

# **UNIVERSITI PUTRA MALAYSIA**

DIRECT ONE-STEP BLOCK METHODS FOR SOLVING SPECIAL SECOND ORDER DELAY DIFFERENTIAL EQUATIONS

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## DIRECT ONE-STEP BLOCK METHODS FOR SOLVING SPECIAL SECOND ORDER DELAY DIFFERENTIAL EQUATIONS

By

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Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

May 2013

## **DEDICATION**

I would like to dedicate this thesis to **my mother**, **Mrs. Selemaiah Binti Darus** who taught me that even the largest task can be accomplished if it is done one step at a time and **my beloved family**, **lecturers and friends** who have supported me all the way since the beginning of my studies.

Finally, this thesis is dedicated to all those who believe in the richness of learning.



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science.

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By

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May 2013

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**Faculty: Science** 

This thesis describes the implementation of one-step block methods of Runge-Kutta type for solving second order Delay Differential Equations (DDEs). The one-step block method will solve the second order DDEs directly without reducing to first order equations. The solution will be approximated simultaneously at more than one point based on equidistant block method and each of these values is calculated independently of each other.

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In this research, the block methods are implemented based on the predictor-corrector scheme using constant step size technique. The derivation of two point, three point, extended two point and extended three point implicit block methods were also described. Those methods were derived using Lagrange interpolation polynomial. The stability polynomials for the methods are obtained and their regions of absolute stability are discussed.

Subsequently, the one-step block methods were then adapted to solve delay differential equations. The basic idea is to apply the same integration method and use interpolation polynomial to approximate the solution in the previous value.

The efficiency of the proposed block method is supported by some numerical results. The results showed that the block methods proposed in this thesis are suitable to solve second order DDEs. Abstrak tesis yang dikemukan kepada senat Universiti Putra Malaysia sebagai memenuhi keperluan ijazah Master Sains.

## KAEDAH BLOK SATU LANGKAH BAGI MENYELESAIKAN PERINGKAT KEDUA KHAS PERSAMAAN PEMBEZAAN LENGAH SECARA TERUS

Oleh

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Tesis ini menerangkan tentang pelaksanaan kaedah blok satu langkah jenis Runge-Kutta bagi menyelesaikan persamaan pembezaan lengah (PPL) peringkat kedua. Kaedah blok satu langkah menyelesaikan peringkat kedua PPL secara terus tanpa mengubahkannya kepada peringkat pertama. Penyelesaian akan dianggarkan secara serentak lebih daripada satu titik berdasarkan kaedah blok sama jarak dan setiap nilai dikira tanpa bergantung pada satu sama lain.

Dalam penyelidikan ini, kaedah blok dilaksanakan berdasarkan pada skema peramalpembetul menggunakan teknik panjang langkah tetap. Penerbitan dua titik, tiga titik, lanjutan dua titik dan lanjutan tiga titik kaedah blok tersirat juga diterangkan. Kaedah ini diterbitkan menggunakan polinomial interpolasi Lagrange. Polinomial kestabilan untuk kaedah ini diperolehi dan rantau kestabilan mutlak dibincangkan. Seterusnya, kaedah blok satu langkah disesuaikan bagi menyelesaikan persamaan pembezaan lengah. Idea asasnya adalah untuk mengaplikasikan kaedah integrasi yang sama dan menggunakan interpolasi untuk menganggar penyelesaian pada nilai ke belakang.

Kecekapan kaedah blok yang dicadangkan ini disokong oleh hasil berangka. Hasil menunjukkan kaedah blok yang dicadangkan dalam kajian ini sesuai bagi menyelesaikan peringkat kedua persamaan pembezaan lengah.

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## DECLARATION

I declare that the thesis is my original work except fot quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.



## **TABLE OF CONTENTS**

	Page
DEDICATION	i
ABSTRACT	ii
ABSTRAK	iv
ACKNOWLEDGEMENTS	vi
APPROVAL	vii
DECLARATION	ix
LIST OF TABLES	xii
LIST OF FIGURES	xiii
LIST OF ABBREVIATIONS	xiv

OTT	DT	T
UHA	AP I	ĿК

TER			
1	INTR	RODUCTION	1
	1.1	Introduction	1
	1.2	Ordinary Differential Equation	2
	1.3	Delay Differential Equation	3
	1.4	Objective of the Thesis	4
	1.5	Scope of Study	5
	1. <mark>6</mark>	Outline of the thesis	5
2	LITE	CRATURE REVIEW	7
	2.1	Introduction	7
	2.2	Assumptions and Definitions	7
	2.3	Stability Properties of Numerical Methods for	12
		Solving Delay Differential Equations	
	2.4	Review of Previous Research	14
3	DIRI	ECT TWO AND THREE POINT ONE-STEP	19
	BLO	CK METHODS FOR SOLVING DELAY	
	DIFF	FERENTIAL EQUATIONS	
	3.1	Introduction	19
	3.2	Lagrange Interpolation Polynomial	19
	3.3	Derivation of the formulae	20
		3.3.1 Two Point Block	20
		3.3.2 Three Point Block	22
		3.3.3 Order of the Method	24
	3.4	P- and Q-Stability	33
		3.4.1 P-Stability Analysis	35
		3.4.2 Q-Stability Analysis	38
	3.5	Algorithm 2PBDDE	40
	3.6	Problem Tested	42

	3.7	Numerical Results	44
	3.8	Discussion	51
_			
4	EXT	ENDED TWO AND THREE POINT DIRECT	53
	ONE	-STEP BLOCK METHODS FOR DELAY	
	DIFF	FERENTIAL EQUATIONS	
	4.1	Introduction	53
	4.2	Derivation of the formulae	54
		4.2.1 Extended Two Point Block	54
		4.2.2 Extended Three Point Block	56
		4.2.3 Order of the Method	58
	4.3	P- and O-Stability	68
		4.3.1 P-Stability Analysis	70
		4.3.2 O-Stability Analysis	72
	44	Algorithm E2PBDDE	74
	4.5	Numerical Results	76
	4.6	Discussion	83
	7.0	Discussion	05
5	CON	CLUSION	85
	5.1	Summary	85
	5.2	Future Research	87
BIBLIOG	RAPHY		88
BIODAT	A OF STI	UDENT	91
PUBLICA	TIONS		92
i Oblici			

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