SOLVING DELAY DIFFERENTIAL EQUATIONS USING EXPLICIT RUNGE-KUTTA METHOD

By

AUNG SAN LWIN

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

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Chairman: **Associate Professor Fudziah Ismail, PhD**

Faculty: **Science and Environmental Studies**

Introduction to delay differential equations (DDEs) and their examples are presented. The General formulation of Explicit Runge-Kutta method when adapted to delay differential equations is described.

Delay Differential Equations are solved by embedded Explicit Runge-Kutta method, which is more attractive from the practical point of view. Embedding technique is used to solve DDEs not just with single delay, but with multiple delays. The technique is also used to get the local truncation error which provides a basis for choosing the next stepsize for the integration. The delay terms are approximated using three techniques of interpolation, which are the divided difference interpolation, Hermite interpolation, and continuous extensions formula of the Runge-Kutta method itself. Numerical results of tested problems based on these interpolations are presented and compared.

Finally, the stability properties of Explicit Runge-Kutta method when applied to DDEs using Lagrange interpolation, Hermite interpolation and continuous extensions Runge-Kutta formula are investigated and their stability regions are illustrated.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Master Sains

**MENYELESAIKAN PERSAMAAN PEMBEZAAN LENGAH MENGGUNAKAN
KAEDAH RUNGE-KUTTA TAK TERSIRAT**

Oleh

AUNG SAN LWIN

Ogos 2004

Pengerusi: Profesor Madya Fudziah Ismail, PhD

Fakulti: Sains dan Pengajian Alam Sekitar

Pengenalan kepada persamaan pembezaan lengah (PPL) dan beberapa contoh diberikan. Formulasi umum bila kaedah Runge-Kutta disesuaikan kepada persamaan pembezaan lengah diterangkan.

Persamaan pembezaan lengah diselesaikan dengan kaedah terbenam tak tersirat Runge-Kutta yang lebih menarik dari segi praktikalnya. Teknik terbenam digunakan bagi menyelesaikan PPL bukan sahaja dengan satu sebutan lengah, tetapi juga dengan sebutan lengah yang lebih. Teknik ini juga digunakan untuk mendapatkan ralat pangkasan tempatan yang memberikan asas untuk pemilihan panjang langkah seterusnya bagi kamiran tersebut. Sebutan lengahnya diperolehi dengan menggunakan tiga teknik interpolasi, iaitu interpolasi beza bahagi, interpolasi Hermite dan juga rumus perluasan

berterusan kaedah Runge-Kutta itu sendiri. Keputusan berangka bagi masaalah yang diuji berdasarkan interpolasi tersebut dipersembahkan dan dibandingkan.

Akhir sekali, ciri-ciri kestabilan kaedah tak tersirat Runge-Kutta bagi menyelesaikan PPL menggunakan interpolasi Langrange, interpolasi Hermite dan rumus perluasan berterusan kaedah Runge-Kutta itu sendiri dikaji, dan rantau kestabilannya diilustrasikan.

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I certify that an Examination Committee met on 9th August 2004 to conduct the final examination of Aung San Lwin on Master of Science thesis entitled "Solving Delay Differential Equations using Explicit Runge-Kutta method" in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

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This thesis submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of the Supervisory Committee are as follows:

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DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

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AUNG SAN LWIN

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