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Application of Iterative Technique (IT) Using SPT N-Values and Correlations for Analysis of Tip and Shaft Capacity for an Axially Loaded Pile in Sand

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ABSTRACT

This is a theoretical method on how to analyze the axial capacity (tip and shaft) of a single pile using iterative technique on SPT-N values and SPT-N correlation. This technique allows the engineer who conducts the analyses to control the input and output information of the analyses in a systematic and organized fashion. This method is also very helpful especially for the development of pile capacity prediction using reliability method. Research on the subject is being carried out at Universiti Putra Malaysia (UPM).

Keywords: SPT N-values, iterative technique (IT), axial pile capacity

INTRODUCTION

The Standard Penetration Test (SPT), developed around 1927 (Bowles 1996), is still one of the most popular and economical means to obtain subsurface information of soil. It has been used in correlation to determine unit weight, (γ), relative density, (D_r), angle of internal friction, (ϕ), and the undrained compressive strength,(q_u) . It has also been used for estimating the stress- strainmodulus,(E), and the bearing capacity of foundations.

Iterative Technique (IT) is an approach that is being used based on the logical assumption that the correlation used to relate N-Values and the sand properties (ϕ , D_r, γ_n) is valid. A very brief description of this technique will be presented here, covering the iteration process, some analysis of results and the suggested correlations. This IT is presented in a stepwise manner to achieve better understanding of this method.

STEP 1. N-Values are firstly corrected for the water table and to account for fine or silty sand below water (Heydinger 1984; Coyle & Costello 1981), as given in Eqn. 1. Water table correction on N-Values is just an example of one correction factor that can be applied on N-Values for discrepancies occurring due to, for instance, difference in equipment's manufacturers, uncertainties in geotechnical parameters and the drive hammer configuration.

$$N' = \frac{1}{2}(N-15)+15 \tag{1}$$

STEP 2. Using a similar method suggested by Wolff (1989) and Ab.Malik (1992), the effective friction angle of sand, (ϕ), can be correlated with the corrected N-Values, (N"), using the relationship as presented by Peck *et al.* (1974), which can be approximated as:

$$\phi = 26.7 + 0.36N" - 0.0014(N")^2 \tag{2}$$

However, because (N") is not available due to lack of overburden stress data, σ_v, where N " is N-Value corrected for overburden stress. Because the unit weight, γ_n, is not usually available, therefore, a preliminary assumption of N' = N" is presumed in Eqn. 2. After the first iteration of LOOP 1 (1^{st} LOOP 1), when N" is available, (ϕ) can be calculated using N". This step requires at least two complete iterations, i.e. second iteration of LOOP 1 (2nd LOOP 1).

STEP 3. Meyerhof (1959) relates the effective angle of shearing resistance (ϕ') and the relative density, (D_r), in equation 3. This equation is rearranged to obtain(D_r) as a function of (ϕ') in Eqn. 4. Where:

$$\phi' = 28 + 0.15\,D_r \tag{3}$$

$$D_r\,\frac{\phi'-28}{0.15} \tag{4}$$

Even though Eqn. (4), can give a rough approximation of D_r, the expression in Eqn. 3 was not derived for the purpose of evaluating relative density from effective angle of internal friction, ϕ'. In other words, $D_r = f(\phi')$ is not a correct function representation.

STEP 4. Now the relative density can be applied in Eqn. 5 to find the probable unit weight, g_n, of sand.

$$D_r = \frac{(\gamma'_n - \gamma'_{min}) \times (\gamma'_{max})}{(\gamma'_{max} - \gamma'_{min}) \times (\gamma'_n)} \tag{5}$$

Where γ_{max} and γ_{min} are arbitrarily chosen values considering medium and dense sand as a preset limit for normal sand condition. Preset values chosen are γ_{max} = 20 kN/m^3 (D_r=0.65) for dense sand and γ_{min} = 17 kN/m^3 (D_r=0.35) for medium dense sand (relative density for most soil is in between 0.35-0.65). The simplification of this formula is as shown in Eqn. 6:

$$\gamma_n = \frac{340}{20 - 3D_r} \tag{6}$$

Eqns. 4, 5 and 6 are applied to find D_r and γ_n', this will be identified as route A. To obtain effective unit weight, γ_n',γ_w (9.8 kN/m^3) is subtracted from the unit weight in Eqn. 6 (for submerged cases).

$$\gamma_n' = \gamma_n - \gamma_w \tag{7}$$

STEP 3 and *STEP 4* are highly debatable because of the assumptions used to derive D_r and γ_n. One method that could overcome this problem is by using empirical values for ϕ', D_r, and γ_n of granular soils based on the N-Values at about 6m depth for normally consolidated sand derived by Shioi and Fukui (1982) to replace Eqns. (4) and (6). These data were fitted using a spreadsheet program and the formula for fine, medium and course sand is as below :

$D_r = 0.07\phi' - 1.92$	}fine sand
$D_r = 0.05\phi' - 1.54$	}medium sand
$D_r = 0.03\phi' - 1.00$	}coarse sand
$\gamma_n = -0.016\phi'^2 + 1.64\phi' - 20.4$	}fine sand
$\gamma_n = -0.019\phi'^2 + 1.90\phi' - 24.7$	}medium sand
$\gamma_n = -0.046\phi'^2 + 3.75\phi'1 - 54.3$	}coarse sand

The alternative equation introduced above will be identified as route B. A comparison of results obtained using route A and B will be discussed in the results and discussion section later in the paper.

STEP 5. It is known from basic soil mechanics theory that the effective overburden stress, σ_v', can be determined as long as the unit weight and the depth of the soil element can be determined accurately. Therefore, overburden stress, σ_v', can be represented as:

$$\sigma_v{}' = \sum_{i=0} [\gamma_n{}' z] \tag{8}$$

STEP 6. The final link for this procedure is completed using a correction factor similarly used by Wolff (1989) and Ab. Malik (1992). Liao and Whitman (1986) introduced this correction factor for effective overburden stress.

$$N" = N'[95.76/\sigma_v{}']^{\frac{1}{2}} ; (\sigma_v{}' \text{ is in kPa}) \tag{9}$$

However, the correlation in Eqn. 9 causes a very rapid increase in N-Values especially for lower overburden stress values (shallow depth). Therefore, the authors have suggested another correlation proposed by Heydinger in 1984, (Coyle *et al.* 1981):

$$N" = N'0.77*[\log(1915.2/\sigma_v{}')] \tag{10}$$

Eqn. 10 will be used in all the analyses of data as presented in Table 1.

STEP 7. Now a loop has been created where the corrected SPT-N value N" can be used to obtain ϕ', D_r and γ_n' by continuous iteration. This loop as shown below is named LOOP 1. This is done simply for identification purposes. Notice that the whole procedure only uses one data input, N-Value.

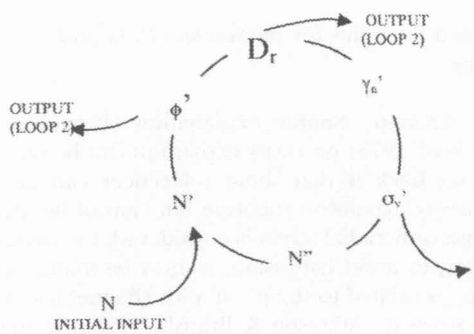

Fig. 1: LOOP 1

Similar IT were used by Fleming *et al.* (1992), to obtain the ultimate tip capacity in sand. Fleming used overburden stress, σ_v', and relative density, D_r, as his initial input. This method will be explained in detail in the next section, but the initial input will be originated from LOOP 1. This means that LOOP 1 and 2 (as discussed in the next section) are linked.

TIP CAPACITY IN SAND

Randolph (1994) and Fleming *et al.* (1992), applied cavity expansion theorem to further develop this method of axial pile capacity analysis. However, Randolph's method of analysis was more refined and found better agreement with numerical solutions compared to Fleming's. Tip capacity in sand was derived using the analogy between spherical cavity expansion and bearing failure by Randolph in 1994 (Gibson 1950). This leads to the relationship between end-bearing pressure, q_b, and limit pressure, p_{lim}, (Eqn. 11).

$$qb = p_{lim} \{ 1 + \tan\phi_{cv}' \tan (45 + \left(\frac{\phi'_{cv}}{2}\right))\} \quad (11)$$

Limit pressure, plim, solutions can be very long and confusing. However, Yu & Houlsby (1991) have published analytical solutions that comply with numerical solutions. Parameters needed to solve the limit pressure equations (Eqns. (11), (28) and (29)) are E, G, ν, c, ϕ', ψ', p_o and m are as explained below:

i. Stress-strain Modulus, E in kPa, Bazaraa, in 1982 (D'Appolonia 1970) is:

Saturated sand,	$E = 250(N + 15)$	(12)
N.C sand,	$E = 500(N + 15)$	(13)
O.C sand,	$E = 40{,}000+1050N$	(14)

ii. Shear modulus, G, in kPa (Randolph 1994) is:

$$G = 40{,}000*\exp(0.7D_r)*(p'/100)^{0.5} \quad (15)$$

iii. Poisson's ratio, n, is a dimensionless factor. From the original expression $G = \frac{E}{2(1+\nu)}$, and using the solution for E and G (Eqns. (12)-(15)) above:

$$\nu = \frac{E}{(2G)-1} \quad (16)$$

Notice that the proposed formulas for parameters E, G and ν are all derived from N-Values using correlations.

iv. Initial mean effective stress p_o. Simple explanation (Briaud 1992) and detailed explanation (Baguelin *et al.* 1978) on cavity expansion can be obtained from various references. (A slight set back is that these references can be used for general understanding of the cavity expansion theorem but cannot be used directly for pile capacity analysis because only radial stress is considered, i.e. pressuremeter analysis, and not axial). However, to avoid confusion, it must be made clear that the initial mean effective stress, p_o' is related to the p' - q' plot (Baguelin *et al.* 1978) and is not similar to overburden stress σ_v. Atkinson & Bransby (1982) defined mean effective stress p' as:

$$p' = \frac{1}{3}(\sigma_v' + 2\sigma_h') \quad (17)$$

v. Factor identifying cavity type, m, for spherical expansion solutions is equal to 2(m = 2).

vi. Two important parameter are still left unsolved, effective angle of friction and dilation, ϕ' and ψ', respectively. To solve these we will use input from LOOP 1. We will require the input of overburden stress, σ_v', bearing capacity factor, N_q, and relative density, D_r, to find the suitable ϕ'_{cv} (Fleming *et al.* 1992 ; Bolton 1986).

Solution for ϕ' and ψ'

Following the work of Bolton (1986, 1987), rigidity index, I_r, is related to relative density, D_r, and mean effective stress, p', Eqns. (18-19).

$$I_r = D_r\,[5.4 - \ln(p'/100)] - 1 \quad \text{for } p' < 150\text{kPa} \tag{18}$$

$$I_r = 5D_r - 1 \quad \text{for } p' \geq 150\text{kPa} \tag{19}$$

Fleming suggested that the mean effective stress at failure p', be approximated using Eqn. (20)

$$p' \approx \sqrt{N_q} * \sigma_v' \tag{20}$$

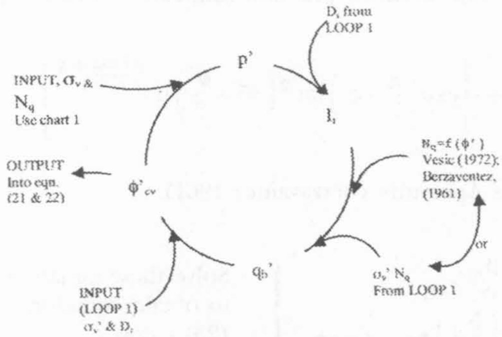

Fig. 2: LOOP 2

Rigidity index, I_r, is obtained for its respective mean effective stress p' and D_r using Eqns. (18)-(19)). Effective friction and dilation angles, ϕ' and ψ', are deduced, which leads to effective friction and dilation angles for cavity expansion solutions as in Eqns. (21) and (22).

$$\phi = \phi'_{cv} + 1.5I_r \tag{21}$$

$$\psi' = 1.875I_r \tag{22}$$

This angle of friction, ϕ and angle of dilation, ψ, is the final input into the limit pressure equation (Eqns. (11), (28) and (29)) required for solving the limit pressure, plim, equation. Simplified formula for input into the plim solution is given by Eqns. (28) and (29). The critical state angle of friction, ϕ_{cv}, relates to conditions where soil shears with zero dilation (i.e. at constant volume).

A logical question that should arise is that, why not apply relative density, D_r, straight into Eqns. (18)-(19) to find the corresponding ϕ and ψ for analysis? This is simply because relative density, D_r, in this analysis must be a function of critical state angle of friction, ϕ_{cv}. How many iterations should be conducted before a good solution is achieved? (i.e. Fleming *et al.* (1992) used three iterations to find the corrected bearing capacity factor, N_q, for tip capacity analysis). Parameters D_r and ϕ'_{cv} is extracted from LOOP 2 to find ϕ' and ψ' using Eqns. (21)-(22).

SHAFT CAPACITY IN SAND

Peak Shaft Capacity Phenomenon and Its Analyses

The peak shaft capacity occurs near the tip of the pile. This phenomenon has been observed using instrumented piles (Vesic 1970; Lehane *et al.* 1993). The peak shaft resistance ratio to the overburden stress, $\left(\frac{\tau_s}{\sigma_v'} = \beta\right)$ that is maximum at the tip of the pile in sand has to be determined. Where β is similar to the effective stress coefficient originally introduced by Burland (1973), Zeevaert (1960), Eide *et al.* (1961) and Chandler (1968). β_{max} is a function of S_t, δ and N_q.

Where;

S_t = 2*exp(-7tan δ)

δ = Angle of friction between pile and soil, can be determined as in Potyoundy (1961).

$$Nq = Nq = \frac{3}{3-\sin\phi'}\left[\exp\left(\frac{\pi}{2}-\phi'\right)\tan^2\left(45+\frac{\phi'}{2}\right)Irr^{\left(\frac{1.33\sin\phi'}{1+\sin\phi'}\right)}\right]$$

or use Chart 1 in the Appendix (Berzavantez 1961)

$$\left.\begin{aligned}\tau_{max} &= \beta_{max} * \sigma_v{}' \\ q_b &= N_q * \sigma_v{}' \\ \tau_{max} &= \left(\frac{N_q}{50}\right) * \sigma_v{}' * \tan\delta\end{aligned}\right\} \quad \text{Solve these equation to obtain equation. (23) - (24)}$$

where $1/50 \cong S_t$

$$\frac{\tau_{max}}{q_b} = \frac{\beta_{max}}{N_q} = S_t \tan\delta \tag{23}$$

so; $\beta_{max} = N_q * S_{t*} \tan\delta$

because, $\beta_{max} = K_{max*} \tan\delta$; (24)

then, Kmax =Nq*St (25)

Shaft Friction Distribution Along Pile

Randolph adopted the work by Toolan *et al.* (1990) to postulate β_{min} which is believed to be linked to the active earth pressure coefficient, K_a. Randolph also adopted a simple distribution of shaft capacity as proposed by Heerema (1980). The shaft capacity distribution is represented by the formula:

$$\frac{\tau_s}{\sigma_v'} = \beta(z) = \beta_{min} + (\beta_{max} - \beta_{min}) * \exp\left[-\mu\left(\frac{L-z}{d}\right)\right] \tag{26}$$

Where, $\beta_{min} \approx K_a = \dfrac{1-\sin\phi'}{1+\sin\phi'}; \beta_{max}$ as in equation. (24)

and K_{max} as in Eqn. (25). Therefore;

$$\frac{\tau_s}{\sigma'_v \tan\delta} = K(z) = K_{min} + (K_{max} - K_{min}) * \exp\left[-u\left(\frac{L-z}{d}\right)\right] \tag{27}$$

Where μ is the rate of exponential ($0.025<\mu<0.1$). Exponential decay, m, can be determined by having a large database of instrumented piles, L is the total embedment length of the pile, z = depth below ground level, d = pile diameter and (L-z)/d is the normalized length of pile driven past that particular location.

Advantages and Improvements Needed

The purpose of this whole procedure is to have only one data input (N-Values) to calculate axial capacity of a single pile in sand using previously proposed correlations. This itself makes this method a versatile tool for analysis. However, this method is only suggested as an option for analyses and not to be used solely. Another advantage is that, if data is unreliable or inaccurate due to testing procedures, then these data can be averaged through the process of iteration. This method also gives a new meaning for determining shaft capacity using empirical correlations, where it considers several important soil parameter (i.e. ϕ', I_r, D_r, etc.) instead of the empirical correlation using N-Values directly into shaft capacity formula (i.e. τ_s = (N/50) $\tan\delta$)

An obstacle faced in the development of this IT is that, since all the formulas and correlation are interlinked, one wrong data input can trigger off a chain reaction of wrongfully interpreted data. Perhaps this is the only setback of this procedure.

TABLE 1
Results of IT using route A and B of (LOOP 1)

Depth / m	SPT-N	N'	ϕ'^A	ϕ'^B	D_r^A	D_r^B	γ_n^A	γ_n^B	N''^A	N''^B
0.762	37	37	38.1	38.1	67.4	49	18.9	17.17	62	59
			43.6	43.1	104.3	68.7	20.15	21.3	60	59
2.286	44	44	39.8	39.8	78.8	55.8	17.2	21.6	57	54
			42.7	42.1	97.8	64.6	17.3	21.5	57	54
3.81	51	33	37.1	37.1	61	64	18.7	19.6	36	36
			38.1	37.8	68	68	18.9	19.9	36	36
5.33	58	37	38.2	38.2	68	70.7	18.9	20.13	37	36
			38.2	37.9	68	68.7	18.9	19.97	37	36
6.555	64	40	38.9	38.9	73	74.8	19.1	20.4	37	37
			38	38	66.5	69.6	18.9	20.04	37	36

RESULTS

For each depth, Table 1, the first iteration (1st LOOP for each depth) is represented by the first row, and the second iteration by the second row.

DISCUSSION AND CONCLUSION

The result in Table 1 shows that the number of iteration required to achieve satisfactory results is 2 iterations (2nd LOOP 1). These are results of data analysed using IT, for the Northwestern pile prediction symposium and the results comply with the results obtained by Heydinger (1984) in the symposium. Compared with the other 24 predictors, Heydinger obtained a predicted to measured pile capacity ratio of more than 90%. However, the application of LOOP 2 in non-homogenus or residual soil (i.e. Malaysian soil) is still being experimented with.

Suggested Correlation for Further Application

Bazaraa (1967) suggested these correlations:

N' = N; for fine to coarse sand
N' = 0.6N; for very fine or silty sand

D' Appolonia *et al.* (1970) and Gibbs *et al.* (1957):

$N' = 4N/(1+4p')$ for $p' \leq 0.75 kg/cm^2$
$N' = 4N/(3.25+p')$ for $p' \geq 0.75 kg/cm^2$

Thornburn *et al.* (1971) & Meyerhof (1976).

$$\left.\begin{array}{l}\tau_s = \overline{N}/50 \text{tons} \\ \tau_t = 4N \text{ tons}\end{array}\right\} \text{Driven pile in sand}$$

$$\left.\begin{array}{l}\tau_s = \overline{N}/60 \text{tons} \\ \tau_t = 2.5N \text{ tons}\end{array}\right\} \text{Driven pile in silt}$$

Limit Pressure, Plim, Solutions

For spherical cavity expansion in the associated Mohr-Coulomb material (i.e. m=2 and $\beta = \alpha$) is E, ν, c, ϕ, ψ, p_o.

$$R\propto = \frac{(m+\alpha)(\alpha-1)p_{lim}}{\alpha(1+m)(\alpha-1)p_o} \tag{28}$$

$$\left(\frac{\eta}{\gamma}\right)(1-\delta)^{\frac{(\beta+m)}{\beta}} = \frac{\xi}{n!(n-\gamma)} * \left[R\propto^{(n-\gamma)} - 1\right] \tag{29}$$

Solving Eqns. (28)-(29) will produce the value of p_{lim}. And this p_{lim} should be replaced in Eqn. (11) to give the tip capacity, q_b. Where symbols are:

$$\alpha = \frac{1+\sin\phi'}{1-\sin\phi'} \qquad M = \frac{E}{1-v^2(2-m)}$$

$$\beta = \frac{1+\sin\psi}{1-\sin\psi} \qquad \gamma = \frac{\alpha(\beta+m)}{m(\alpha-1)\beta}$$

$$\eta = \frac{(\beta+m)(1-2v)*[(\alpha-1)p_o]*[1+(2-m)v]}{E(\alpha-1)\beta}$$

$$\xi = \left\{\frac{\left[1-v^2(2-m)\right]*(1+m)\delta}{(1-v)*(\alpha-1)\beta} * \left[\alpha\beta + m(1-2v) + 2v - \frac{mv(\alpha+\beta)}{1-v(2-m)}\right]\right\}$$

Appendix:

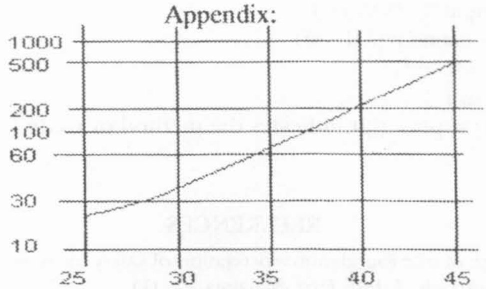

Chart 1. Berzavantez et al. 1961

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NOTATION

c	soil cohesion
D_r	relative density
E	Young's modulus of soil
G	shear modulus of soil
I_r	rigidity index
K_a	active earth pressure coefficient
K_{max}	maximum ratio of radial effective stress at pile surface to in situ vertical effective stress
m	factor identifying cavity type
N	SPT N-Values
N'	SPT blow count after initial correction (for 1st LOOP 1)
N'	SPT blow count after correcting for overburden stress (for 2nd LOOP 1)
N_q	bearing capacity factor

p_{lim} limit pressure for spherical cavity expansion
p_o initial total stress
q_b tip bearing capacity (kN)
q_u unconfined compressive strength (kN/m^2)
S_t ratio of radial effective stress to end bearing pressure in vicinity of the pile
β_{max} maximum ratio of shaft friction to effective overburden stress
β_{min} minimum ratio of shaft friction to effective overburden stress
ϕ' effective angle of shearing resistance
ϕ'_{cv} critical state friction angle
γ_n' effective unit weight of soil (kN/m^3)
γ_n total unit weight of soil (kN/m^3)
γ_w unit weight of water (kN/m^3)
ν Poisson's ratio of soil
$\sigma'h$ effective horizontal stress
$\sigma'v$ effective overburden stress
τ_{max} unit peak shaft capacity (kN/m^2)
τ_s unit shaft friction capacity(kN/m^2)
τ_t unit tip capacity (kN/m^2)
ψ' dilation angle of soil
N''^A, N''^B as in Table 1; superscript indicates the method or route (A or B) used to derive g_n' and D_r.

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