Non-linear Dependence in the Malaysian Stock Market

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ABSTRAK
Kajian ini mengkaji secara empirik kewujudan ketidaklinearan dalam pasaran saham Malaysia dengan mengaplikasikan ujian Brock-Dechert-Scheinkman (BDS) dan bispektrum Hinich. Hasil keputusan BDS menunjukkan bahawa ciri siri pulangan di pasaran saham Malaysia didorong oleh mekanisme ketidaklinearan. Aplikasi seterusnya dengan menggunakan ujian bispektrum Hinich juga menyokong hasil ujian BDS. Hasil keputusan kajian ini memberi implikasi kuat terhadap kerja penyelidikan yang melibatkan pasaran saham Malaysia kerana kewujudan ketidaklinearan menyarankan bahawa penggunaan kaedah linear adalah tidak sesuai untuk membuat inferens.

ABSTRACT
This study empirically investigates the presence of non-linearity in the Malaysian stock market, employing the Brock-Dechert-Scheinkman (BDS) and Hinich bispectrum tests. The BDS results reveal that the characteristics of the returns series in the Malaysian stock market are driven by non-linear mechanisms. Subsequent application of the Hinich bispectrum test confirms the results of the BDS test. The result of the present study has strong implications on the empirical work involving the Malaysian stock market as the existence of non-linearity suggests the inappropriateness of using linear methods for drawing inferences.

Keywords: Non-linearity, BDS test, Hinich bispectrum test, stock market, Malaysia

INTRODUCTION
It is an accepted fact that financial economics has been dominated over the past few decades by linear paradigm, with linear models being widely employed in the time series analysis of financial data. However, with the development and adaptation of more sophisticated econometric techniques, this assumption of linearity, which has been made as an approximation of the real world, is now found to be inappropriate. Specifically, the adequacy of conventional linear models has been challenged in recent years with abundant evidence emerging in the literature to suggest non-linearity is a universal phenomenon, at least for time series data of stock prices. This growing body of research covers stock markets of the U.S. (Hinich and Patterson 1985; Scheinkman and LeBaron 1989; Hsieh 1991), U.K. (Abhyankar et al. 1995; Opong et al. 1999), Germany (Kosfeld and Robé 2001), G-7 countries

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In the literature, there is no generally agreed definition for ‘non-linearity’. Following Ammermann and Patterson (2003: 177), any time series model that cannot be written in the form of a linear ARMA or ARIMA model, i.e., any type of model that exhibits some form of serial dependency other than simple correlation or autocorrelation, is, by definition, a non-linear model.
NON-UNARITY random walk. The naive Greece (Barkoulas and Travlos 1998; Panas 2001), eleven African markets (Joe and Menyah 2003), and random sample of world stock markets (De Gooijer 1989; Ammermann and Patterson 2003). The above stylized fact of stock returns is hardly surprising as Antoniou (Sarantis 2001), Turkey (Antoniou et al. 1997), Greece (Barkoulas and Travlos 1998; Panas 2001), eleven African markets (Joe and Menyah 2003), and random sample of world stock markets (De Gooijer 1989; Ammermann and Patterson 2003). The above stylized fact of stock returns is hardly surprising as Antoniou (Sarantis 2001), Turkey (Antoniou et al. 1997) and Sarantis (2001) listed several possible factors that might induce significant non-linearity in stock markets. Among them are difficulties in executing arbitrage transactions, market imperfections, irrational investors’ behaviour, diversity in agents’ beliefs, and heterogeneity in investors’ objectives. However, from our survey on the literature of the Malaysian stock market, it was found that the issue of non-linearity did not receive much attention from researchers in their empirical work. This was a shock finding since the first evidence of non-linearity in stock returns was reported by Hinich and Patterson (1985) 20 years back. It could be that Malaysian researchers were not aware of the profound implications resulting from the existence of non-linearity on their empirical analysis or little testing has been done due to lack of computer codes to implement the tests (Patterson and Ashley 2000: 1). These two possibilities motivate the writing of the present paper.

**IMPLICATIONS OF NON-LINEARITY**

To raise the awareness of Malaysian researchers, this paper provides a brief discussion on the implications of non-linearity on empirical analysis. Generally, testing for non-linearity can be viewed as a general test of model adequacy for linear models (Hinich and Patterson 1989). In this regard, the existence of non-linearity calls into question the adequacy of linear models, and hence invites the development of non-linear time series models. On the theoretical front, there has been an emergence of non-linear models over the past two decades to capture the complex features of financial time series and subsequently provide more superior forecasts than their linear counterparts or the naive random walk. The growth in this area is indeed phenomenal with literally unlimited numbers of non-linear models being documented in extant literature. Those that have generated much attention from researchers include the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models (for recent survey, refer to Engle 2002; Li et al. 2002) and Smooth Transition Autoregressive (STAR) models (a survey of recent developments is provided by van Dijk et al. 2002).

On the empirical front, the existence of non-linearity casts doubt on the robustness of empirical results and statistical inferences drawn from linear methods. In this regard, several studies have demonstrated the weaknesses of those popular time series tests that are constructed on the basis of linear autoregressive models, such as the stationarity tests, the causality and cointegration tests, under those circumstances when the underlying generating process is non-linear in nature. Sarntis (2000), Kapetanios et al. (2003) and Liew et al. (2004) illustrated that the adoption of linear stationarity tests are inappropriate in detecting mean reversion if the true data-generating process is in fact a stationary non-linear process. The empirical findings of Sarantis (2001) highlighted the risk of drawing wrong inferences on causal relationships when non-linearity is ignored and non-causality tests based on linear models are employed. The Monte Carlo simulation evidence in Bierens (1997) indicated that the standard linear cointegration framework presents a mis-specification problem when the true nature of the adjustment process is non-linear and the speed of adjustment varies with the magnitude of the disequilibrium. All the aforementioned studies highlight the fact that it is imperative to test for non-linearity to determine the nature of the underlying series before deciding on the appropriate empirical methods. If non-linearity prevails, then non-linear methods should be employed in subsequent empirical analysis. To date, progress in this area has been encouraging, with more advanced statistical tools being developed such as the non-linear stationarity tests (Sarno 2001; Chortareas et al. 2002; Kapetanios et al. 2003), non-linear causality tests (Baek and Brock 1992; Brooks and Hinich 1999; Skalin and Teräsvirta 1999), non-parametric cointegration tests (Bierens 1997; Breitung 2002) and non-linear cointegration test (Kapetanios 2003). However, existing studies involving the Malaysian stock market have yet to adopt the above research framework.

In the literature on the Malaysian stock market, one of the most active research areas focuses on the investigation of her informational efficiency in terms of weak-form. Browsing through prior work reveals that the empirical evidence is inconclusive. On the one hand, most studies reported the market is weak-form
efficient, for instance, Barnes (1986), Laurence (1986), Saw and Tan (1989), Annuar et al. (1991, 1993), Kok and Lee (1994) and Kok and Goh (1995). On the other hand, empirical evidence of inefficiency cannot be suppressed, which is documented in Yong (1989, 1993). Another recent study by Lai et al. (2003) using the variance ratio test also reveals the non-randomness of successive price changes in Bursa Malaysia. Though the empirical results on the Malaysian stock market are mixed, one notable similarity of all the aforementioned studies is the application of standard statistical tests- serial correlation test, runs test, variance ratio test and unit root tests, to uncover linear serial dependencies or autocorrelation in the data. However, the lack of linear dependencies does not imply that the series are random as there might be other more complex forms of dependencies which cannot be detected by these standard methodologies. A possible hidden pattern that went undetected in earlier studies is that of the non-linear dependency structure. Even the influential paper of Fama (1970: 394) acknowledged this possibility, "Moreover, zero covariances are consistent with a fair game model, but as noted earlier, there are other types of nonlinear dependence that imply the existence of profitable trading systems, and yet do not imply nonzero serial covariances."

The prevalence of non-linearity in stock markets has at least two important implications on the weak-form efficient market hypothesis (EMH). Firstly, the existence of non-linearity implies the potential of predictability in stock returns (Antoniou et al. 1997; Patterson and Ashley 2000). In this regard, the empirical work of Andrada-Felix et al. (2003) has demonstrated the profitability of non-linear trading rules. Furthermore, in testing the primary hypothesis that graphical technical analysis methods may be equivalent to non-linear forecasting methods, Clyde and Osler (1997) found that technical analysis works better on nonlinear data than on random data, and the use of technical analysis can generate higher profits than a random trading strategy if the data generating process is non-linear. This finding of non-linear predictable patterns would certainly be at odds with the weak-form EMH, which postulates that even non-linear combinations of previous prices are not useful predictors of future prices (Brooks 1996; Brooks and Hinich 1999; McMillan and Speight 2001). Secondly, those conventional linear statistical tests based on autocorrelation coefficients and runs tests are not capable of capturing non-linearity, as they are designed to uncover linear patterns in the data. Specifically, if the returns generating process is non-linear and a linear model is used to test for efficiency, then the hypothesis of no predictability may be wrongly accepted (De Gooijer 1989; Hsieh 1989; Antoniou et al. 1997; Joe and Menyah 2003; Liew et al. 2003). It is possible then that those favourable evidences of efficiency in the Malaysian stock market are the outcome of using linear models in markets characterized by inherent non-linearity, and hence the findings should be met with a dose of scepticism. Given the profound implications of non-linearity on model adequacy and its subsequent statistical inferences in various aspects of financial applications, the present study attempts to document the existence of non-linearity in the Malaysian stock market.

**EMPIRICAL TESTS FOR NON-LINEARITY**

In the literature, there is a wide variety of tests designed to detect non-linearity, each developed to serve as diagnostic test procedure to identify the presence of varying forms of nonlinear structure which are undetected by conventional time series techniques. Barnett and Serletis (2000) highlighted that none of the tests for non-linearity completely dominates the others. This is supported by the Monte Carlo experiments conducted by Ashley et al. (1986), Ashley and Patterson (1989), Hsieh (1991), Liu et al. (1992), Lee et al. (1993), Brock et al. (1996), Barnett et al. (1997) and Ashley and Patterson (2001). In this case, the available non-linearity tests can be utilized in a complementary

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2 Besides the empirical work on market linkages and weak-form efficiency discussed earlier, the implications of non-linearity on other financial applications are no lesser. For instance, pricing derivative securities such as options and futures with martingale methods may not be appropriate. Statistical inferences concerning asset pricing models based on standard testing procedures may no longer be valid. On the theoretical level, it invites the development of non-linear pricing models to account for non-linear behaviour.

3 Barnett and Serletis (2000) and Patterson and Ashley (2000) provided a review of those non-linearity tests that are widely employed in the literature.
way, rather than competing. Several studies have advocated that the application of a battery of non-linearity tests in a sequential way can provide deeper insight into the nature of non-linear generating mechanism of a time series (see, for example, Barnett et al. 1995, 1997; Barnett and Serletis 2000; Ashley and Patterson 2001).

Among those existing tests for non-linearity, the most popular one is the Brock-Dechert-Scheinkman (BDS) test developed by Brock et al. (1987). This test has been extensively employed by researchers for the detection of non-linearity in financial time series data (see, for example, Hsieh 1989, 1991; Scheinkman and LeBaron 1989; De Grauwe et al. 1993; Steurer 1995; Brooks 1996; Al-Loughani and Chappell 1997; Mahajan and Wagner 1999; Opong et al. 1999; Serletis and Shintani 2003). Though the sampling distribution of the BDS test statistic is not known, either in finite samples or asymptotically, under the null of non-linearity, it is possible to use the BDS test to produce a test of linearity against the broad alternative of non-linearity. In particular, after the linear structure has been removed by fitting the best possible linear model, the BDS test can then be used to test the residuals for remaining non-linear dependence. The issue that needs to be addressed is whether such a method of linear filtering will change either the asymptotic or the finite sample distribution of the BDS test statistic. Brock (1987) proved that using residuals in linear models instead of raw data does not alter the asymptotic distribution of the BDS test statistic. The simulations results in Hsieh (1991) provided further support. In practice, to remove the linear structure in the data, the class of ARIMA or Box-Jenkins models can be used to fit a linear model to a time series. According to Barnett et al. (1995: 304), filtering out all possible linear possibilities with certainty is difficult, but nevertheless pre-filtering by ARIMA fit is often viewed as a reputable means of pre-whitening. However, for simplicity, the AR(\(p\)) model has been widely used in the literature for filtering linear dependence from time series data prior to testing for non-linearity (see, for example, Hsieh 1989, 1991; Steurer 1995; Brooks 1996; Barkoulas and Travlos 1998; Opong et al. 1999; Mahajan and Wagner 1999). Brooks (1996: 309) justified the use of this simplified autoregressive procedure, arguing that the process of log differencing has already removed the unit root in the series, and since any moving average model can also be represented by an infinite order autoregression, the class of possible linear specifications is restricted to those of an autoregressive form. Though applying the BDS test to the residuals of a filtered data will give strong support for the conclusion of non-linearity, it conveys very little information as to what kind of non-linear process that generated the data. This is because the BDS test has great power against vast class of non-linear processes (Hsieh 1991; Barnett et al. 1997; Ashley and Patterson 2001). With high power against such a vast class of alternatives, the BDS test can only be used as a “non-linearity screening test”. In fact, this is the limitation of previous studies that only provide evidence of non-linearity, assuming at the outset that the non-linearity takes a particular form.

Another popular non-linear test is the Hinich bispectrum test (Hinich 1982), which involves estimating the bispectrum of the observed time series (for empirical applications, see, for example, De Grauwe et al. 1993; Abhyankar et al. 1995; Brooks 1996; Vilasuso and Cunningham 1996; Ammermann and Patterson 2003; Lim et al. 2003a). Unlike the BDS test, the Hinich bispectrum test provides a direct test for a non-linear generating mechanism, irrespective of any linear serial dependencies that might be present. Thus, pre-whitening is not necessary in using the Hinich approach. Even if pre-whitening is done anyway, the adequacy of the pre-whitening is irrelevant to the validity of the test. Ashley et al. (1986) presented an equivalence theorem to prove that the Hinich linearity test statistic is invariant to linear filtering of the data, even if the filter is estimated. Thus, the linearity test can be applied to the original returns series, or to the residuals of a linear model with no loss of power. In terms of implementation, the bispectrum test produces a test statistic having known asymptotic sampling distribution under the respective null hypotheses of linearity and Gaussianity. However, the alternative hypothesis is not as broad as that for the BDS test. With the bispectrum test, the alternative hypothesis is all

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Rejection of the null of 'independent and identical distribution' (i.i.d.) indicates the presence of non-linearity (since linear dependence has been filtered out), while the non-rejection implies no evidence of non-linearity.
non-linear processes having non-flat bispectrum. In other words, the bispectrum test has no power against those forms of non-linearity that display flat bispectrum and non-flat higher order polyaestra (Barnett et al. 1997). Thus, this approach appears to have limitations when the data fails to reject the null of linearity. Failure of rejection does not imply the acceptance of linearity for it might be due to some non-linear processes against which the bispectrum test has low power. Thus, a further test is needed in this case to determine the presence of non-linearity.

To overcome the above-mentioned limitations, both the BDS and Hinich bispectrum test can be used in a complementary, rather than competing way. Moreover, the application of both the BDS and Hinich bispectrum tests in a sequential way can provide a deeper insight into the types of non-linear processes (Barnett et al. 1995, 1997; Barnett and Serletis 2000; Ashley and Patterson 2001). In this study, the differing power of the BDS and Hinich bispectrum tests in detecting GARCH-type models is utilized as an alternative framework for determining the adequacy of GARCH-type models in characterizing the underlying data-generating process for the series under study. Specifically, the low power of the Hinich bispectrum test relative to the BDS test for the GARCH-type models suggests that the bispectrum test is useful as a marker for these GARCH models. This is supported by the Monte Carlo experiments conducted by Barnett et al. (1997) in which the bispectrum test wrongly accepts linearity for data simulated from the ARCH and GARCH models. The fact that the Hinich bispectrum test has low power against ARCH and GARCH is well acknowledged in the literature (see, for example, Hsieh 1989, 1991; Krager and Kugler 1993; Abhyankar et al. 1995; Opong et al. 1999; McMillan and Speight 2001; Caporale et al. 2005). Another popular framework to examine the validity of specifying a GARCH error structure is the Hinich portmanteau bicorrelation test (see, for example, Hinich and Patterson 1995; Brooks and Hinich 1998; Brooks et al. 2000; Lim et al. 2003b).

In following sections, the paper reviews some major development in the Malaysian stock market, describes the data and procedures. The results are then summarized and used to draw conclusions and implications.

THE MALAYSIAN STOCK MARKET

In Malaysia, the Kuala Lumpur Stock Exchange (KLSE) is the only body approved by the Ministry of Finance, under the provisions of the Securities Industry Act, 1983, as the stock exchange in the country. The KLSE is a self-regulatory organization with its own memorandum and articles of association, as well as rules which govern the conduct of its members in securities dealings.
The KLSE is also responsible for the surveillance of the market place, and for the enforcement of its listing requirements which spell out the criteria for listing, disclosure requirements and standards to be maintained by listed companies.

Although the history of KLSE can be traced to the 1930s, public trading of shares in Malaysia only began in 1960 when the Malayan Stock Exchange (MSE) was formed. When the Federation of Malaysia was formed in 1963, with Singapore as a component state, the MSE was renamed the Stock Exchange of Malaysia (SEM). With the secession of Singapore from the Federation of Malaysia in 1965, the common stock exchange continued to function but as the Stock Exchange of Malaysia and Singapore (SEMS).

The year 1973 was a major turning point in the development of the local securities industry, for it saw the split of SEMS into The Kuala Lumpur Stock Exchange Berhad (KLSEB) and the Stock Exchange of Singapore (SES). The split was opportune in view of the termination of the currency interchangeability arrangements between Malaysia and Singapore. Although the KLSEB and SES were deemed to be separate exchanges, all the companies previously listed on the SEMS continued to be listed on both exchanges.

When the Securities Industry Act 1973 was brought into force in 1976, a new company called the Kuala Lumpur Stock Exchange (KLSE) took over the operations of KLSEB as the stock exchange in Malaysia, to provide a central market place for buyers and sellers to transact business in shares, bonds and various other securities of Malaysian listed companies. On 1 January 1990, following the decision on the “final split” of the KLSE and SES, all Singapore-incorporated companies were delisted from the KLSE and vice-versa for Malaysian companies listed on the SES.

The year 2004 represents another major milestone in the development of the Malaysian securities industry with the demutualisation of KLSE. The demutualisation process took place with the passing of the Demutualisation Bill by the Dewan Rakyat on 11 September 2003, together with other related amendments to the securities law. This was followed by the passing of the Bill by the Dewan Negara on 5 November 2003. As a result of the exercise, KLSE ceases to be a non-profit entity limited by the guarantee of its members, and becomes a public company limited by shares. On 20 April 2004, KLSE was officially renamed Bursa Malaysia, and there is no abbreviation or translation for its usage since it is a brand name for the exchange.

The KLSE computes an index for each of the main sectors traded on the bourse—industrial, finance, property, tin and plantation sectors—and the second board. However, the most widely followed, by far, is the Kuala Lumpur Composite Index (KLCI). The KLCI was introduced in 1986 after it was found that there was a need for a stock market index which would serve as a more accurate indicator of the performance of the Malaysian stock market and the economy. At that time, there was effectively no index which represented the entire market. The KLCI satisfies stringent guidelines and was arrived at only after rigorous screening of the component companies that were eventually selected to compose the index. In 1995, the number of component companies was increased to 100 and will be limited to this number although the actual component companies may change from time to time. The KLCI is constructed by using the value weighted average method, where the weight used is the price of the stock multiplied by the number of ordinary shares outstanding.

**METHODOLOGY**

In this paper, the BDS test, as the first run test, is applied to the residuals of a pre-filtered linear model. If the null of ‘independent and identical distribution’ (i.i.d.) cannot be rejected, there is little point in continuing, since the BDS test provides strong evidence against the presence of non-linearity. If the null is instead rejected, the Hinich bispectrum test can then be used to permit the class of relevant non-linearity to be narrowed. In particular, the Hinich bispectrum test is useful as a marker for the GARCH-type models. Since linearity has been ruled out by the BDS test, the non-rejection of the null by the Hinich test might be due to the presence of non-linear processes which the Hinich test has low power against, specifically the GARCH-type models (Hsieh 1989; Brooks 1996; Barnett *et al.* 1997). On the other hand, rejection of the null hypothesis by the Hinich bispectrum test provides evidence against the adequacy of GARCH-type models for the series under study. In other words, the series are more likely being generated by a non-linear process that is of a form in addition to, or instead of GARCH-type.
The Data

In this study, we utilize the daily closing values of the Kuala Lumpur Composite Index (KLCI) obtained from the Daily Diary at Bursa Malaysia for the sample period of 2/1/90 to 31/10/2001. The price series obtained from the database are used to compute a set of continuously compounded percentage returns for the KLCI, using the relationship:

\[
\tau_t = 100 \times \ln \left( \frac{P_t}{P_{t-1}} \right) \tag{1}
\]

where \(P_t\) is the closing price of the stock on day \(t\), and \(P_{t-1}\) the rate on the previous trading day.

One possible justification for using returns rather than raw data is that the raw data is likely to be non-stationary. Stationarity is a pre-requisite for both the BDS and Hinich bispectrum tests. Hsieh (1991) pointed out that non-stationarity in the data series can cause a rejection of the null hypothesis of independent and identical distribution (i.i.d.) on the basis of the BDS test. On the other hand, non-stationarity may cause a spurious rejection of the null of linearity in the bispectrum test (Hinich and Patterson 1985).

Brock-Dechert-Scheinkman (BDS) Test

Brock, Dechert and Scheinkman (Brock et al. 1987) developed a statistical test and the BDS statistic. The original BDS paper took the concept of the correlation integral\(^8\) and transformed it into a formal test statistic which is asymptotically distributed as a normal variable under the null hypothesis of independent and identically distributed (i.i.d.) against an unspecified alternative. In principle, no distributional assumption on the underlying data generating process is needed in using the BDS test as a test statistic for i.i.d. random variables. Though the estimation is non-parametric, the test statistic is asymptotically distributed as a standard normal variable, with zero mean and unit variance. Hence, the significance of the test statistic is readily determined from standard normal tables.

A revision of this original paper has been done in Brock et al. (1996).

The BDS test is based on the correlation integral as the test statistic. Given a sample of i.i.d. observations, \(\{x_i, t = 1, 2, \ldots, n\}\), Brock et al. (1987, 1996) showed that:

\[
W_{m,n}(\varepsilon) = \sqrt{n} \frac{T_{m,n}(\varepsilon)}{V_{m,n}(\varepsilon)} \tag{2}
\]

has a limiting standard normal distribution, where \(W_{m,n}(\varepsilon)\) is the BDS statistic. \(n\) is the sample size, \(m\) is the embedding dimension, and the metric bound, \(\varepsilon\), is the maximum difference between pairs of observations counted in computing the correlation integral. \(T_{m,n}(\varepsilon)\) measures the difference between the dispersion of the observed data series in a number of spaces with the dispersion that an i.i.d. process would generate in these same spaces, that is \(C_{m,n}(\varepsilon) - C_{1,n}(\varepsilon)^m\). \(T_{m,n}(\varepsilon)\) has an asymptotic normal distribution with zero mean and variance \(V_{m,n}(\varepsilon)^9\).

This BDS test has an intuitive explanation. The correlation integral \(C_{m,n}(\varepsilon)\) is an estimate of the probability that the distance between any two \(m\)-histories, \(x^m = (x_0, x_1, \ldots, x_{m-1})\) and \(x^m = (x_0, x_1, \ldots, x_{m-1})\) of the series \(\{x_i\}\) is less than \(\varepsilon\), that is, \(C_{m,n}(\varepsilon) \rightarrow \text{prob}\{|x_{i+1} - x_{j+1}| < \varepsilon, \text{for all } i = 0, 1, \ldots, m-1\}, \text{as } n \rightarrow \infty\).

If the series \(\{x_i\}\) are independent, then, for \(1 < d < m\), \(C_{m,n}(\varepsilon) \rightarrow \prod_{i=0}^{m-1} \text{prob}\{|x_{i+1} - x_{j+1}| < \varepsilon\}, \text{as } n \rightarrow \infty\).

Furthermore, if the series \(\{x_i\}\) are also identically distributed, then \(C_{m,n}(\varepsilon) \rightarrow C_{1,n}(\varepsilon)^m\), as \(n \rightarrow \infty\). The BDS statistic therefore tests the null hypothesis that \(C_{m,n}(\varepsilon) = C_{1,n}(\varepsilon)^m\), which is the null hypothesis of i.i.d.\(^10\)

The need to choose the values of \(\varepsilon\) and \(m\) can be a complication in using the BDS test. For a given \(m\), \(\varepsilon\) cannot be too small because \(C_{m,n}(\varepsilon)\) will capture too few points. On the other hand, \(\varepsilon\) cannot be too large because \(C_{m,n}(\varepsilon)\) will capture too many points. For this reason, we adopt the approach used by advocates of this test. In particular, we set \(\varepsilon\) as a proportion of standard deviation of the data, \(\sigma\). Hsieh and LeBaron

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\(^8\) In Grassberger and Procaccia (1983), the correlation integral was introduced as a measure of the frequency with which temporal patterns are repeated in the data. For example, the correlation integral \(C(\varepsilon)\) measures the fraction of pairs of points of a time series \(\{x_i\}\) that are within a distance of \(\varepsilon\) from each other.

\(^9\) \(V_{m,n}(\varepsilon)\) can be estimated consistently by \(V_{m,n}(\varepsilon)\). For details, refer to Brock et al. (1987, 1996).

\(^{10}\) The null of i.i.d. implies that \(C_{m,n}(\varepsilon) = C_{1,n}(\varepsilon)^m\) but the converse is not true.
(1988a, b) have performed a number of Monte Carlo simulation tests regarding the size of the BDS statistics under the null of i.i.d. and the alternative hypotheses. The Monte Carlo evidence showed that the ‘best’ choice of $\varepsilon$ is between 0.50 and 1.50 times the standard deviation.

On the other hand, at our chosen setting of $\varepsilon$, we produce the BDS test statistics, $W_{xx}(\varepsilon)$ for all settings of embedding dimensions from 2 to 5. Though most researchers computed the BDS statistics for embedding dimensions varying from 2 to 10 (see, for example, Hsieh 1989; De Grauwe et al. 1993; Brooks 1996; Mahajan and Wagner 1999; Opong et al. 1999), it is important to take note that the small samples properties of BDS test degrade as one increases the embedding dimension. Specifically, the Monte Carlo simulations in Brock et al. (1991) demonstrated that as the dimension goes beyond 5, the small samples properties of BDS degrade, mainly due to the reduction of non-overlapping observations as $m$ grows. Thus, only BDS test statistics for embedding dimensions of 2 to 5 are given much consideration in this study.

**Hinich Bispectrum Test**

Hinich (1982) laid out a statistical test for determining whether an observed stationary time series $\{x\}$ is linear. It is possible that $\{x\}$ is linear without being Gaussian, but all of the stationary Gaussian time series are linear. The Hinich (1982) test involves estimating the bispectrum of the observed time series to test for the null hypothesis of Gaussianity and linearity.

In this section, we provide a brief description of the testing procedures presented by Hinich (1982). Let $\{x\}$ denote a third order stationary time series, where the time unit $t$ is an integer. The third-order cumulant function of $\{x\}$ in the time domain is defined to be $C_{xxx}(r, s) = E[x_{m}, x_{n}, x_{l}]$ for each $(r, s)$ when $E[x_{t}] = 0$, in which $s \leq r$ and $r = 0, 1, 2, ...$.

Since third-order cumulants are hard to interpret, and their estimates are even harder to fathom, the bispectrum in the frequency domain is calculated, which is the double Fourier transform of the third-order cumulant (or bicovariance) function.

The bispectrum at frequency pair $(f_{1}, f_{2})$, denoted as $B_{xxx}(f_{1}, f_{2})$, is the double Fourier transform of $C_{xxx}(r, s)$:

$$B_{xxx}(f_{1}, f_{2}) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} C_{xxx}(r, s) \exp[-2\pi i (r f_{1} + s f_{2})]$$

assuming that $|C_{xxx}(r, s)|$ is summable. The symmetries of $C_{xxx}(r, s)$ translate into symmetries of $B_{xxx}(f_{1}, f_{2})$ that yield a principal domain for the bispectrum, which is the triangular set $\Omega = \{(f_{1}, f_{2}) : 0 < f_{1} < 1/2, f_{2} < f_{1}, 2f_{1} + f_{2} < 1\}$.

The use of the bispectrum has an intuitive explanation. If $\{x\}$ is linear and Gaussian, the bispectrum is flat at zero over all frequencies $(f_{1}, f_{2}) \in \Omega$. However, if $\{x\}$ is linear but not Gaussian, then the bispectrum is non-zero, and is instead a constant independent of frequency. Hence, if the bispectrum is non-constant and a function of frequency, then a non-linear process is implied. In this regard, Brillinger (1965) proved that once a consistent estimator of the bispectrum is calculated, linearity and Gaussianity tests can be performed.

Instead of estimating the bispectrum as given in Equation (3), Hinich (1982) provided an equivalent approach that yields a consistent estimator of the bispectrum. Suppose we have a sample of $N$ observations: $\{x_{0}, x_{1}, ..., x_{N}\}$. Let $f_{c} = c/N$ for $c = 0, 1, ..., N - 1$. For each pair of integers $j$ and $k$, define:

$$F(j, k) = X(f_{j}) X(f_{k}) X^{*}(f_{j-k}) / N$$

where $X(f_{j}) = \sum_{t=0}^{N-1} x_{t} \exp(-2\pi i f_{j} t)$ and $*$ denotes the complex conjugate.

A consistent estimator of the bispectrum is formed by averaging the $F(j, k)$ in a square of $M$ points whose centers are defined by the lattice $L = \{(2m-1)/2, (2\nu-1)/2 : m = 1, ..., c$ and $\nu \leq N/2M - c/2 + 3/4\}$ in the principal domain. For squares that lie completely inside the principle domain, a consistent estimator of the bispectrum is:

$$\hat{B}_{xxx}(f_{m}, f_{n}) \propto \sum_{j=-m+1}^{m-1} \sum_{k=-n+1}^{n-1} F(j, k)$$

If a square has points outside the principal domain, those points are not included in the average. $B_{xxx}(f_{m}, f_{n})$ is a consistent and asymptotically complex normal estimator of the bispectrum $B_{xxx}(f_{m}, f_{n})$ in Equation (3) if the sequence $(f_{m}, f_{n})$ converges to $(f_{1}, f_{2})$. 

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One important consideration in the estimation of bispectrum is the parameter $M$, the frame size. The choice of $M$ governs the trade-off between the bias and variance of the estimator. In this regard, the larger (smaller) the $M$, the smaller (larger) the finite sample variance, but the larger (smaller) the sample bias. Due to this trade-off, there is no unique value for $M$. Hinich (1982) and Ashley et al. (1986) recommended the upper bound value of $M$ should be $M = N^{3/2}$. In this study, we set $M$ equal to 25.\(^{11}\)

The estimated standardized bispectrum is given by

$$
\hat{Z}(f_m, f_n) = \frac{\hat{B}_{xx}(f_m, f_n)}{\left[N / M^2\right]^{1/2} \left[\hat{S}_{xx}(g_m)\hat{S}_{xx}(g_n)\hat{S}_{xx}(g_{m+n})\right]^{1/2}}
$$

where $g_u = (2u-1)M / 2N$ for each integer $u$.

The $\hat{S}_{xx}(\cdot)$ in Equation (6) are estimates of the regular power spectrum, which is the Fourier transform of the second-order moment (or autocovariance) and is a function of only one frequency. The power spectrum of $\{x_t\}$ at frequency $g$ is given by:

$$
\hat{S}_{xx}(g) = \sum_{s=-\infty}^{\infty} C_{xx}(s) \exp[-2\pi i gs]
$$

where $C_{xx}(s) = E[x_{t+s}, x_t]$ is the second-order moment or autocovariance function.

Once again, Gaussianity and linearity of $\{x_t\}$ are tested through the null hypotheses that the estimated standardized bispectrum is zero over all frequencies $(f_m, f_n)$ and that the bispectrum is constant over all frequencies respectively. Though the bispectrum has been understood for at least 40 years dated back to the paper by Hasselman et al. (1963), the absence of statistical tests for significance of bispectrum estimates was identified as one of the problems that have severely limited its progress. In this regard, Hinich (1982) provided a streamlined and practical procedure that utilizes the asymptotic properties of the bispectrum estimator, with the test statistics for both hypotheses reduced to:

$$
\hat{Z} = 2 \sum_{m \neq n} \left| \hat{X}(f_m, f_n) \right|^2
$$

Under the null hypothesis of Gaussianity, the test statistic is distributed asymptotically as a standard normal. On the other hand, under the null of linearity, the test statistic is distributed approximately as a $\chi^2$ random variable with two degrees of freedom. Hinich (1982) and Ashley et al. (1986) recommended the use of the 80 percent quantile of the empirical distribution, scaled by a function of the variance of the series, to provide asymptotically standard normal variable. However, in this study, we use 90 percent quantile to get a more plausible result instead of the 80 percent.\(^{12}\)

**RESULTS AND ANALYSIS**

**Descriptive Statistics**

Table 1 provides summary statistics for the returns series in order to get a better view of some of the important statistical features. The means are quite small. The KLCI returns series exhibit some degree of positive or right-skewness. On the other hand, the distributions of returns for all the series are highly leptokurtic, in which the tails of its distribution taper down to zero more gradually than do the tails of a normal distribution. Not surprisingly, given the non-zero skewness levels and excess kurtosis demonstrated within these series of returns, the Jarque-Bera (JB) test strongly rejects the null of normality.

**Unit Root Tests**

One area that deserves our attention is the stationarity of the returns series, which is a prerequisite for both the BDS and Hinich bispectrum tests. The results from the Augmented Dickey Fuller (ADF) test in Table 2 show that the null hypothesis of a unit root can be rejected for KLCI returns series, which is the

\(^{11}\) Hinich recommended a reduction in the frame size to 25 for our sample sizes in order to improve the power of the test.

\(^{12}\) In a personal communication, Hinich recommended the use of 90 percent quantile.
first difference of the price series of KLCI, even beyond \( m=5 \), the small sample properties are not very good (in terms of normal approximations) at sample sizes comparable to ours.

It is obvious that the BDS statistics generated all lie in the extreme positive tail of the standard normal distribution. Specifically, all of the values are significant at least at the 5% level of significance, especially at the suggested dimensions of 2 to 5. According to Brock et al. (1991), the large BDS statistics can arise in two ways. It can either be that the finite sample distribution under the null of i.i.d. is poorly approximated by the asymptotic normal distribution, or the BDS statistics are large when the null hypothesis of i.i.d. is violated. From the various Monte Carlo simulations, Brock et al. (1991) ruled out the first possibility, thus suggesting that our large BDS statistics in Table 3 provide strong evidence of departure from the i.i.d. null. In other words, these results indicate that the KLCI returns series are not truly random since some patterns show up more frequently than would be expected in a truly random series.

**TABLE 2**
Unit root test results for KLCI

<table>
<thead>
<tr>
<th></th>
<th>Level</th>
<th>First Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trend</td>
<td>No Trend</td>
</tr>
<tr>
<td><strong>Augmented Dickey Fuller (ADF)</strong></td>
<td>-2.07(53)</td>
<td>-6.66(52)**</td>
</tr>
<tr>
<td><strong>Philips-Perron (PP)</strong></td>
<td>-1.61(8)</td>
<td>-51.66(8)**</td>
</tr>
</tbody>
</table>

**Notes:** The null hypothesis is that the series contains unit root. The critical values for rejection are -3.97 for models with a linear time trend and -3.43 for models without a linear time trend at a significant level of 1% (**). Values in brackets indicate the chosen lag lengths.
TABLE 3  
BDS test results on KLCI returns series

<table>
<thead>
<tr>
<th>m</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
<th>1.25</th>
<th>1.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3.90</td>
<td>3.20</td>
<td>2.66</td>
<td>2.19</td>
<td>1.82*</td>
</tr>
<tr>
<td>3</td>
<td>5.72</td>
<td>4.53</td>
<td>3.64</td>
<td>2.95</td>
<td>2.42</td>
</tr>
<tr>
<td>4</td>
<td>7.45</td>
<td>5.73</td>
<td>4.45</td>
<td>3.50</td>
<td>2.83</td>
</tr>
<tr>
<td>5</td>
<td>9.72</td>
<td>7.18</td>
<td>5.35</td>
<td>4.07</td>
<td>3.21</td>
</tr>
<tr>
<td>6</td>
<td>12.70</td>
<td>8.91</td>
<td>6.35</td>
<td>4.66</td>
<td>3.59</td>
</tr>
<tr>
<td>7</td>
<td>16.50</td>
<td>10.96</td>
<td>7.49</td>
<td>5.30</td>
<td>3.98</td>
</tr>
<tr>
<td>8</td>
<td>21.90</td>
<td>13.63</td>
<td>8.83</td>
<td>6.02</td>
<td>4.39</td>
</tr>
<tr>
<td>9</td>
<td>29.05</td>
<td>16.90</td>
<td>10.35</td>
<td>6.77</td>
<td>4.82</td>
</tr>
<tr>
<td>10</td>
<td>39.45</td>
<td>21.13</td>
<td>12.16</td>
<td>7.62</td>
<td>5.27</td>
</tr>
</tbody>
</table>

Notes: Asymptotically, the computed BDS statistics, $W_{mn}(\epsilon) \sim N(0,1)$ under the null of i.i.d. The BDS test is taken as a two-tailed test. The critical values are 2.58 and 1.96 for the 1% and 5% levels of significance respectively. All the BDS statistics, except one (with asterisk*) are significant at least at 5% level of significance.

**Testing for Non-linearity**

In this section, we use both the BDS and Hinich bispectrum tests to detect non-linear departure from the i.i.d. null in the KLCI returns series.

**BDS Test**

The rejection of the i.i.d. null by the BDS test can be due to non-white linear and non-white non-linear dependence. To make sure that the data is in fact picking up non-linear dependencies, the linear structure has to be removed by fitting the best possible linear model. The BDS test can then be used to test the residuals for remaining non-linear dependencies.

To achieve that, we filtered the data by the following autoregression to account for possible linear dependence:

$$r_t = \beta_0 + \beta_1 D_{M,t} + \beta_2 D_{T,t} + \beta_3 D_{W,t} + \beta_4 D_{TH,t} + \beta_5 D_{Ho1,t} + \sum_{i=1}^{P} \beta_i r_{t-i} + \epsilon_t$$  
(9)

where $D_{M,t}$, $D_{T,t}$, $D_{W,t}$, $D_{TH,t}$, $D_{Ho1,t}$ are dummy variables for Monday, Tuesday, Wednesday and Thursday respectively to capture day-of-the-week effects. $D_{Ho1,t}$ is a dummy variable to capture any holiday returns effects, whose values denoted the number of trading days missed due to holidays since the last day during which trading occurred (excluding weekends). The lag length of the AR($p$) terms was chosen so that $Q,(50)$ test is not significant at the 10% level. It should be emphasized that the objective is not to build a statistically adequate empirical model, but rather to choose an acceptable specification, which will remove autocorrelation effects and linear holiday and day-of-the-week effects from the returns series. For the KLCI returns series, the identified model is AR(6).

After fitting the best possible linear model, the BDS test can then be used to test the residuals for remaining non-linear dependence. Table 4 reports the results of the BDS test on the residuals of the fitted AR($p$) model. The results show that the KLCI returns series exhibit highly significant BDS statistics even after autocorrelation effects and linear holiday and day-of-the-week effects have been filtered out, thereby indicating the existence of strong non-linear dependencies within these data series.

However, there is always a worry that the rejection of the null by the BDS test could be due to the possibility of imperfect pre-whitening. This concern is well directed since much of the Monte Carlo research that has been published on the BDS test (see, for example, Brock et al.

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13 For the linear seasonality effects, our regression results show only the presence of Monday effects in the KLCI returns series. Other incorporated dummies are not significant, even at the 10% level of significance.

## TABLE 4
BDS test results on residuals of AR(\(p\)) fit for KLCI returns series

<table>
<thead>
<tr>
<th>(m)</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
<th>1.25</th>
<th>1.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3.80</td>
<td>3.09</td>
<td>2.56</td>
<td>2.09</td>
<td>1.78*</td>
</tr>
<tr>
<td>3</td>
<td>5.49</td>
<td>4.31</td>
<td>3.48</td>
<td>2.80</td>
<td>2.37</td>
</tr>
<tr>
<td>4</td>
<td>7.15</td>
<td>5.50</td>
<td>4.29</td>
<td>3.37</td>
<td>2.80</td>
</tr>
<tr>
<td>5</td>
<td>9.40</td>
<td>6.95</td>
<td>5.22</td>
<td>3.95</td>
<td>3.20</td>
</tr>
<tr>
<td>6</td>
<td>12.49</td>
<td>8.74</td>
<td>6.26</td>
<td>4.56</td>
<td>3.59</td>
</tr>
<tr>
<td>7</td>
<td>16.38</td>
<td>10.83</td>
<td>7.41</td>
<td>5.21</td>
<td>3.98</td>
</tr>
<tr>
<td>8</td>
<td>21.85</td>
<td>13.52</td>
<td>8.77</td>
<td>5.92</td>
<td>4.40</td>
</tr>
<tr>
<td>9</td>
<td>29.19</td>
<td>16.79</td>
<td>10.31</td>
<td>6.67</td>
<td>4.82</td>
</tr>
<tr>
<td>10</td>
<td>39.50</td>
<td>20.93</td>
<td>12.13</td>
<td>7.52</td>
<td>5.28</td>
</tr>
</tbody>
</table>

Notes: Asymptotically, the computed BDS statistics, \(W_m(\varepsilon) \sim N(0,1)\) under the null of i.i.d. The BDS test is taken as a two-tailed test. The critical values are 2.58 and 1.96 for the 1% and 5% levels of significance respectively. All the BDS statistics, except one (with asterisk*) are significant at least at 5% level of significance.

Kian-Ping Lim, Muzafar Shah Habibullah & Hock-Ann Lee

1991) has emphasized the pre-testing issue and the potential dependence of the properties of the test on the prior linear filter. Some of the test’s sensitivity to non-linearity could be a result of remaining linear dynamics in the data.

**Hinich Bispectrum Test**

The Hinich bispectrum test is a good complement for the BDS test (Abhyankar et al. 1995). This bispectrum test provides a direct test for a non-linear generating mechanism, irrespective of any linear serial dependencies that might be present. Thus, pre-whitening is not necessary in using the Hinich approach. Even if pre-whitening is done the adequacy of the pre-whitening is irrelevant to the validity of this test (Ashley et al. 1986).

In this section, the Hinich bispectrum test is applied to both the original KLCI returns series and also the residuals of the AR(\(p\)) fit. Table 5 reports the results for the bispectrum Gaussianity test. It is obvious that the null is strongly rejected, irrespective of whether the returns series or the residuals are employed.

Although Gaussianity and linearity tests are linked, a rejection of Gaussianity does not necessarily rule out linearity. As mentioned earlier, the null of linearity examines whether the estimated standardized bispectrum is constant over all frequencies, whereas Gaussianity requires constant at zero over all frequencies. The linearity test provided by Hinich (1982) is able to detect non-constant bispectrum which suggests a non-linear generating process for the returns series. Table 5 reports the \(p\)-value for the 90 percent quantile bispectrum linearity test. The results reject the null hypothesis of a linear generating mechanism at the conventional level of significance for KLCI returns series. These indicate the existence of non-linear dependencies within the daily returns, at least in the form that can be detected by the bispectrum test.

## TABLE 5
Gaussianity and linearity test results on KLCI returns series

<table>
<thead>
<tr>
<th></th>
<th>KLCI Returns Series</th>
<th>KLCI Returns Series Residuals of AR((p)) Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussianity Test</td>
<td>0.0000*</td>
<td>0.0000*</td>
</tr>
<tr>
<td>Linearity Test</td>
<td>0.0228</td>
<td>0.0228</td>
</tr>
</tbody>
</table>

Notes: Both test statistics are distributed as \(N(0,1)\) and are taken as a one-sided test.

* Denotes very small value.
Non-linear Dependence in the Malaysian Stock Market

CONCLUDING REMARKS

The outcomes of our econometric investigation support the presence of non-linearity in the data generating process of KLCI returns series. In the first run test, the BDS results reveal that there is in fact a non-linear mechanism that drives the returns series under investigation. Subsequent application of the Hinich bispectrum test confirms the results of the earlier BDS test. It is important to note that the rejection of the null of linearity in the bispectrum test is a strong support for the presence of non-linearity (Barnett et al. 1997). Moreover, the Hinich bispectrum test is able to provide a direct test for a non-linear generating mechanism, irrespective of any linear serial dependencies that might be present. Consequently, when this test rejects the null, one need not worry about the possibility that the linear pre-whitening model has failed to remove all linear serial dependence in the data (Ashley and Patterson 2001). This has helped us to cast away our worries that the rejection of the null in the BDS test could be due to the possibility of imperfect pre-whitening.

These bispectrum test results, however, do yield additional information beyond merely confirming the results of the earlier BDS test. Since the bispectrum test has relatively low power against GARCH-type models (Hsieh 1989; Brooks 1996; Barnett et al. 1997), the results not only suggest the inadequacy of linear models for the underlying KLCI returns series, but provide further insight into the types of non-linear process, or at least determine the adequacy of the GARCH models that are widely employed in the financial world. In particular, the findings reveal that the returns series are more likely being generated by a process that is of a form in addition to, or instead of GARCH-type. Furthermore, the present study has strong implications on the empirical work involving the Malaysian stock market as the existence of non-linearity highlights the risk of drawing wrong inferences from linear methods.

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REFERENCES


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