



UNIVERSITI PUTRA MALAYSIA

**SOLVING THIRD-ORDER BOUNDARY VALUE PROBLEM BY DIRECT
METHODS**

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**SOLVING THIRD-ORDER BOUNDARY VALUE PROBLEM BY DIRECT
METHODS**

By

AHMAD SHAH ABDULLAH BIN AHMAD ZULKIFLI

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia,
in Fulfilment of the Requirement for the Degree of Master of Science**

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DEDICATION

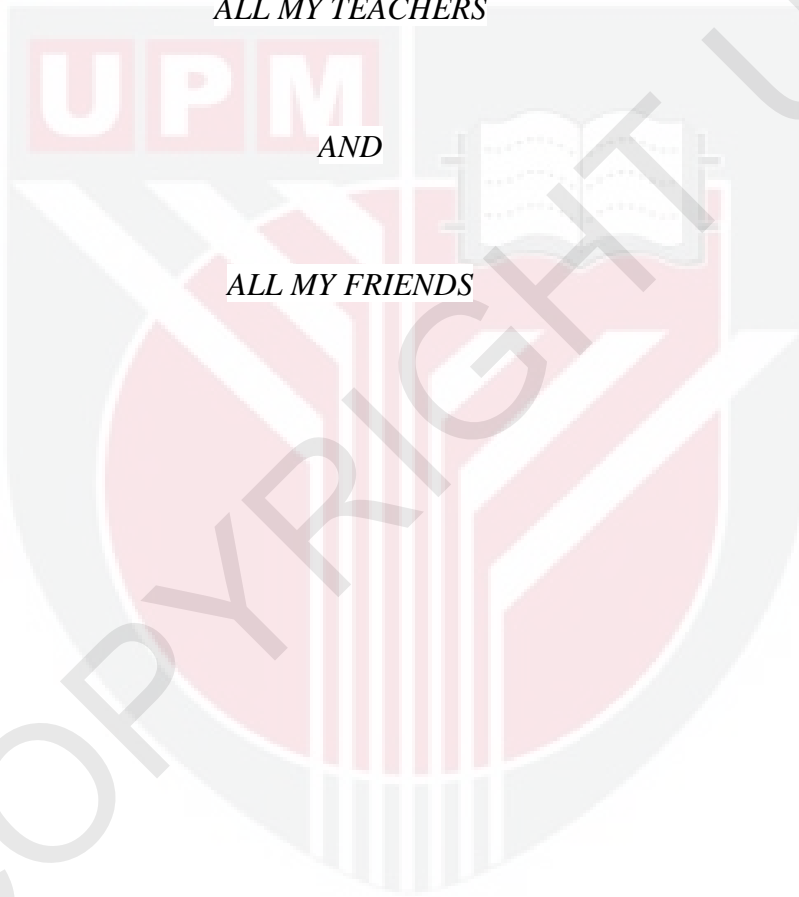
TO

MY FAMILY,

ALL MY TEACHERS

AND

ALL MY FRIENDS



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

SOLVING THIRD-ORDER BOUNDARY VALUE PROBLEM BY DIRECT METHODS

By

AHMAD SHAH ABDULLAH BIN AHMAD ZULKIFLI
February 2014

Chairman: Associate Professor Zanariah Abdul Majid, PhD

Faculty: Institute for Mathematical Research

In this research, the direct method of multistep method is developed for the numerical solution of nonlinear boundary value problems (BVPs) of Type 1 and Type 2 directly. Most of the existing research involving BVPs will reduce the problem to a system of first order Ordinary Differential Equations (ODEs). However, the proposed method will solve the third-order BVPs directly without reducing to first-order ODEs with constant step size using the shooting technique. On- point and two-point direct block method of Adam Moulton have been derived. These methods consists the predictor and corrector method where the predictor is one order less than the corrector. In the numerical results, one-point direct methods have advantages in accuracy and for two-point direct block methods have advantages in timing calculation. The results clearly show that the proposed method is suitable for solving third-order nonlinear BVPs.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

**PENYELESAIAN MASALAH NILAI SEMPADAN PERINGKAT KETIGA
MENGUNAKAN KAEDAH LANGSUNG**

Oleh

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Dalam kajian ini, kaedah langsung bagi kaedah multilangkah dibangunkan untuk penyelesaian berangka bagi masalah nilai sempadan (MNS) tak linear dari Jenis 1 and Jenis 2 secara langsung. Kebanyakan penyelidikan yang sedia ada melibatkan MNS akan menurunkan masalah kepada sistem Persamaan Pembezaan Biasa (PPB) peringkat pertama. Walau bagaimanapun, kaedah yang dicadang akan menyelesaikan MNS peringkat ketiga secara langsung tanpa menurunkan masalah ke sistem PPB peringkat pertama dengan saiz langkah malar menggunakan teknik menembak. Kaedah blok langsung Adam Moulton satu titik dan dua titik telah diterbitkan. Kaedah ini terdiri kaedah peramal dan pembedul yang mana peramal adalah kurang satu peringkat daripada pembedul. Dalam keputusan berangka, kaedah langsung satu titik mempunyai kelebihan dari segi ketepatan dan kaedah blok langsung dua titik mempunyai kelebihan dalam pengiraan masa. Keputusan jelas menunjukkan kaedah yang dicadangkan adalah sesuai bagi penyelesaian MNS tak linear peringkat ketiga.

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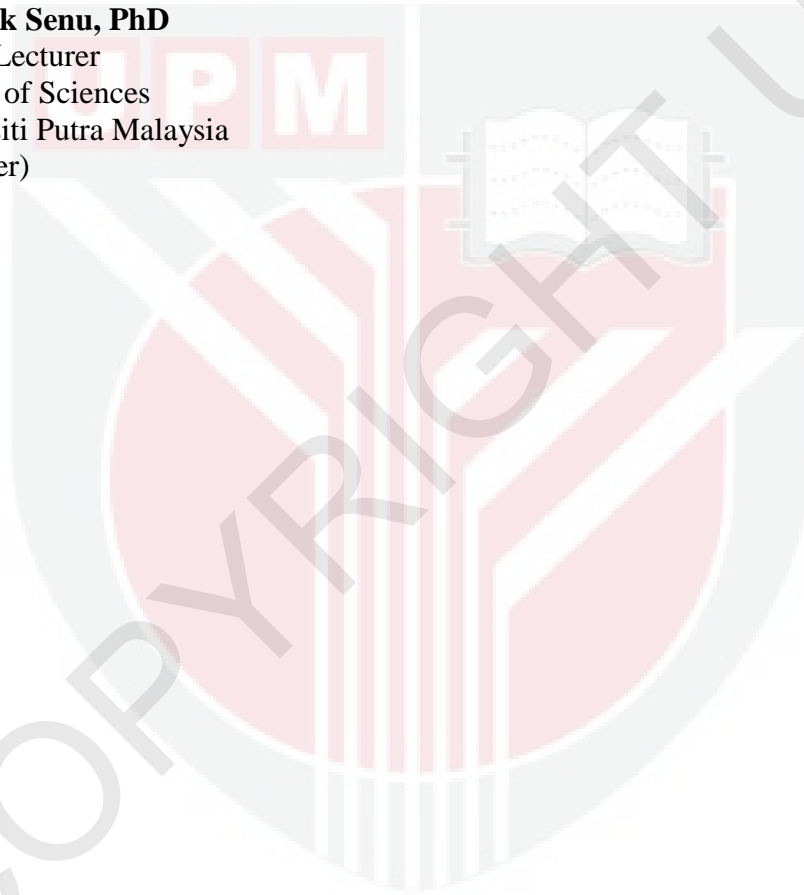
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DECLARATION

Declaration by graduate student

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