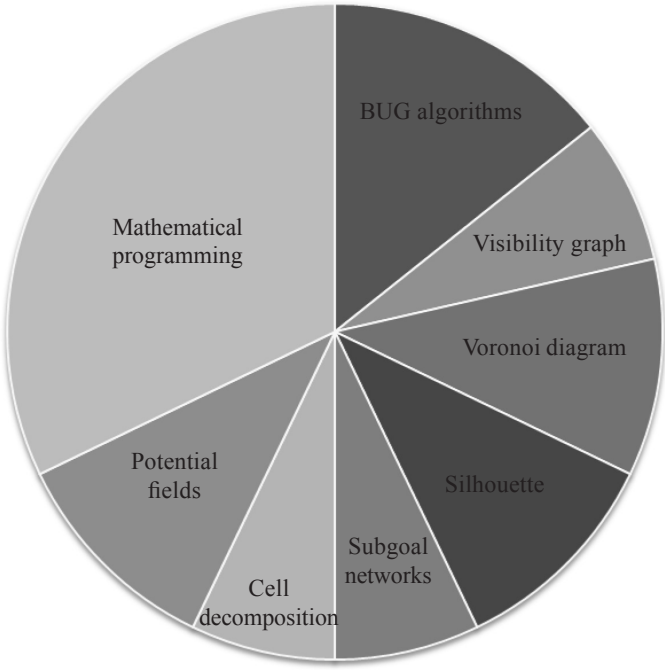

Fig. 4: The portion of each conventional approach

TABLE 3
Heuristic approaches in motion planning.

Approach	80's (%)	90's (%)	00-10 (%)	Total (%)
Neural networks	33.3	21.2	13.2	17.6
Genetic algorithm	33.3	18.2	15.8	17.6
Simulated annealing	0.0	9.1	5.3	6.8
Ant colony optimization	0.0	9.1	13.2	10.8
Particle swarm optimization	0.0	6.1	13.2	9.5
Stigmergy	0.0	3.0	5.3	4.1
Wavelets	0.0	6.1	2.6	4.1
Tabu search	0.0	6.1	2.6	4.1
Fuzzy logic	33.3	21.2	28.9	25.7
Total	100	100	100	100

CURRENT ISSUES AND CHALLENGES IN THE FIELD OF MOTION PLANNING

According to the papers that were reviewed in this paper and the comparisons made between the different approaches in the field of robot motion planning, it is clear that some algorithms are more useful, and thus, there are better opportunities to do some research about them. As shown in Tables 1-3, these approaches are fuzzy logic, neural networks, genetic algorithm and mathematical programming. On the other hand, it seems that each approach, especially the conventional ones,

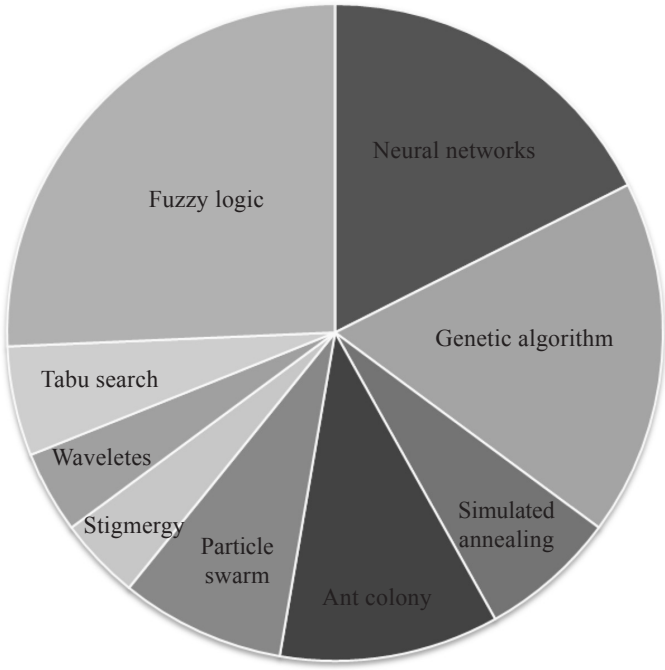

Fig. 5: The portion of each heuristic approach

suffer from many drawbacks. In order to improve the efficiency of these approaches, one of the best ways is to combine some of them together.

The overall paradigm of the most practical motion planners has seven main elements (Ahirkencheikh & Seireg, 1994), namely, Discretization of configuration space, Collision detection, Search Mechanism, Uncertainty, Completeness, Time and Local minima. There are some areas in motion planning which seem to have come to an end. For example, Potential-fields-Based approaches are well understood and that one should not simply report yet another minor variant of it, unless, of course, it has some significant contribution as is the case with harmonic potentials (Gupta & Del Pobil, 1998).

After analyzing the existing researches in the field of motion planning, the following new intellectual challenges, new applications, and emerging issues for motion planning are proposed:

- A crucial open issue: How does motion planning interact with perception and control modules? How can we merge the geometric model-based approaches with the reactive sensor-based approaches?
- Dealing with a partially unknown environment calls for the use of input from the real world. Online sensor-based motion planning should integrate vision and motion planning. This raises another important issue, i.e., How do we combine sensing with motion planning in an incremental way? This issue has recently received some attention.
- The new emerging field of micro-scale robots will present fertile ground and novel issues for motion planning research. At such microscopic scales, thermodynamical laws will need to be taken into account.

- New applications should be explored to keep this field alive. The motion planning-type algorithms have a great application potential in virtual prototyping, mechanical design and ergonomics. CAD tools are becoming increasingly inexpensive and common. There are possible applications of the motion planning-type algorithms in consumer technology, including computer animation and virtual reality.
- The advent of the so-called service robotics applications poses new major challenges. Since planning is a prediction of the future, dealing with incompleteness, error in information robustness in uncertainty will become key issues in practical motion planning.
- Since motion planning is just a part of the humans interaction system with the world, an open future direction is how to integrate motion planning approaches with the other related problems, such as grasp planning, manipulation, and fine motion planning.

Each of the existing approaches for the motion planning has its own advantages and drawbacks. This is because each algorithm is for a specific goal and considers the priority among different performance criteria. There are several measures for the performance of an algorithm, such as time for path traversal, velocity of manipulator links or joint, energy, actuator forces and proximity of obstacles. A mathematical comparison between the time complexity and path length of some of the main algorithms is provided in Table 4.

TABLE 4
A mathematical comparison

Approach	Advantage	Disadvantage
Potential fields	Real-Time	Not complete
Cell decomposition	Complete, Sound	Heavy computation, time
Visibility graph	Complete and yields minimum length paths	Heavy computation, generates semi-free paths to the obstacles, time
Voronoi diagram	Complete and generates roadmap with maximum distance	Possibly inefficient paths, time
Heuristic approaches	Les time, parallel search	Not complete, not sound
Exact cell decomposition	Complete	Heavy computation, time
Approximate cell decomposition	Sound and useful when only a coarse representation of workspace is available	Not complete
Bug 1	Complete	Long paths, time
Bug 2	Complete	Long paths, time

CONCLUSION

In this paper, after analyzing about 150 papers in the field of robot motion planning approaches, the amount of the existing works for each approach has been identified and classified. This paper divides the motion planning algorithms into two major groups, namely, the Conventional Approaches and Heuristic Approaches. The conventional approaches are BUG Algorithms, Roadmap, Cell Decomposition, Potential Fields, and, Mathematical programming, whereas the heuristic approaches include the Neural Network, Genetic Algorithms, Particle Swarm Optimization,

Ant Colony, Stigmergy, Wavelet Theory, Fuzzy Logic and Tabu Search. After a brief introduction of each approach, the important works in each field were presented. A complete discussion of the portion of each approach in the field of robot motion planning is also presented, including different comparative figures and charts.

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