



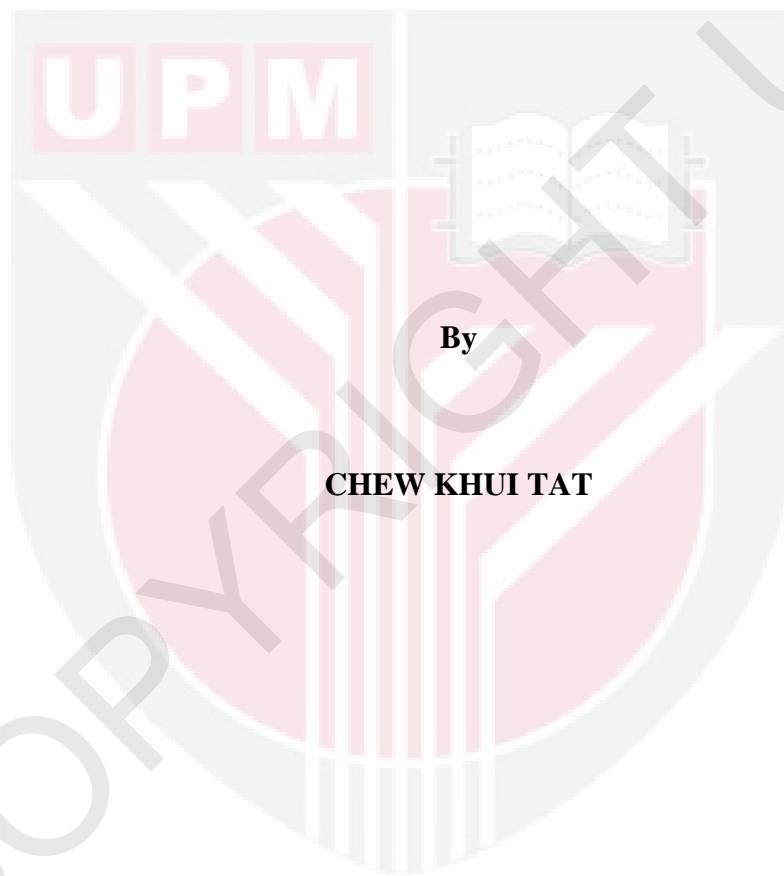
UNIVERSITI PUTRA MALAYSIA

**NUMERICAL SOLUTION OF SECOND ORDER LINEAR TWO-POINT
BOUNDARY VALUE PROBLEM USING DIRECT MULTISTEP METHOD**

CHEW KHUI TAT

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By

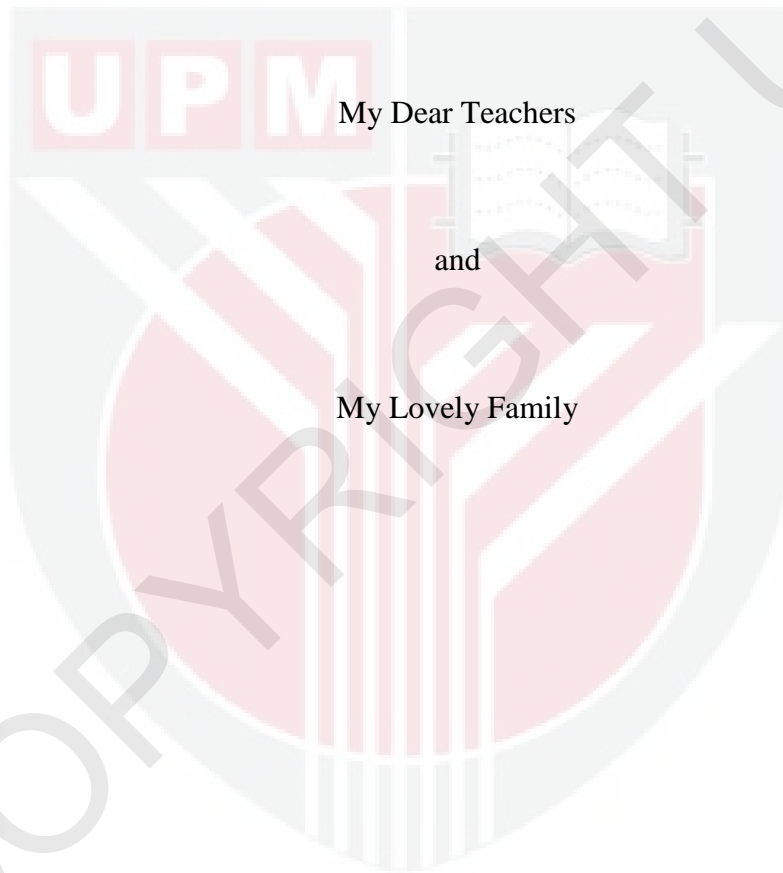
CHEW KHUI TAT

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia,
in Fulfilment of the Requirement for the
Degree of Master of Science**

November 2012

DEDICATION

To



My Dear Teachers

and

My Lovely Family

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in
fulfilment of the requirement for the degree of Master of Science

**NUMERICAL SOLUTION OF SECOND ORDER LINEAR TWO-POINT
BOUNDARY VALUE PROBLEM USING
DIRECT MULTISTEP METHOD**

By

CHEW KHUI TAT

November 2012

Chairman: Associate Professor Zanariah Abdul Majid, PhD

Faculty: Science

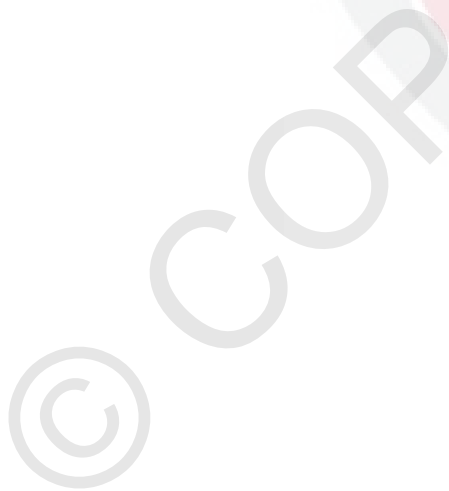
In this thesis, direct multistep methods are developed for solving second order linear two-point boundary value problems. The proposed direct multistep methods consist of one point direct method and two point direct block method. These methods are then used together with linear shooting technique in solving second order linear two-point boundary value problems using constant step size.

Most of the existing research involving second order linear two-point boundary value problems will reduce the problems to a system of first order ordinary differential equation. This approach will enlarge the system of first order ordinary differential equation and needs more computation work. The advantage of direct multistep methods proposed in this thesis solve second order linear two-point boundary value problems directly without reducing it to first order ordinary differential equation.

Moreover, the direct multistep methods are also implemented to solve linear boundary value problems with singular perturbation. The algorithms for solving

second order linear two-point boundary value problems and linear boundary value problems with singular perturbation are then executed in programing code which is written in C language. The numerical results showed that the performance of the developed methods gave good results in terms of maximum error and execution time.

In conclusion, the proposed methods in this thesis are suitable for solving second order linear two-point boundary value problems and linear boundary value problems with singular perturbation.



Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia Sebagai memenuhi keperluan ijazah Master Sains

**PENYELESAIAN BERANGKA UNTUK MASALAH NILAI SEMPADAN
DUA TITIK LINEAR PERINGKAT KEDUA MENGGUNAKAN KAEDAH
TERUS MULTILANGKAH**

By

CHEW KHUI TAT

November 2012

Pengerusi: Profesor Madya Zanariah Abdul Majid, PhD

Fakulti: Sains

Di dalam tesis ini, kaedah terus multilangkah dibangunkan untuk menyelesaikan masalah nilai sempadan dua titik linear peringkat kedua. Kaedah terus multilangkah yang dicadangkan mengandungi kaedah terus satu titik dan kaedah terus blok dua titik. Kaedah tersebut akan digunakan bersama dengan teknik penembakan linear untuk menyelesaikan masalah nilai sempadan dua titik linear peringkat kedua dengan menggunakan saiz langkah malar.

Kebanyakan penyelidikan terdahulu yang melibatkan masalah nilai sempadan dua titik linear peringkat kedua akan menurunkan masalah kepada sistem persamaan pembezaan biasa peringkat pertama. Pendekatan ini akan membesarkan sistem persamaan perbezaan biasa peringkat pertama malahan memerlukan lebih kerja pengiraan. Kelebihan kaedah terus multilangkah dalam tesis ini adalah dengan menyelesaikan masalah nilai sempadan dua titik linear peringkat kedua secara terus tanpa menurunkan masalah kepada sistem persamaan pembezaan biasa peringkat pertama.

Selepas itu, kaedah terus multilangkah juga akan dilaksanakan untuk menyelesaikan masalah nilai sempadan linear dengan pengusikan singular. Algoritma bagi menyelesaikan masalah nilai sempadan dua titik linear peringkat kedua dan nilai sempadan linear dengan pengusikan singular akan dilaksanakan dengan menggunakan kod pengaturcaraan yang ditulis dalam bahasa C. Hasil berangka menunjukkan bahawa prestasi kaedah yang dibangunkan memberikan hasil yang baik dari segi ralat dan masa pelaksanaan penyelesaian.

Kesimpulannya, kaedah yang diusulkan dalam tesis ini adalah bersesuaian untuk menyelesaikan masalah nilai sempadan dua titik linear peringkat kedua dan nilai sempadan linear dengan pengusikan singular.

ACKNOWLEDGEMENT

I would like to express the deepest appreciation to my committee chair, Associate Professor Dr. Zanariah Abdul Majid, for her wise council, guidance, invaluable advice and constant encouragement throughout my research. Without her guidance and persistent help this research would not have been possible. I wish to give my sincere thanks to the members of supervisory committee, Prof. Dato' Dr. Mohamed Suleiman and Dr. Norazak Senu for their comments and helps.

A gratefully acknowledged the financial support of Fundamental Research Grant Scheme (FRGS) from Ministry of Higher Education and Graduate Research Fellowship (GRF) from Universiti Putra Malaysia.

I dedicate this thesis to my parents and my sisters, for their supports and encouragement.

I certify that a Thesis Examination Committee has met on 19 November 2012 to conduct the final examination of Chew Khui Tat on his thesis entitled “Numerical Solution of Second Order Linear Two-Point Boundary Value Problem Using Direct Multistep Method” in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the degree of Master of Science.

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Date: 23 January 2013

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of Supervisory Committee were as follows:

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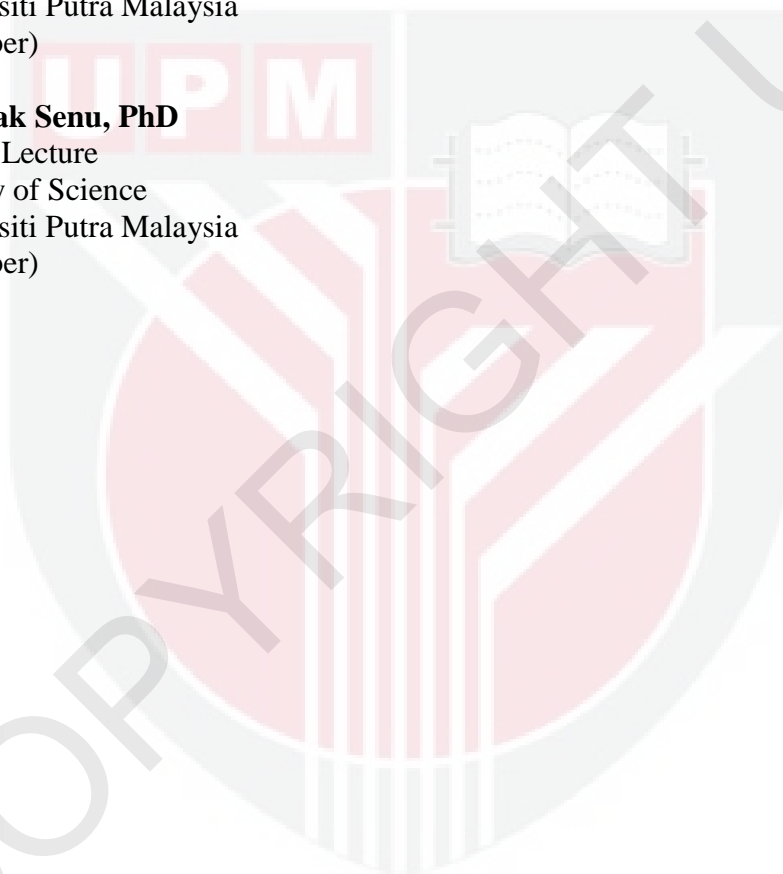
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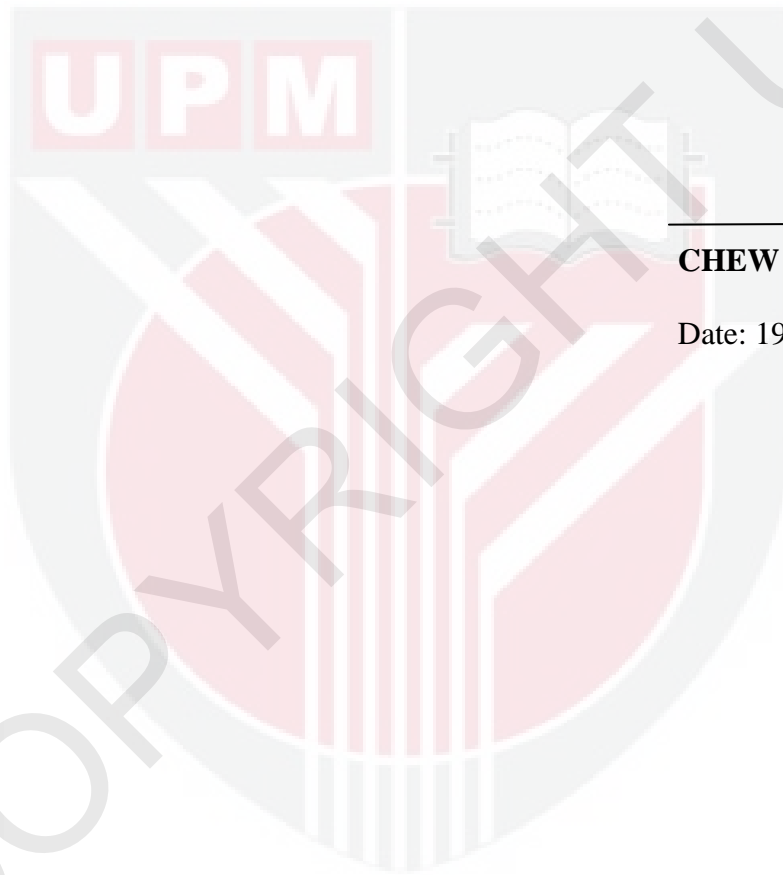
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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.



CHEW KHUI TAT

Date: 19 November 2012

TABLE OF CONTENTS

	Page
DEDICATION	i
ABSTRACT	ii
ABSTRAK	iv
ACKNOWLEDGEMENTS	vi
APPROVAL	vii
DECLARATION	ix
LIST OF TABLES	xii
LIST OF FIGURES	xvi
LIST OF ABBREVIATIONS	xviii
CHAPTER	
1 INTRODUCTION	
1.1 Introduction	1
1.2 Objective of the thesis	2
1.3 Scope and limitation of Study	3
1.4 Outline of the thesis	4
2 LITERATURE REVIEW	
2.1 Introduction	6
2.2 Boundary Value Problem	6
2.3 Existence Theory	8
2.4 Singular Perturbation Problem	9
2.5 Multistep Method	10
2.6 Review of Previous Works	11
3 SOLVING LINEAR BOUNDARY VALUE PROBLEM BY ONE POINT DIRECT METHOD USING CONSTANT STEP SIZE	
3.1 Introduction	16
3.2 Lagrange Interpolation Polynomial	16
3.3 Derivation of One Point Direct Method	17
3.4 Linear Shooting Technique	35
3.5 Algorithm of 1PDM code	37
3.6 Tested Problems	40
3.7 Numerical Results	42
3.8 Discussion	58
4 SOLVING LINEAR BOUNDARY VALUE PROBLEM BY MODIFIED DIRECT ADAMS MOULTON METHOD USING CONSTANT STEP SIZE	
4.1 Introduction	61
4.2 Derivation of Two Point Direct Block Method	61
4.3 Algorithm of 2PDM code	93
4.4 Numerical Results	96
4.5 Discussion	112

5	SOLVING LINEAR BOUNDARY VALUE WITH SINGULAR PERTURBATION PROBLEM BY DIRECT METHOD	
5.1	Introduction	115
5.2	Implementation	115
5.3	Tested Problems	119
5.4	Numerical Results	120
5.5	Discussion	133
6	CONCLUSION	
6.1	Summary	135
6.2	Future Work	136
	BIBLIOGRAPHY	137
	BIODATA OF STUDENT	140
	LIST OF PUBLICATIONS	141

