UNIVERSITI PUTRA MALAYSIA

NUMERICAL SOLUTION OF SECOND ORDER LINEAR TWO-POINT BOUNDARY VALUE PROBLEM USING DIRECT MULTISTEP METHOD

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NUMERICAL SOLUTION OF SECOND ORDER LINEAR TWO-POINT BOUNDARY VALUE PROBLEM USING DIRECT MULTISTEP METHOD

By

CHEW KHUI TAT

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirement for the Degree of Master of Science

November 2012

DEDICATION

To

My Dear Teachers

and

My Lovely Family

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

NUMERICAL SOLUTION OF SECOND ORDER LINEAR TWO-POINT BOUNDARY VALUE PROBLEM USING DIRECT MULTISTEP METHOD

By

CHEW KHUI TAT

November 2012

Chairman: Associate Professor Zanariah Abdul Majid, PhD

Faculty: Science

In this thesis, direct multistep methods are developed for solving second order linear two-point boundary value problems. The proposed direct multistep methods consist of one point direct method and two point direct block method. These methods are then used together with linear shooting technique in solving second order linear two-point boundary value problems using constant step size.

Most of the existing research involving second order linear two-point boundary value problems will reduce the problems to a system of first order ordinary differential equation. This approach will enlarge the system of first order ordinary differential equation and needs more computation work. The advantage of direct multistep methods proposed in this thesis solve second order linear two-point boundary value problems directly without reducing it to first order ordinary differential equation.

Moreover, the direct multistep methods are also implemented to solve linear boundary value problems with singular perturbation. The algorithms for solving

second order linear two-point boundary value problems and linear boundary value problems with singular perturbation are then executed in programing code which is written in C language. The numerical results showed that the performance of the developed methods gave good results in terms of maximum error and execution time.

In conclusion, the proposed methods in this thesis are suitable for solving second order linear two-point boundary value problems and linear boundary value problems with singular perturbation.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia Sebagai memenuhi keperluan ijazah Master Sains

PENYELESAIAN BERANGKA UNTUK MASALAH NILAI SEMPADAN DUA TITK LINEAR PERINGKAT KEDUA MENGGUNAKAN KAEDAH TERUS MULTILANGKAH

By

CHEW KHUI TAT

November 2012

Pengerusi: Profesor Madya Zanariah Abdul Majid, PhD

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Di dalam tesis ini, kaedah terus multilangkah dibangunkan untuk menyelesaikan masalah nilai sempadan dua titik linear peringkat kedua. Kaedah terus multilangkah yang dicadangkan mengandungi kaedah terus satu titik dan kaedah terus blok dua titik. Kaedah tersebut akan digunakan bersama dengan teknik penembakan linear untuk menyelesaikan masalah nilai sempadan dua titik linear peringkat kedua dengan menggunakan saiz langkah malar.

Kebanyakan penyelidikan terdahulu yang melibatkan masalah nilai sempadan dua titik linear peringkat kedua akan menurunkan masalah kepada sistem persamaan pembezaan biasa peringkat pertama. Pendekatan ini akan membesarkan sistem persamaan perbezaan biasa peringkat pertama malahan memerlukan lebih kerja pengiraan. Kelebihan kaedah terus multilangkah dalam tesis ini adalah dengan menyelesaikan masalah nilai sempadan dua titik linear peringkat kedua secara terus tanpa menurunan masalah kepada sistem persamaan pembezaan biasa peringkat pertama.

Selepas itu, kaedah terus multilangkah juga akan dilaksanakan untuk menyelesaikan masalah nilai sempadan linear dengan pengusikan singular. Algoritma bagi menyelesaikan masalah nilai sempadan dua titik linear peringkat kedua dan nilai sempadan linear dengan pengusikan singular akan dilaksanakan dengan menggunakan kod pengaturcaraan yang ditulis dalam bahasa C. Hasil berangka menunjukkan bahawa prestasi kaedah yang dibangunkan memberikan hasil yang baik dari segi ralat dan masa pelaksanaan penyelesaian.

Kesimpulannya, kaedah yang diusulkan dalam tesis ini adalah bersesuaian untuk menyelesaikan masalah nilai sempadan dua titik linear peringkat kedua dan nilai sempadan linear dengan pengusikan singular.

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I dedicate this thesis to my parents and my sisters, for their supports and encouragement.

I certify that a Thesis Examination Committee has met on 19 November 2012 to conduct the final examination of Chew Khui Tat on his thesis entitled "Numerical Solution of Second Order Linear Two-Point Boundary Value Problem Using Direct Multistep Method" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the degree of Master of Science.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

CHEW KHUI TAT

Date: 19 November 2012

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