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Derivation of Bearing Capacity Equation for a Two Layered System of Weak Clay Layer Overlaid by Dense Sand Layer

Abdulhafiz O. Al-Shenawy & Awad A. Al-Karni
King Saud University, Civil Eng. Dept., P. O. Box 800
Riyadh, 11421, Saudi Arabia

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ABSTRAK

Pengiraan keupayaan galas muktamad tapak cetek ke atas sistem tanah dua lapisan bergantung pada corak permukaan kegagalan yang dibangunkan di bawah tapak. Bagi lapisan tanah liat lemah yang dilapisi oleh lapisan pasir tumpat atas, kajian terdahulu menganggap bahawa permukaan kegagalan adalah kegagalan ricih menebuk melalui lapisan pasir atas dan mod kegagalan Prandtl dalam lapisan tanah liat lemah bawah. Dengan mengadaptasi anggapan ini dalam kajian ini, persamaan keupayaan galas muktamad diperoleh sebagai satu fungsi sifat-sifat tanah, kelebaran tapak, dan ketebalan tanah atas. Kertas ini membentangkan kajian berparameter yang terperinci bagi parameter reka bentuk termasuk kesan sudut geseran, nisbah ketebalan lapisan pasir dengan kelebaran tapak, nisbah kedalaman pembenaman dengan kelebaran tapak, dan nisbah kejeleketan tanah liat dengan keluaran berat unit tanah liat oleh kelebaran tapak. Carta reka bentuk dibangunkan dalam bentuk tiada dimensi untuk julat parameter reka bentuk yang sangat luas. Carta yang baru memberi pilihan lain kepada sesiapa yang percaya bahawa carta reka bentuk dibangunkan berdasarkan analisis had atas lebih anggar keupayaan galas disebabkan sifat penyelesaian had atas. Carta reka bentuk baru adalah terhad kepada tapak cetek.

ABSTRACT

Calculation of the ultimate bearing capacity of shallow footing on a two layered system of soil depends on the pattern of the failure surface that develops below the footing. For a weak clay layer overlaid by a top dense sand layer, previous studies assumed that the failure surface is a punching shear failure through the upper sand layer and Prandtl's failure mode in the bottom weak clay layer. By adapting this assumption in this study, the ultimate bearing capacity equation was derived as a function of the properties of soils, the footing width, and the topsoil thickness. The paper presents a detailed parametric study of the design parameters including the effect of angle of friction, the ratio of the thickness of sand layer to the footing width, the ratio of the depth of embedment to the footing width, and the ratio of the clay soil cohesion to the product of the clay unit weight by the footing width. Design charts were developed in dimensionless form for very wide ranges of design parameters. The available method based on the limit equalibrium analysis was developed in dimensionlised form and for a limited range of design parametrs. The new charts give another option for those who believe that the design charts developed based on the upper limit analysis overestimate the bearing capacity due to the very nature of the upper bound solution. The new design charts are limited to shallow footings.

Keywords: Shallow footing, bearing capacity, two layered system, weak clay layer, dense sand layer, design chart

INTRODUCTION

The function of a foundation is to transfer the load of the superstructure to the underlying soil formation without overstressing the soil. The soil must be capable of carrying the load for structure(s) placed upon it without shear failure and with the resulting settlement being tolerable for that structure.

Many investigations on the subject of ultimate bearing capacity have been carried out during the past century. Subsequently, numerous proposals have been advanced regarding considerations, criteria, and procedures for evaluation of the ultimate bearing capacity of soils. Among the very early contributors were Prandtl (1921) who developed a solution for a surface strip footing over a perfectly plastic cohesive-frictional weightless half-space. Reissner (1924) extended the solution of Prandtl to include the effect of a uniform surcharge load on the resistance of penetration of ultimate applied load. Since real soils possess weight, Terzaghi (1943) was the first to introduce the concept of ultimate bearing capacity and presented a comprehensive theory for the evaluation of such capacity of shallow foundations. Subsequently, the bearing capacity theory went through many modifications to account for different features such as foundation shape, load inclination, ground slope, nonsymmetrical loads, and water table. The general bearing capacity theories proposed by Meyerhof (1963), Hansen (1970), Vesic (1973) and others are now routinely used in foundation design.

The bearing capacity theories mentioned above involve cases in which the soil supporting the foundation is homogeneous and extends to a considerable depth. However, in practice, layered soil profiles are often encountered. For layered clayey soil, Button (1953) was the first to analyse footings on layered soils of different cohesion. Many other studies were conducted for clayey layers including those of Sivareddy and Srinivasan (1967), Brown and Meyerhof (1969), Desai and Reese (1970a, b) and Merifield *et al.* (1999). In another case, many authors studied the bearing capacity of a sand layer overlaying a clay layer. These studies were conducted by Meyerhof (1974), Meyerhof and Hanna (1978), Hanna and Meyerhof (1980), Hardy and Townsend (1982), Okamura *et al.* (1997), Kenny and Andrawes (1996), Burd and Frydman (1997), and Michalowski and Shi (1995). For footings resting over a two-layer c-ϕ soil, the ultimate bearing capacity was studied by Purushothamaraj *et al.* (1974), Satyanarayana and Garg (1980), Florkiewicz (1989), and Azam and Wang (1991).

In this study, design charts were developed using the punching shear model in a dimensionless form since those of Hanna and Meyerhof (1980) were not presented in a nondimensionlised form, which limits their application. The new design charts were developed for very wide ranges of design parameters. The presented charts here may be useful in overcoming the problem of the design charts that were developed by Michalowski and Shi (1995) which may overestimate the bearing capacity by a significant amount because of the very nature of the upper bound solution on which the derivation is based.

FORMULATION

According to *Fig. 1*, a shallow footing with width B is resting on a two-layered soil. The top layer is a dry granular soil with thickness H, unit weight γ, and angle of friction ϕ. The bottom layer is assumed to be a normally consolidated clay with undrained cohesion c_u. The punching shear failure mechanism is assumed to be developed here which is characterised by the formation of two vertical shear bands inside the granular layer and a Prandtl-type failure in the cohesive layer as shown by *Fig. 1*. By assuming a constant vertical stress (σ_{zz}) acting along the width (B) of horizontal slice with thickness *dz* at depth *z* from the base of the footing (see *Fig. 2*), and considering equilibrium of the vertical forces then

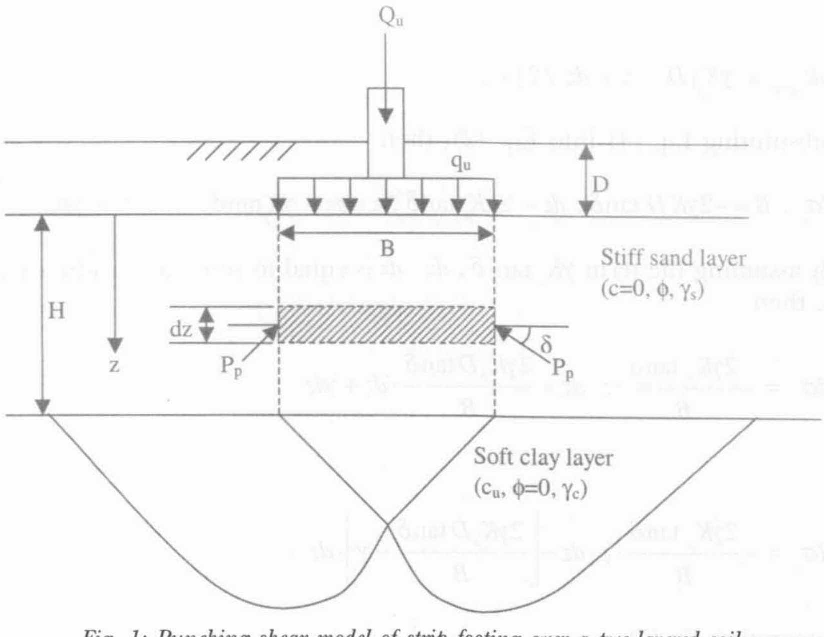

Fig. 1: Punching shear model of strip footing over a two-layered soil

$$\sigma_{zz} \,.\, B - (\sigma_{zz} + d\sigma_{zz}).\, B - 2dP \,.\, sin\delta + \gamma Bdz = 0 \tag{1}$$

which gives

$$-d\sigma_{zz} \,.\, B - 2dP_p \sin\delta + \gamma Bdz = 0 \tag{2}$$

where dP_p is the passive force which inclines upwards at an angle δ to the horizontal and equal to $dP_{p(h)}/\cos\delta$ where

$$dP_{p(h)} = p.\, dz = \left[\gamma DK_P + \gamma\left(z + \frac{dz}{2}\right)K_P\right].\, dz \tag{3}$$

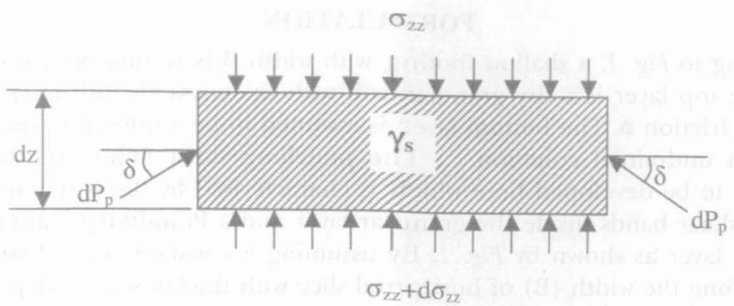

Fig. 2: Applied forces on a strip dz of the failure zone at depth z from the base of footing

or

$$dP_{p(h)} = \gamma K_p[D + z + dz/2].dz \tag{4}$$

By substituting Eq. (4) into Eq. (2), then

$$d\sigma_{zz} . B = -2\gamma K_p D \tan\delta . dz - 2\gamma K_p \tan\delta . z . dz - \gamma K_p \tan\delta . dz . dz + \gamma B dz \tag{5}$$

By assuming the term $\gamma K_p \tan\delta . dz . dz$ is equal to zero and dividing Eq. (5) by B, then

$$d\sigma_{zz} = -\frac{2\gamma K_p \tan\delta}{B} \cdot z \cdot dz - \frac{2\gamma K_p D \tan\delta}{B} \cdot dz + \gamma dz \tag{6}$$

or

$$d\sigma_{zz} = -\frac{2\gamma K_p \tan\delta}{B} \cdot z \cdot dz - \left[\frac{2\gamma K_p D \tan\delta}{B} - \gamma\right] \cdot dz \tag{7}$$

By integrating Eq.(7)

$$\sigma_{zz} = -\frac{\gamma K_p \tan\delta}{B} \cdot z^2 - \left[\frac{2\gamma K_p D \tan\delta}{B} - \gamma\right] z + C \tag{8}$$

where C is the integration constant, and Eq. (8) can be rewritten as

$$\sigma_{zz} = -A_1 . z^2 - A_2 z + C \tag{9}$$

where

$$A_1 = \frac{\gamma K_p \tan\delta}{B} \tag{10}$$

and

$$A_2 = \left\lfloor \frac{2\gamma K_p D \tan\delta}{B} - \gamma \right\rfloor \tag{11}$$

By applying the boundary conditions, at z=0; σ_{zz}=q_u and from Eq. (9), then

$$C = q_u \tag{12}$$

and at z=H, $\sigma_{zz} = q_{up}$ where $q_{up} = 5.14c_u + \gamma(D + H)$ and from Eq. (9), then

$$q_{up} = -A_1 H^2 - A_2 H + C \tag{13}$$

or

$$C = q_{up} + A_1 H^2 + A_2 H \tag{14}$$

Then from Eq.(12) and Eq. (14)

$$q_u = 5.14c_u + \gamma(D + H) + A_1 H^2 + A_2 H \tag{15}$$

By substituting with A_1 and A_2, Eq. (15) can be written in a dimensionless form as:

$$\frac{q_u}{\gamma B} = 5.14\frac{c_u}{\gamma B} + \frac{D}{B} + K_p \tan\delta\left(\frac{H}{B}\right)^2 + 2K_p \tan\delta\left(\frac{D}{B}\right)\left(\frac{H}{B}\right) \tag{16}$$

Evaluation of δ/ϕ

The effect of the parameter δ/ϕ is a major concern here. Using a high value of δ/ϕ will lead to unconservative results. *Fig. 3* shows a comparison between the results of the variation of $(H/B)_{critical}$ with D/B at δ/ϕ of 1.0 and 0.67, where $(H/B)_{critical}$ is the ratio of the depth of the sand layer below the footing base to the footing width at which the clay layer has no effect on the bearing capacity. A significant effect is shown. For example, the $(H/B)_{critical}$ at D/B=0 is increased from 3.5 to 4.77 when δ/ϕ is decreased from 1.0 to 0.67, respectively. The difference between the two values increases as D/B increases. The figure also shows a comparison with the results of Michalowski and Shi (1995). The results of Michalowski and Shi (1995) show that the value of δ/ϕ is close to one when H/B is small and reduces as H/B increases. This conclusion is in agreement with suggestion of Meyerhof (1974). However, the experimental results of Hanna and Meyerhof (1980) suggested lower values of δ/ϕ. By using the values of δ/ϕ suggested by Hanna and Meyerhof (1980) the following relationships were developed

$$\delta/\phi = a + b\,\frac{q_2}{q_1} + c\left(\frac{q_2}{q_1}\right)^2 \tag{17}$$

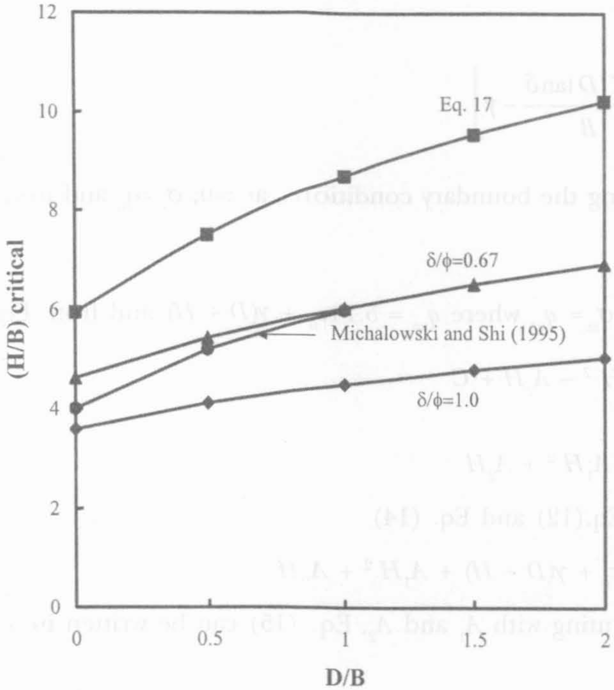

Fig. 3: Effect of δ/ϕ on the variation of $(H/B)_{critical}$ with D/B

where

$$a = 0.00829\left(\frac{q_2}{q_1}\right) - 0.00872 \tag{18}$$

$$b = 0.000744\left(\frac{q_2}{q_1}\right) + 1.0621 \tag{19}$$

$$c = -0.009\left(\frac{q_2}{q_1}\right) - 0.0515 \tag{20}$$

and q_1 is the upper layer (sand layer) bearing capacity and q_2 is the lower layer (clay layer) bearing capacity. By using the relationship in Eq. (17), the variation of $(H/B)_{critical}$ with D/B is shown in *Fig. 3.* Since the value of $(H/B)_{critical}$ for Michalowski and Shi (1995) is based on the upper bound analysis, it overestimates the bearing capacity as discussed before in the introduction. However, the value of $(H/B)_{critical}$ based on Eq. 17 gives even more conservative results for the bearing capacity.

The results in *Figs. 4 a* and *b* show the variation of $(H/B)_{critical}$ with $c_u/\gamma B$. Figs. *4 a* and *b* compare the results of Michalowski and Shi (1995) with the results of Eq. (16) at δ/ϕ equal to one and δ/ϕ from Eq. (17), respectively. In *Fig. 4a* the results are close in values but differ in trend since those of Michalowski and Shi (1995) are concave up, while the curves of δ/ϕ equal to one are concave down. However, in *Fig. 4b* the results are different in values but agree in trend as the results of Michalowski and Shi (1995) are concave up, and the ones for δ/ϕ from Eq. (17) are concave up also. It may be concluded that using Eq. (17) to calculate δ/ϕ and using it in Eq. (16) is more reliable since the difference in values shown in *Fig. 4b* is due to the overestimation associated with the upper bound solution. Thus, Eq. 17 is used in solving Eq. 16 and in developing the design charts.

Parametric Study

The bearing capacity of the shallow foundation resting on layered soils, with an upper dense sand layer and bottom soft clay layer, depends mainly on five parameters. These parameters include ϕ, H/B, D/B, and δ/ϕ, which are related to the sand layer and the parameter $c_u/\gamma B$ which is related to the clay layer. These parameters are presented in dimensionless form to generalise their effect

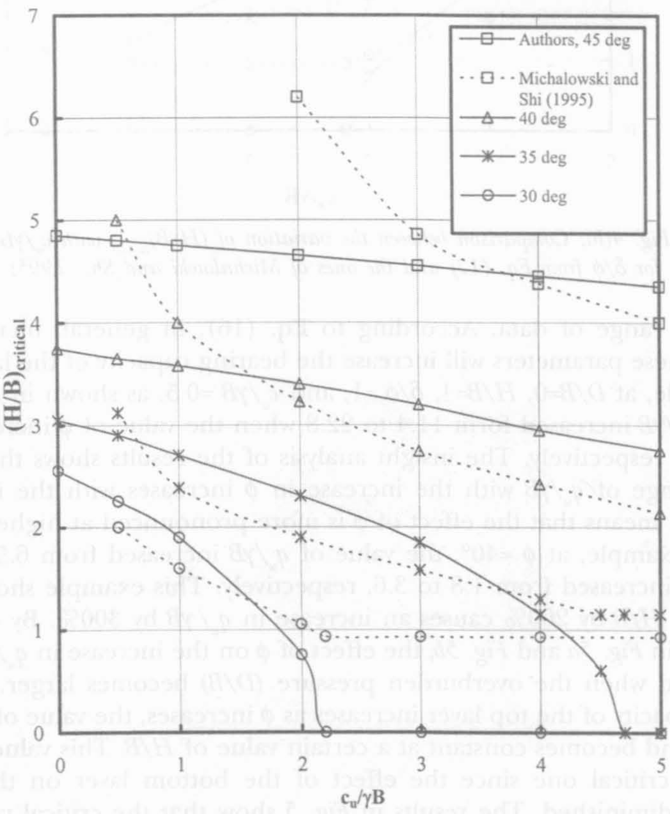

Fig. 4(a): Comparison between the variation of $(H/B)_{critical}$ with $c_u/\gamma B$ for δ/ϕ = 1.0 and the ones of Michalowki and Shi (1995)

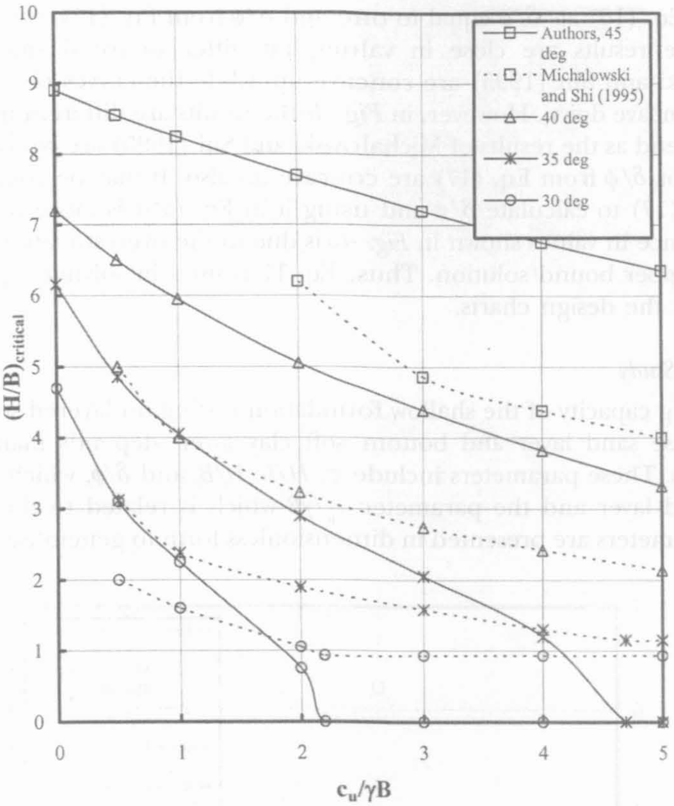

Fig. 4(b): Comparison between the variation of $(H/B)_{critical}$ with $c_u/\gamma B$ for δ/ϕ from Eq. (17) and the ones of Michalowki and Shi (1995)

for a wide range of data. According to Eq. (16), in general, increasing the values of these parameters will increase the bearing capacity of the layered soil. For example, at D/B=0, H/B=4, δ/ϕ =1, and $c_u/\gamma B$ =0.5, as shown in *Fig. 5a*, the value of $q_u/\gamma B$ increased form 11.4 to 22.8 when the value of ϕ increased from 30° to 40°, respectively. The insight analysis of the results shows that the rate of the change of $q_u/\gamma B$ with the increase in ϕ increases with the increase in H/B, which means that the effect of ϕ is more pronounced at higher values of H/B. For example, at ϕ =40°, the value of $q_u/\gamma B$ increased from 6.54 to 18.46 when H/B increased from 1.8 to 3.6, respectively. This example shows that an increase in H/B by 200% causes an increase in $q_u/\gamma B$ by 300%. By comparing the results in *Fig. 5a* and *Fig. 5b*, the effect of ϕ on the increase in $q_u/\gamma B$ is more pronounced when the overburden pressure *(D/B)* becomes larger. Since the bearing capacity of the top layer increases as ϕ increases, the value of $q_u/\gamma B$ also increases and becomes constant at a certain value of H/B. This value of H/B is called the critical one since the effect of the bottom layer on the bearing capacity is diminished. The results in *Fig. 5* show that the critical value of H/B increases as ϕ increases. From *Fig. 6*, the effect of the angle of friction (ϕ) on

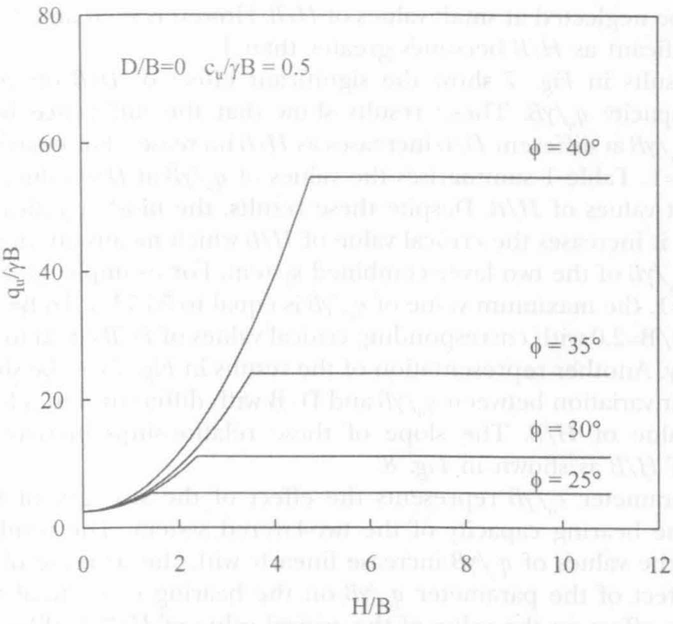

Fig. 5(a): Effect of angle of friction (ϕ) on the variation of $q_u/\gamma B$ with H/B at D/B = 0 and $c_u/\gamma B$ = 1

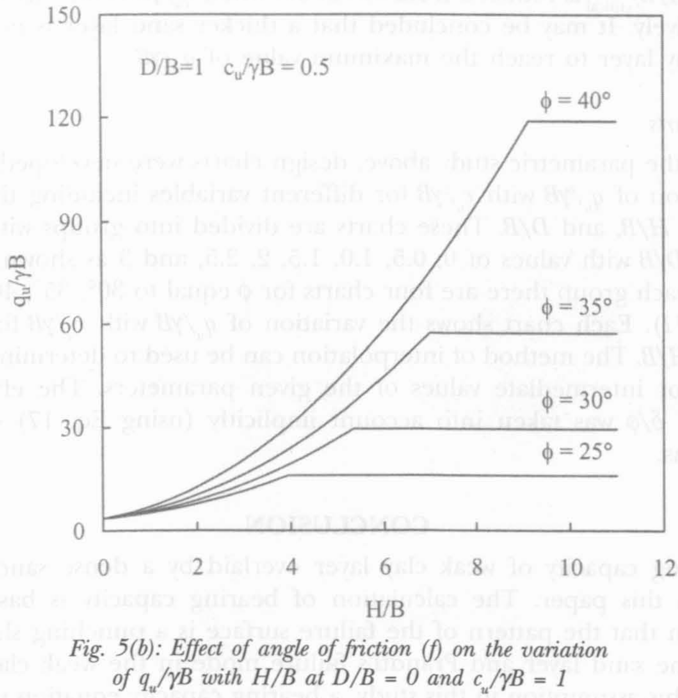

Fig. 5(b): Effect of angle of friction (f) on the variation of $q_u/\gamma B$ with H/B at D/B = 0 and $c_u/\gamma B$ = 1

$q_u/\gamma B$ can be neglected at small values of H/B. However, such an effect becomes more significant as H/B becomes greater than 1.

The results in *Fig. 7* show the significant effect of D/B on the ultimate bearing capacity $q_u/\gamma B$. These results show that the difference between the values of $q_u/\gamma B$ at different D/B increases as H/B increases. For example, at ϕ=40 and $q_u/\gamma B$ =1, Table 1 summarises the values of $q_u/\gamma B$ at D/B values of 0 and 2 at different values of H/B. Despite these results, the most significant effect of D/B is that it increases the critical value of H/B which means an increase in the values of $q_u/\gamma B$ of the two layer combined system. For example, at ϕ=40 and $q_u/\gamma B$ =1, δ/ϕ=1, the maximum value of $q_u/\gamma B$ is equal to 54.71 at D/B=0 and equal to 182 at D/B=2.0 with corresponding critical values of H/B equal to 6 and 10.6, respectively. Another representation of the results in *Fig. 7* can be shown in *Fig. 8* as a linear variation between $q_u/\gamma B$ and D/B with different slopes for each line of each value of H/B. The slope of these relationships increases with the increase of H/B as shown in *Fig. 8*.

The parameter $c_u/\gamma B$ represents the effect of the strength of the bottom layer on the bearing capacity of the two-layered system. The results in *Fig. 9* show that the values of $q_u/\gamma B$ increase linearly with the increase of $c_u/\gamma B$. The greatest effect of the parameter $q_u/\gamma B$ on the bearing capacity of the layered system is its effect on the value of the critical value of H/B. Unlike th effect of the angle of friction, the critical value of H/B is reduced as the value of $q_u/\gamma B$ increases as shown in *Fig. 10*. For example at ϕ=40°, D/B =0, and δ/ϕ=1, the value of $(H/B)_{\text{critical}}$ is reduced from 6.0 to 3.8 when $q_u/\gamma B$ is increased from 1 to 4, respectively. It may be concluded that a thicker sand layer is needed for a weaker clay layer to reach the maximum value of $q_u/\gamma B$.

Design Charts

Based on the parametric study above, design charts were developed to present the variation of $q_u/\gamma B$ with $c_u/\gamma B$ for different variables including the angle of friction ϕ, H/B, and D/B. These charts are divided into groups with different values of D/B with values of 0, 0.5, 1.0, 1.5, 2, 2.5, and 3 as shown in *Figs. 11* to 17. In each group there are four charts for ϕ equal to 30°, 35°, 40°, and 45° (see *Fig. 11*). Each chart shows the variation of $q_u/\gamma B$ with $c_u/\gamma B$ for different values of H/B. The method of interpolation can be used to determine the value of $q_u/\gamma B$ for intermediate values of the given parameters. The effect of the parameter δ/ϕ was taken into account implicitly (using Eq. 17) during the calculations.

CONCLUSION

The bearing capacity of weak clay layer overlaid by a dense sand layer was studied in this paper. The calculation of bearing capacity is based on the assumption that the pattern of the failure surface is a punching shear failure through the sand layer and Prandtl's failure mode in the weak clay layer. By adapting this assumption in this study, a bearing capacity equation was derived

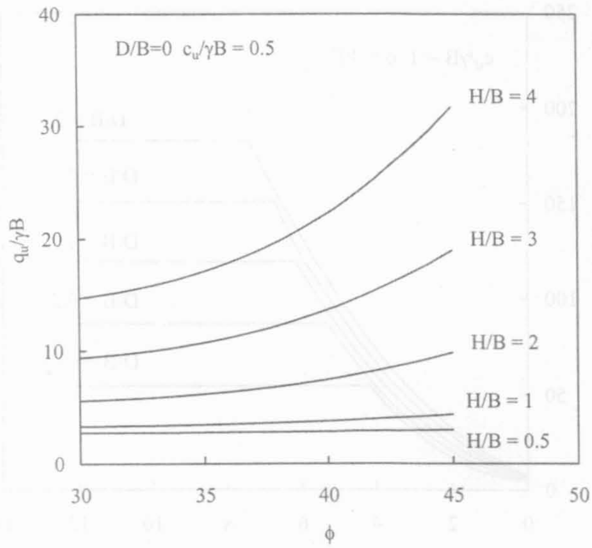

Fig. 6(a): Effect of H/B on the variation of $q_u/\gamma B$ with angle of friction (ϕ) at D/B = 0 and cu/γB = 0.5

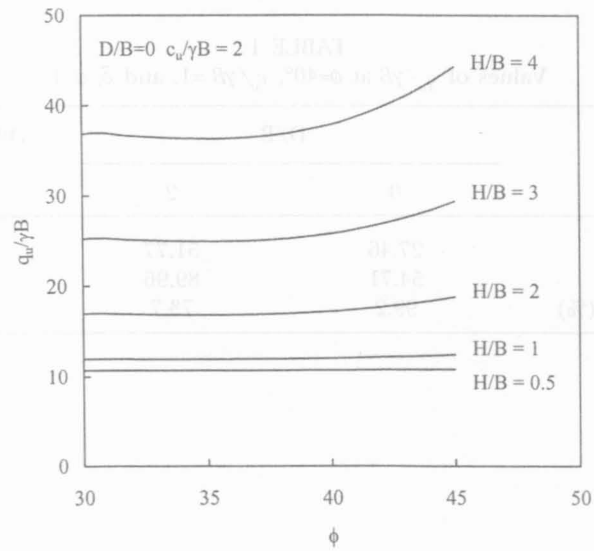

Fig. 6(b): Effect of H/B on the variation of $q_u/\gamma B$ with angle of friction (ϕ) at D/B = 0 and $c_u/\gamma B$ = 2

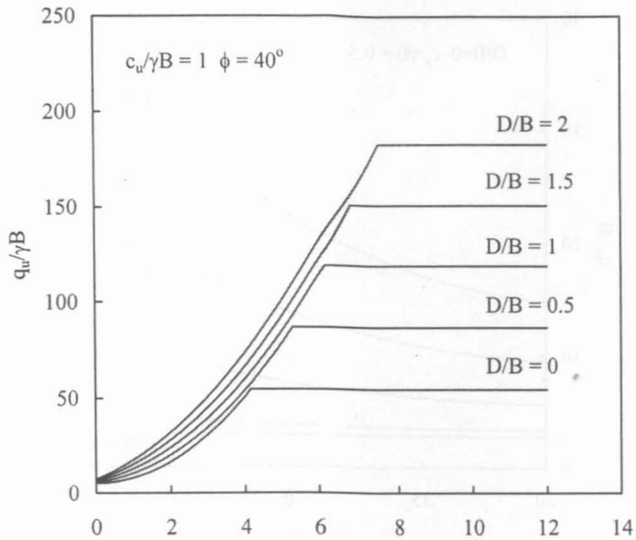

Fig. 7: Effect of D/B on the variation of $q_u/\gamma B$ with H/B at f = 40° and $c_u/\gamma B$ = 1

TABLE 1
Values of $q_u/\gamma B$ at ϕ=40°, $c_u/\gamma B$ =1, and δ/ϕ=1

H/B	D/B		Difference (%)
	0	2	
4	27.46	51.77	88.5
6	54.71	89.96	64.4
Difference (%)	99.2	73.7	

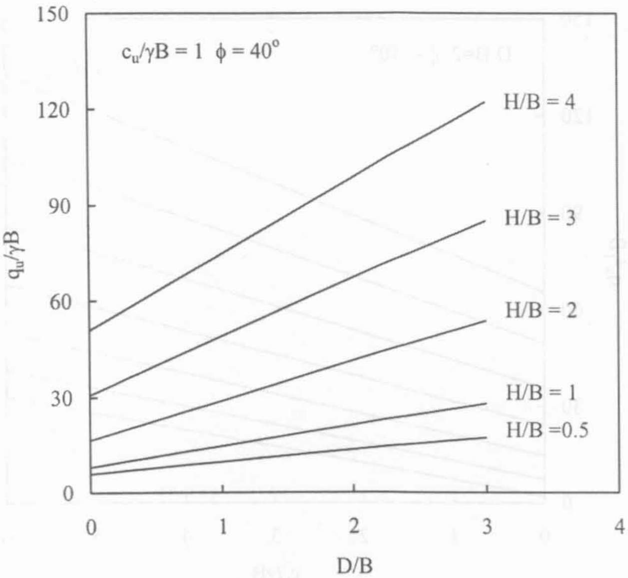

Fig. 8: Variation of qu/gB with D/B at different values of H/B at ϕ = 40° and $c_u/\gamma B$ = 1

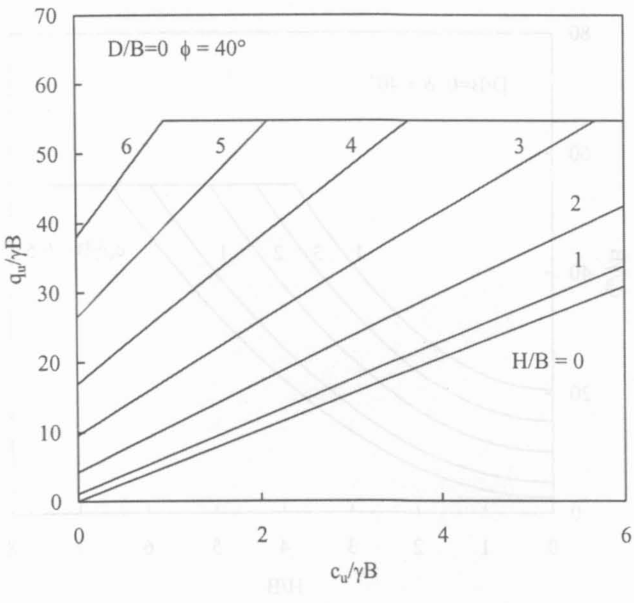

Fig. 9(a): Variation of $q_u/\gamma B$ with $c_u/\gamma B$ at different values of H/B at ϕ = 40° and D/B = 0

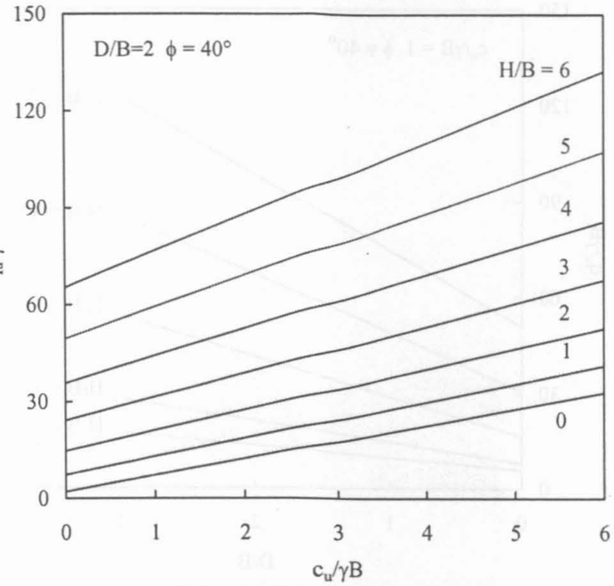

Fig. 9(b): Variation of $q_u/\gamma B$ with $c_u/\gamma B$ at different values of H/B at ϕ = 40° and D/B = 2

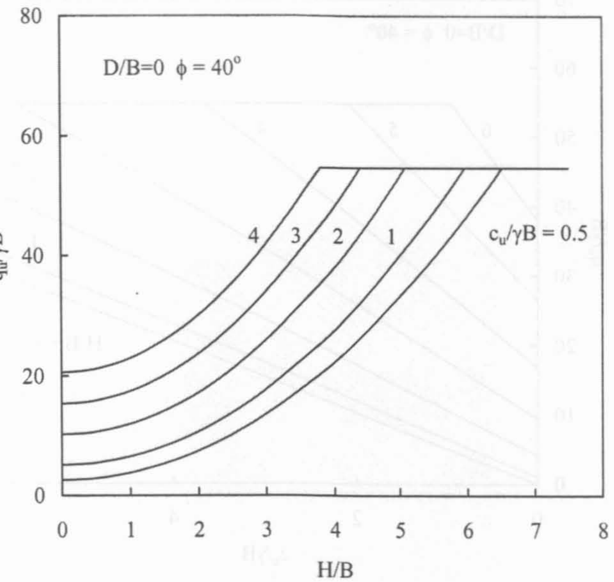

Fig. 10: Variation of $q_u/\gamma B$ with $c_u/\gamma B$ at different values of H/B at f = 40° and D/B = 2

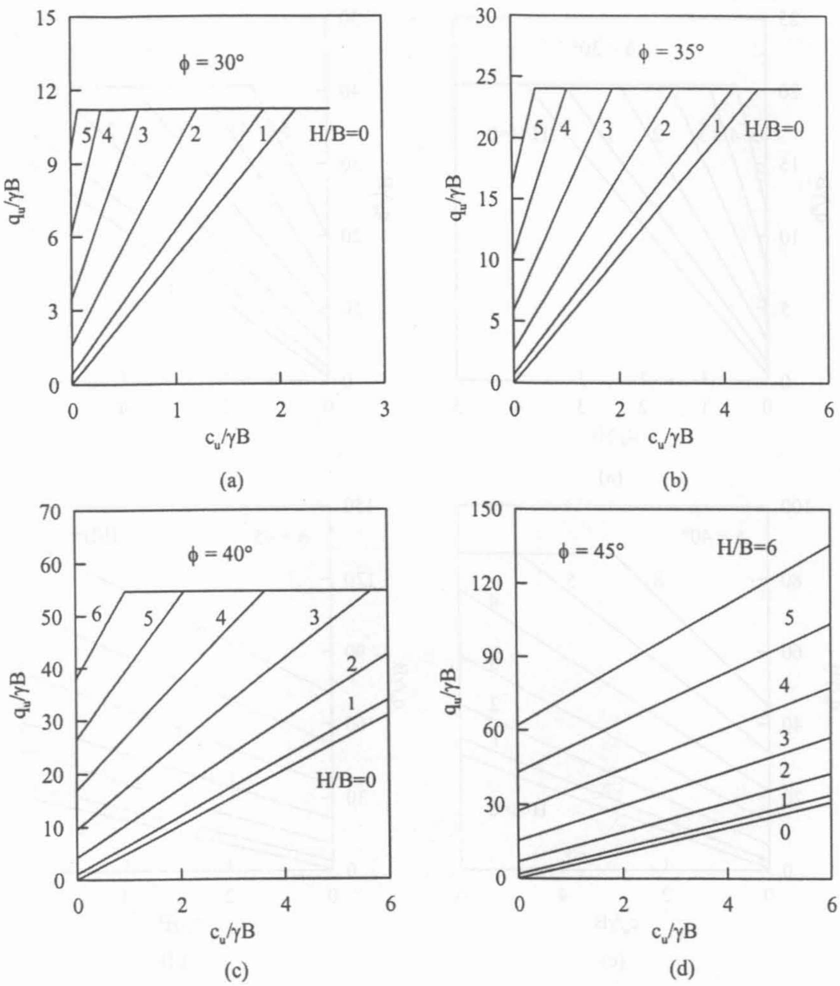

Fig. 11: Variation of $q_u/\gamma B$ with $c_u/\gamma B$ for sand-clay foundation soil at $D/B = 0$: (a) $\phi = 30°$, (b) $\phi = 35°$, (c) $\phi = 40°$ and (d) $\phi = 45°$

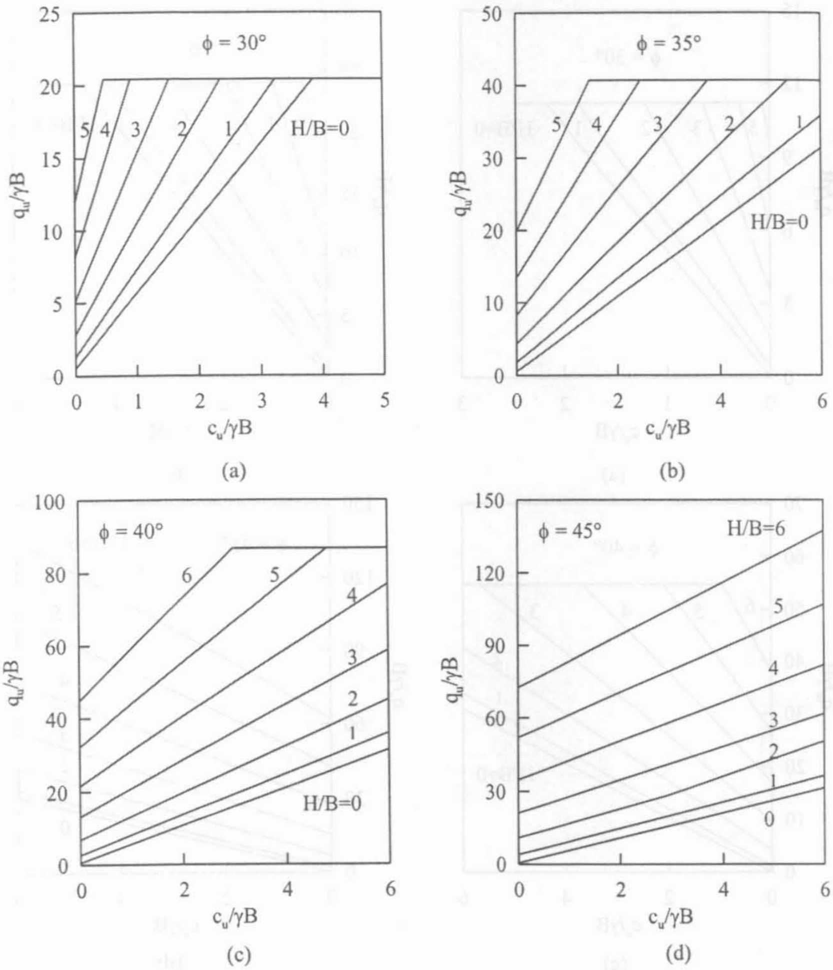

Fig. 12: Variation of $q_u/\gamma B$ with $c_u/\gamma B$ for sand-clay foundation soil at $D/B = 0.5$: (a) $\phi = 30°$, (b) $\phi = 35°$, (c) $\phi = 40°$ and (d) $\phi = 45°$

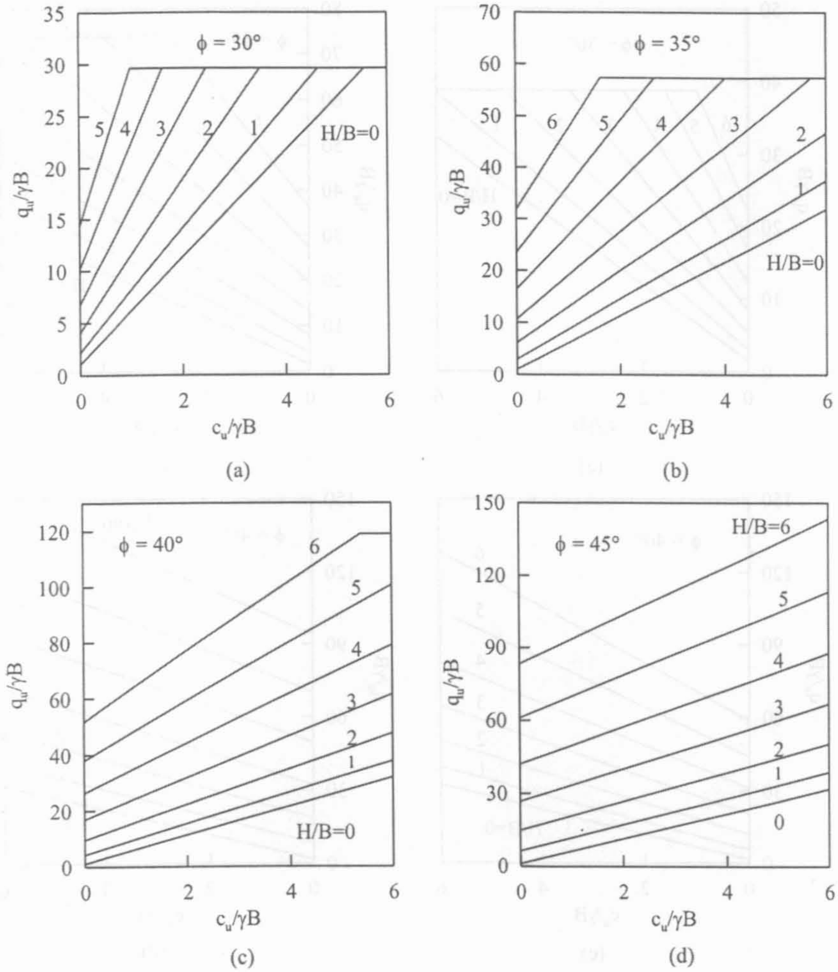

Fig. 13: Variation of $q_u/\gamma B$ with $c_u/\gamma B$ for sand-clay foundation soil at $D/B = 1.0$: (a) $\phi = 30°$, (b) $\phi = 35°$, (c) $\phi = 40°$ and (d) $\phi = 45°$

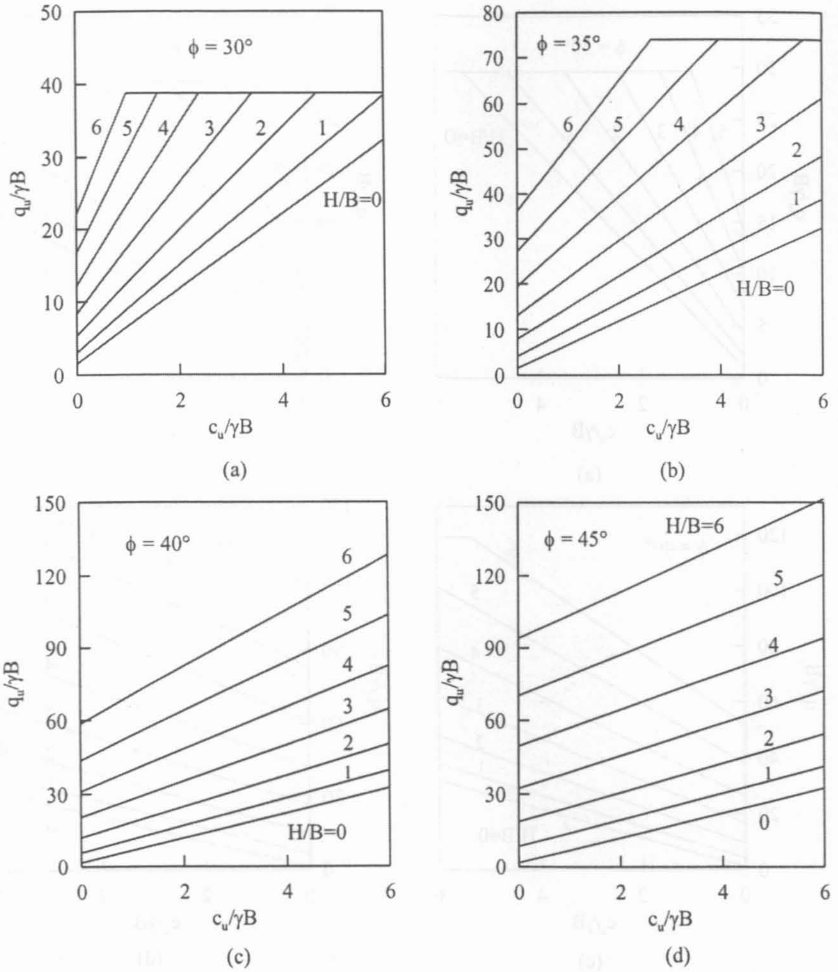

Fig. 14: Variation of $q_u/\gamma B$ with $c_u/\gamma B$ for sand-clay foundation soil at $D/B = 1.5$: (a) $\phi = 30°$, (b) $\phi = 35°$, (c) $\phi = 40°$ and (d) $\phi = 45°$

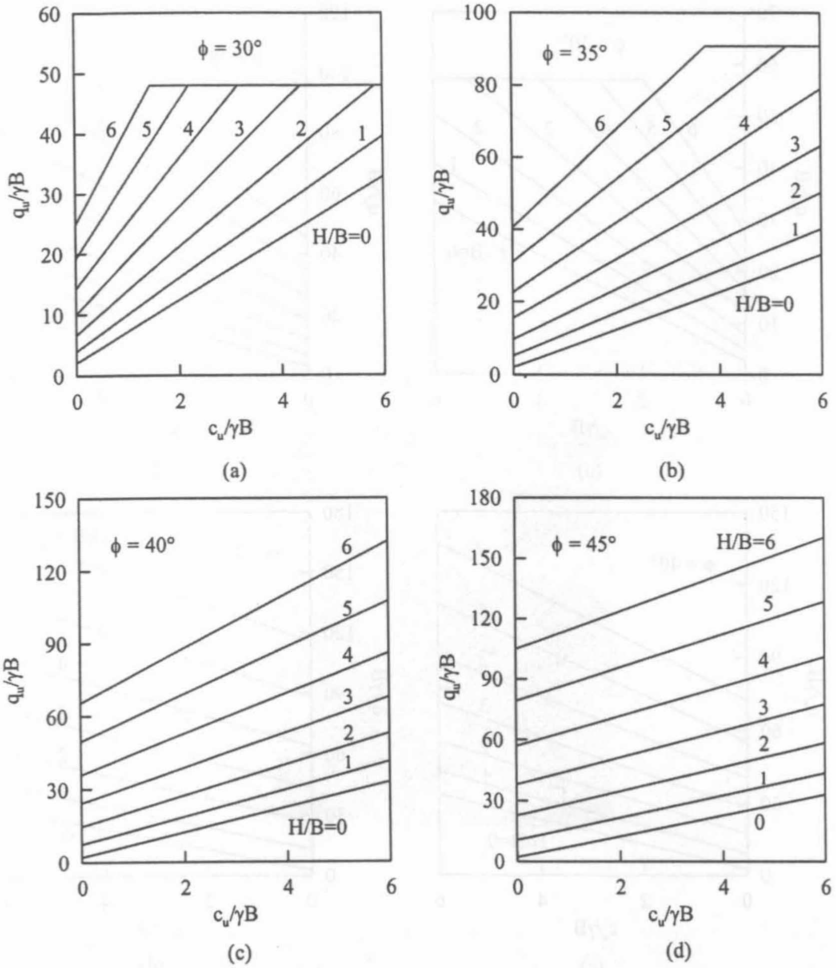

Fig. 15: Variation of $q_u/\gamma B$ with $c_u/\gamma B$ for sand-clay foundation soil at $D/B = 2.0$: (a) $\phi = 30°$, (b) $\phi = 35°$, (c) $\phi = 40°$ and (d) $\phi = 45°$

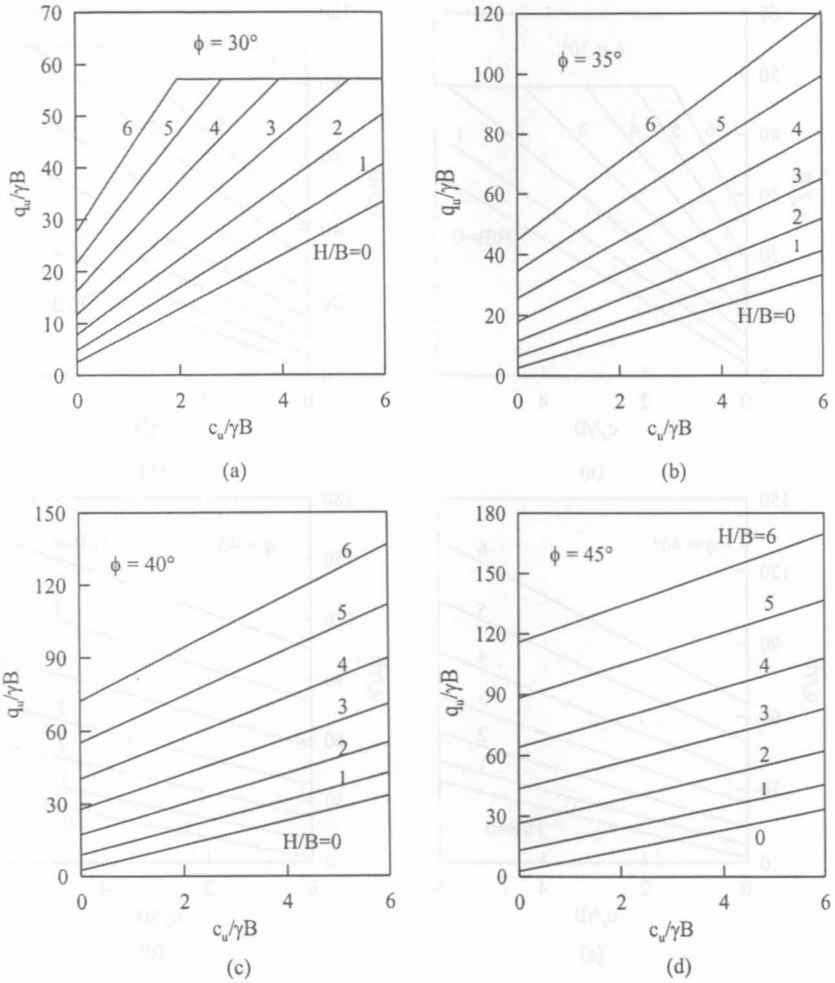

Fig. 16: Variation of $q_u/\gamma B$ with $c_u/\gamma B$ for sand-clay foundation soil at $D/B = 2.5$: (a) $\phi = 30°$, (b) $\phi = 35°$, (c) $\phi = 40°$ and (d) $\phi = 45°$

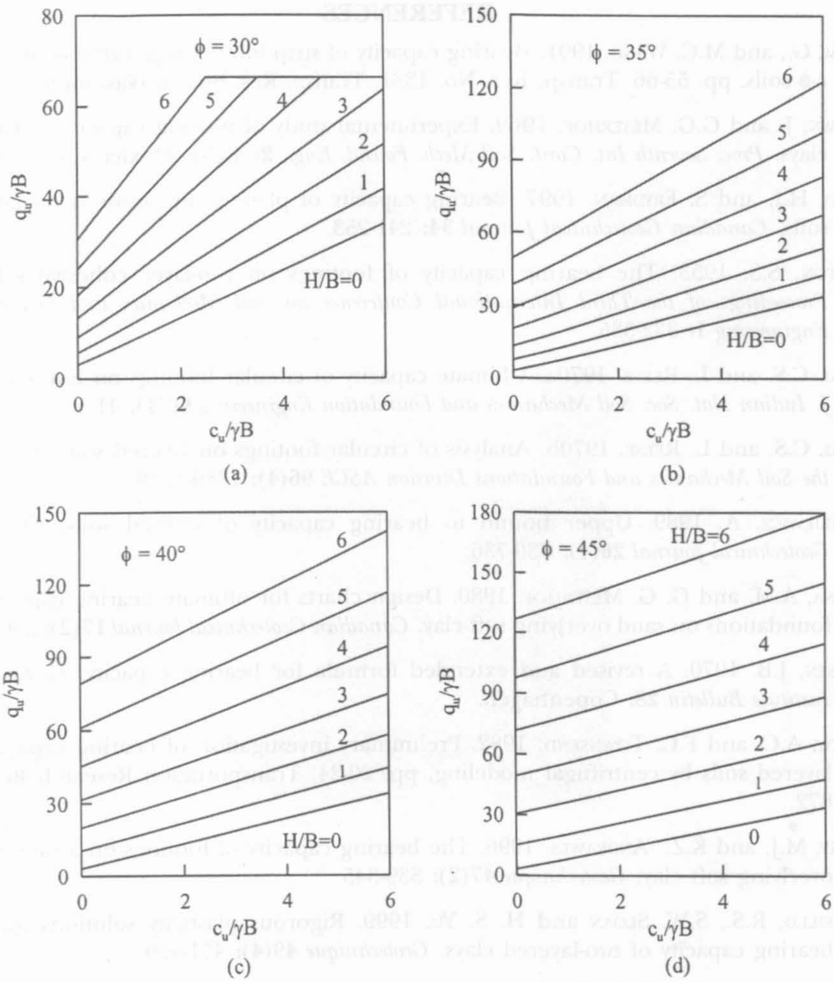

Fig. 17: Variation of $q_u/\gamma B$ with $c_u/\gamma B$ for sand-clay foundation soil at $D/B = 3.0$: (a) $\phi = 30°$, (b) $\phi = 35°$, (c) $\phi = 40°$ and (d) $\phi = 45°$

as a function of the properties of soils, the footing width, and the topsoil thickness. The effect of the punching shear parameter (δ) was considered and evaluated from empirical relationships that were developed based on the experimental results of Hanna and Meyerhof (1980). Based on this analysis, design charts were developed using the punching shear model in a dimensionless form since those of Hanna and Meyerhof (1980) were not presented in a nondimensionalised form, which limits their application. The presented charts here may be useful in overcoming the problem of design charts that were developed by Michalowski and Shi (1995) which may overestimate the bearing capacity by a significant amount depending on the very nature of the upper bound solution on which the derivation is based.

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