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On the Meridional Circulation of the Atmosphere

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ABSTRAK

Beberapa aspek kitaran meridional atmosfera adalah di kaji. Diketahui bahawa kehadiran kitaran meridional adalah ditentukan secara primer oleh eddy. Disahkan juga bahawa kitaran meridional adalah disebabkan oleh geseran di permukaan zon baratan di dalam kes barokliniti yang kuat.

ABSTRACT

Some aspects of the meridional circulation of the atmosphere are considered. It is corroborated that: (a) the existence of the meridional circulation is primarily determined by the eddies, and (b) that the meridional circulation is set up by surface friction in the zone of the westerlies in the case of strong baroclinicity.

Keywords: meridional circulation, forcing function, zonal momentum angular momentum.

INTRODUCTION

Following Necco (1980) the meridional circulation comprises:

1) an equatorial cell. This is a direct cell. Meaning that warm air ascends close to the Equator and cool air descends at thirty degrees of latitude.
2) an extratropical cell, also known as Ferrell cell. This is an indirect cell, due to the fact that the descending (ascending) air is the warm (cool) one.
3) a direct polar cell.

Several aspects of the meridional circulation are addressed. For this purpose, zonal averages of both the u-momentum and the heat equation are taken. A partial differential equation (PDE) is obtained, where the forcing functions of the PDE produce the mean meridional circulation.

THE PROBLEM

The u momentum equation states that:

$$\partial u/\partial t + \partial (u\,u)/\partial x + \partial (u\,v)/\partial y + \partial(u\,\omega)/\partial p - f\,v + g\,\partial z/\partial x + F_x = 0 \qquad (1)$$

The zonal average, denoted by [ ], is taken. The departure from the zonal average is denoted by ('). Therefore, any variable is denoted by:

$$(\ ) = [\ ] + (\ )' \qquad (2)$$

The zonal average of equation (1) is:

$$\partial [u]/\partial t + \partial([u][v])/\partial y + \partial([u][\omega])/\partial p + \partial([u'v'])/\partial y + \partial([u'\omega'])/\partial p - f [v] + [F_x] = 0 \quad (3)$$

The partial derivative of equation (3) with respect to pressure is taken. It yields:

$$\partial/\partial t (\partial[u]/\partial p) + \partial^2([u][v])/\partial y \partial p + \partial^2 ([u][\omega])/\partial p^2 + \partial/\partial p (\partial[u'v']/\partial y) + \partial/\partial p (\partial[u'\omega']/\partial p) - f \partial[v] /\partial p + \partial [Fx]/\partial p = 0 \quad (4)$$

The temperature equation is:

$$\partial \theta/ \partial t + \partial (\theta u)/\partial x + \partial (\theta v)/\partial y + \partial (\theta \omega)/ \partial p = (Q \theta)/T \quad (5)$$

The zonal average of equation (5) yields:

$$\partial [\theta]/ \partial t + \partial([\theta][v])/ \partial y + \partial([\theta][\omega])/\partial p + ([\theta'v']/\partial y + ([\theta'\omega']/\partial p = ([Q]\theta)/T \quad (6)$$

The partial derivative with respect to y of equation (6) is taken. It follows that:

$$\partial/\partial t (\partial[\theta]/(y) + \partial^2([\theta][v])/\partial y^2 + \partial^2 ([\theta][\omega])/\partial p \partial y + \partial^2 [\theta'v']/\partial y^2 + \partial^2 [\theta'\omega']/\partial p \partial y = (\theta/T) \partial [Q]/\partial y \quad (7)$$

On the other hand:

$$u_g = -(g/f) \partial z/ \partial y \quad (8)$$

It follows that:

$$\partial u_g /\partial p = - (g/f) \partial (\partial z/ \partial p)/ \partial y \quad (9)$$

However:

$$\partial z/\partial p = - (R T/ g p \theta) \theta \quad (10)$$

Therefore:

$$\partial u_g /\partial p = - (R T/ f p \theta) \partial \theta/ \partial y \quad (11)$$

Upon multiplication of equation (7) by (R T/ f p θ) and making usage of equation (11) it follows that:

$$\partial/\partial t (\partial ug /\partial p) + (R T/ f p \theta)\{\partial^2([\theta][v])/\partial y^2 + \partial^2 ([\theta][\omega])/\partial p \partial y\} + (R T/ f p \theta)\{\partial^2 [\theta'v']/\partial y^2 + \partial^2 [\theta'\omega']/\partial p \partial y\} = (R/f p) \partial [Q]/\partial y \quad (12)$$

Upon subtraction of equation (4) from equation (12) it follows that:

$$(R T/ f p \theta)\{\partial^2([\theta][v])/\partial y^2 - \partial^2 ([u][\omega])/\partial p^2\} + (R T/f p \theta)\{\partial^2 ([\theta][\omega])/\partial p \partial y\} - \{\partial^2([u][v])/\partial y \partial p\} + f \partial[v] /\partial p = -(R T/f p \theta) \{\partial^2 [\theta'v']/\partial y^2 + \partial^2 [\theta'\omega']/\partial p \partial y\} + (R/f p) \partial [Q]/ \partial y + \partial [F_x]/\partial y + \partial/\partial p (\partial[u'v']/ \partial y) + \partial/\partial p (\partial[u'\omega']/\partial p) \quad (13)$$

It is interesting to note that:

$$(R\,T/ f\,p\,\theta)\{\partial^2([\theta][v])/\partial y^2\} = (R\,T/ f\,p\,\theta)\,\partial/\partial y\,([\theta]\partial[v]/\partial y + [v]\,\partial[\theta]/\partial y) \tag{14}$$

$$\partial^2([u][v])/\partial y\,\partial p = \partial/\partial p\,([v]\,\partial[u]/\partial y + [u]\,\partial[v]/\partial y) \tag{15}$$

$$(R\,T/ f\,p\,\theta)\{\partial^2([\theta][\omega])/\partial p\,\partial y\} = (R\,T/ f\,p\,\theta)\partial/\partial y\,\{[\omega]\partial[\theta]/\partial p + [\theta]\partial[\omega]/\partial p\} \tag{16}$$

$$\partial^2([u][\omega])/\partial p^2 = \partial/\partial p\,([\omega]\partial[u]/\partial p + [u]\,\partial[\omega]/\partial p) \tag{17}$$

Upon substitution of equations (14) to (17) into equation (13) and making usage of the zonally averaged continuity equation:

$$\partial[v]/\partial y + \partial[\omega]/\partial p = 0 \tag{18}$$

it is obtained:

$$(R\,T/ f\,p\,\theta)\partial\{([v]\,\partial[\theta])/\partial y\}/\partial y - \partial\{[v]\partial[u]/\partial y\}/\partial p + (R\,T/ f\,p\,\theta)\,\partial/\partial y\,\{[\omega]\partial[\theta]/\partial p\} - \partial/\partial p\,([\omega]\partial[u]/\partial p + f\,([v]\;/\partial p = -(R\,T/ f\,p\,\theta)\{\partial^2[\theta'v']/\partial y^2 + \partial^2[\theta'\omega']/\partial p\,\partial y\} + \partial/\partial p\,(\partial[u'v']/\partial y) + \partial/\partial p\,(\partial[u'\omega']/\partial p) + (R/f\,p)\,\partial[Q]/\partial y + \partial[F_x]/\partial p \tag{19}$$

It is noted that:

$$(R\,T/ f\,p\,\theta)\partial\{([v]\,\partial[\theta])/\partial y\}/\partial y = (R\,T/ f\,p\,\theta)\,\{(\partial[v]/\partial y)\,(\partial[\theta])/\partial y) + [v]\,\partial^2[\theta])/\partial y^2\} \tag{20}$$

$$-\partial\{[v]\partial[u]/\partial y\}/\partial p = -(\partial[v]/\partial p)(\partial[u]/\partial y) - [v]\,\partial\{\partial[u]/\partial y\}/\partial p = -(\partial[v]/\partial p)(\partial[u]/\partial y - (RT/ f\,p\,\theta)\,[v]\,\partial^2[\theta])/\partial y^2 \tag{21}$$

$$(RT/ f\,p\,\theta)\,\partial\{[\omega]\partial[\theta]/\partial p\}/\partial y = (RT/ f\,p\,\theta)\,(\partial[\omega]/\partial y)(\partial[\theta]/\partial p) + [\omega]\partial(\partial[\theta]/\partial y)/\partial p \tag{22}$$

$$-\partial/\partial p\,([\omega]\partial[u]/\partial p) = -(\partial[\omega]/\partial p)(\partial[u]/\partial p) - [\omega]\partial/\partial p\,(\partial[u]/\partial p) = -(\partial[\omega]/\partial p)(\partial[u]/\partial p) - (RT/ f\,p\,\theta)[\omega]\partial/\partial p(\partial[\theta])/\partial y) \tag{23}$$

Upon substitution of equations (20) to (23) into (19) and making use of equation (11) it yields:

$$(\partial[u]/\partial p)(\partial[v]/\partial y) - (\partial[v]/\partial p)(\partial[u]/\partial y + (RT/ f\,p\,\theta)\,(\partial[\omega]/\partial y)(\partial[\theta]/\partial p) - (\partial[u]/\partial p)(\partial[\omega]/\partial p) + f\,\partial[v]/\partial p = -(R\,T/f\,p\,\theta)\,\partial/\partial y\,\{\partial[\theta'v']/\partial y + \partial[\theta'\omega']/\partial p\} + \partial/\partial p\,\{\partial[u'v']/\partial y + \partial[u'\omega']/\partial p\} + (R/f\,p)\,(\partial[Q]/\partial y) + \partial[F_x]/\partial p \tag{24}$$

Let:

$$[v] = \partial\psi/\partial p \tag{25a}$$

and

$$[\omega] = -\partial\psi/\partial y \tag{25b}$$

Upon usage of equations (25), equation (24) becomes:

$$(\partial [u]/\partial p)\ (\partial^2 \psi/\partial p \partial y) - \partial [u]/\partial y\ (\partial^2 \psi/\partial p^2) - (RT/f\ p\ \theta)(\partial [\theta]/\partial p)$$
$$(\partial^2 \psi/\partial y^2) - (\partial [u]/\partial p)\ (\partial^2 \psi/\partial y\ (p) + f\ (\partial^2 \psi/\partial p^2) = (R\ T/f\ p\ \theta)\partial/\partial$$
$$y\{\partial [\theta' v']/\ \partial y + \partial [\theta' \omega']/\partial p\} + \partial/\ \partial p\ \{\partial [u'v']/\partial y + \partial [u'\omega']/\partial p\} +$$
$$(R/f\ p)\ (\partial [Q]/\partial y) + \partial [F_x]/\partial p \qquad (26)$$

It follows that:

$$-(RT/f\ p\ \theta)(\partial [\theta]/\partial p)(\partial^2 \psi/\partial y^2) + 2\ (\partial [u]/\partial p)(\partial^2 \psi/\partial y\ \partial p) +$$
$$(f - \partial[u]/\partial y)(\partial^2 \psi/\partial p^2) = \partial H/\partial y + \partial \chi/\partial p \qquad (27)$$

Equation (27) may also be written as:

$$A\ (\partial^2 \psi/\partial y^2) + 2\ B\ (\partial^2 \psi/\partial y\ \partial p) + C\ (\partial^2 \psi/\partial p^2) = \partial H/\partial y + \partial \chi/\partial p \qquad (28)$$

where :

$$A = -(RT/f\ p\ \theta)(\partial [\theta]/\partial p) \qquad (29a)$$

$$B = (\partial [u]/\partial p) = (RT/f\ p\ \theta)(\partial [\theta]/\partial y) \qquad (29b)$$

$$C = (f - \partial[u]/\partial y) \qquad (29c)$$

where equations (29a), (29b) and (29c) represent the static stability factor, the free symmetric convection criterion and the Hemlholtz dynamical inertial instability factor, respectively. The respective values of H and χ are:

$$H = (R\ [Q]/f\ p) - (R\ T/f\ p\ \theta)\{\partial [\theta' v']/\ \partial y + \partial [\theta' \omega']/\partial p\} \qquad (30a)$$

$$\chi = [F_x] + \partial [u'v']/\partial y + \partial [u'\omega']/\partial p \qquad (30b)$$

Equation (28) is elliptic, parabollic or hyperbolic depending on whether:

$$\delta^2 = A\ C - B^2 = -(RT/f\ p\ \theta)(\partial[\theta]/\partial p)(f - \partial[u]/\partial y) - (\partial [\theta]/\partial y)^2 (RT/f\ p\ \theta)^2 \qquad (31)$$

is positive, zero or negative.

DISCUSSION AND CONCLUSIONS

The factors $\partial H/\partial y$ and $\partial \chi/\partial p$ act as forcing functions which produce mean meridional circulation in the atmosphere. H measures the net effect of the non-adiabatic heating Q (e.g. radiation and turbulent heat convection); while χ measures the net effect of the convergence of the eddy transfer and the frictional dissipation of the zonal momentum. The meridional circulation of the atmosphere depends very much on the prevailing eddy motion, because these eddies are very efficient agents in transporting heat and momentum. Thus, the meridional circulation in the atmosphere must be considered as a secondary process; its existence is primarily determined by the eddy process. The eddy process produces a horizontal convergence of zonal momentum in middle latitudes and divergence in low and high latitudes. Since the dissipation of the zonal momentum takes place through ground friction, the zonal momentum accumulated in the upper atmosphere by the horizontal eddy transports must be brought down to compensate the low-level dissipation. Although the baroclinically unstable atmospheric disturbances are capable of transporting zonal momentum downward, it appears that the vertical eddy transport is smaller than required. This gives an unbalanced χ function which acts as a forcing function to produce mean meridional circulation.

Since the large-scale eddies in the stratosphere produce horizontal flux of zonal momentum, and since the eddies are unable to bring these accumulation of momentum downward due to the fact that the mean zonal current decreases upward above the tropopause, a reverse horizontal transport by some mean meridional cell is needed in the stratosphere. Such circulations are also produced through the forcing function $\partial \chi/\partial p$.

The meridional circulation produced by the averaged radiational heating is a very weak, direct cell. The maximum southward velocity on the earth's surface, being less than 3 cm sec^{-1}. Therefore, it is much weaker than the one produced by $\partial \chi/\partial p$.

From equation (28) it is inferred that $\partial H/\partial y$ and $\partial \chi/\partial p$ act as a forcing function. For our problem, the proper boundary condition is simply that ψ should remain constant along the boundary surfaces, i. e. $\psi = 0$.

This statement is relatively important: "the solution of an elliptic homogeneous P.D.E. has no extreme value inside a closed surface. Therefore, the solution must be identically zero if it satisfies the above mentioned boundary requirement". On the other hand, the solutions of the P. D. E. of the homogeneous type may vanish at the boundaries. Therefore, the free mean meridional convection may exist only when equation (28) is of hyperbolic type. At least in the part of the region inside the bounding surfaces.

Following Kuo (1956), the distribution of the unbalanced momentum function $\partial\chi/\partial p$ is given in Table 1. From these values it is seen that the magnitude of the term f $\partial\chi/\partial p$ is about 10^{-12} to 10^{-11} m. sec^{-3} Hpa^{-1}. Since data are still lacking in higher and lower latitudes, and since the values given in Table 1 may not be very representative, this equation is only integrated roughly, to give a probable pattern of the forced mean meridional circulation, which is represented in *Fig. 1.* It may be seen that this distribution of the forcing function $\partial\chi/\partial p$ produces an indirect Ferrell cell in middle latitudes and two Hadley cells: one in low latitudes and the other one at high latitudes in the troposphere.

In the stratosphere, the eddies (∂^2 [u'v']$/\partial p\partial y$) are positive in low latitudes and negative in middle latitudes, so the meridional circulations induced by this term are in opposite sense as those of the troposphere.

In the atmosphere, surface friction represents a source (sink) of angular momentum in the area of the easterly (westerly) winds at the Earth's surface. The effect of surface friction within the zone of strong surface westerlies in

TABLE 1.
Distribution , in the northern hemisphere, of the forcing function in 10^{-8} m sec^{-2} Hpa^{-1}, following Kuo (1956)

Lat	65	60	55	50	45	40	35	30	25	20	15
150 Hpa.	3	3.	-1.	-9.	-11	-17	-12	-3	10	13	2.
200 Hpa	2	2	3	1	-12	-9	-10	-1	7	6	3
300 Hpa	3	3	4	5	4	1	2	1	-1	-5	-4
500 Hpa	4	4	3	4	6	5	4	1	-2	-4	-5
700 Hpa	3	4	4	4	3	3	1	-1	-1	-2	-3
850 Hpa	1	3	4	5	4	2	-1	-2	-2	-3	-4
1000 Hpa	3	5	5	6	5	3	0	-2	-3	-4	-4

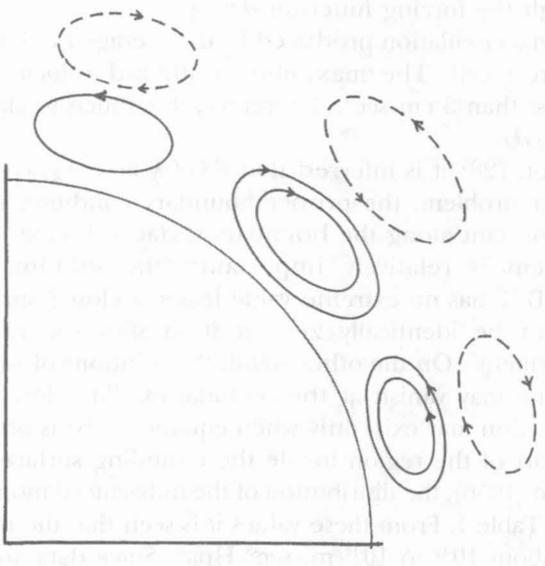

Fig 1. Pattern of the mean forced meridional circulation

middle latitudes may be represented by a number of points-sink of angular momentum. Therefore, it may be concluded that the meridional circulation is set up by surface friction in the zone of the westerlies in the case of strong baroclinicity as is shown in *Fig 2.*

If we attempt to apply the theory of meridional circulations to atmospheric motion, we are faced with the difficulty that the atmosphere is not a symmetric vortex around the Earth's axis, since the fields of motion and state vary from one meridional plane to another one. One way to overcome this difficulty is to apply the theory of meridional circulations to the symmetric vortex formed by averaging over all longitudes.

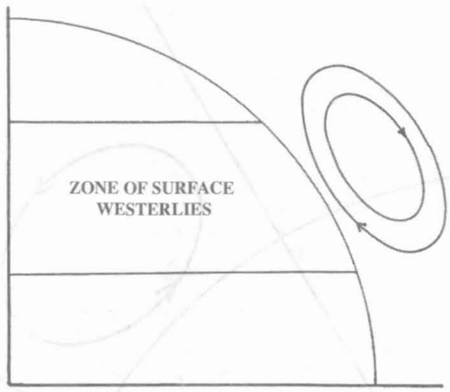

Fig 2. Meridional circulation induced by surface friction in the zone of the surface westerly winds

Assume that $\partial \omega / \partial y < 0$. It has a negative contribution in each term on the LHS of equation (28). Therefore, if A $\partial^2 \psi / \partial y^2$) conforms to $\partial \omega / \partial y < 0$ we get a circulation similar to the direct meridional circulation.

Recalling that:

$$\partial^2 \psi / \partial p^2 = \partial [v] / \partial p \approx - \partial [v] / \partial z \tag{32}$$

It follows that $\partial [v] / \partial z > 0$, if $\partial^2 \psi / \partial p^2 < 0$. Whenever this happens, a direct meridional circulation is obtained.

Finally, if $(R T / f \theta \rho) \neq 0$, then it will play the same role as $\partial [Q] / \partial y$ and set up a heat flux and a direct meridional circulation.

Around 30° N, $[\theta' v']$ is positive; while at 60° N it is negative. Therefore, the expression:

$$\partial [\theta' v'] / \partial y + \partial [\theta' \omega'] / \partial p \tag{33}$$

is negative (positive) at 30° N (60° N). It may be deduced that the term $\partial \omega / \partial y > 0$ sets up an indirect Ferrell cell (*Fig. 3*).

The direct circulation is very weak in its transport of heat from low to high latitudes. Thus, it cannot maintain a steady motion. As a consequence of the continued differential heating we will have baroclinic instability and a consecutive outbreak of wave disturbances and large scale eddies that can greatly modify the appearance of the circulation. These eddies contribute to give the large convergence of momentum in middle latitudes and divergence in low latitudes, which would set up that three cell circulation.

Upon considering the friction near the surface, it is observed that the $[F_x] > 0$ in middle latitudes and negative in high and low latitudes. (*Fig. 4a*). It follows naturally that the variation of $[F_x]$ with height reverses its sign (*Fig. 4b*).

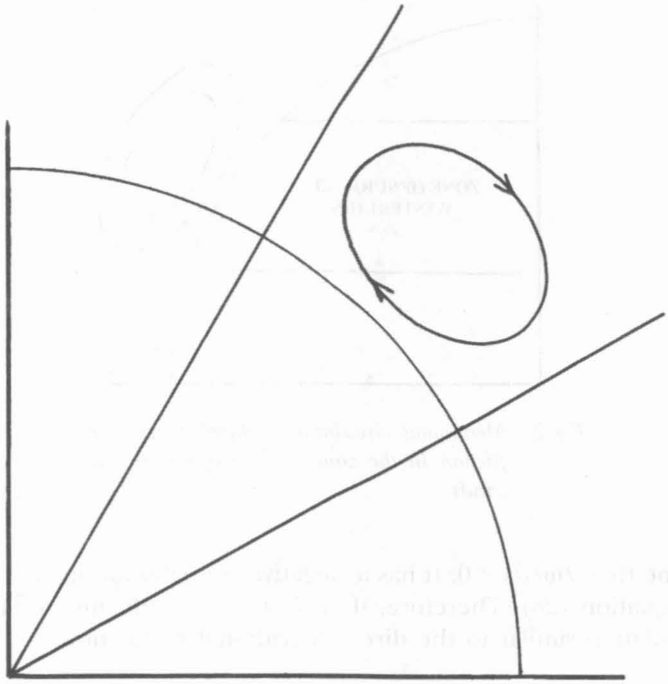

Fig 3. Indirect Ferrell cell induced by $\partial \omega / \partial y > 0$

Therefore, it may be concluded that friction alone may set up a three cell circulation (*Fig. 4c*).

A low pressure system in the vertical is studied. The winds in the tropics are slowed down. As a consequence of this, they are forced inwards. By continuity, the wind is forced to rise (*Fig.5*).

It is desirable to determine whether the zonal winds control meridional circulation, or if it is the other way around.

Observations show that easterlies (westerlies) are predominant in the troposphere (stratosphere) at low latitudes (*Fig. 6*).

If $[u] > 0$, then $[u]$ must decrease, giving the circulation pattern observed in *Fig. 6* in mid latitudes. That is, westerlies in the troposphere and easterlies in the stratosphere. Similar reasoning applies to explain the circulation of the two Hadley cells.

In the equation

$$d\,[v]/dt = -\,f\,[u] - dq(\zeta)/dy \tag{34}$$

the last term represents the eddy flux of heat. This represents heating at 60° N and cooling at about 30° N. That is, the poleward transport of heat.

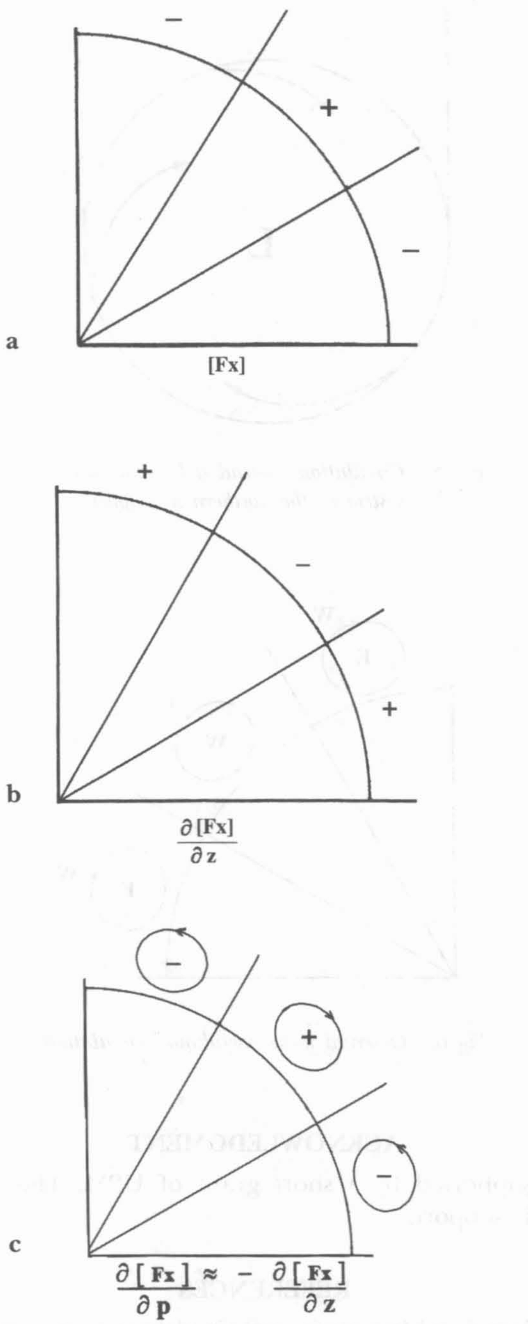

Fig 4. (a) Zonal average of F_x, i.e. $[F_x]$; (b) vertical variation of $[F_x]$ with height, i.e. $\partial[F_x]/\partial z$; (c) circulation induced by $\partial[F_x]/\partial p \approx -\partial[F_x]/\partial z$

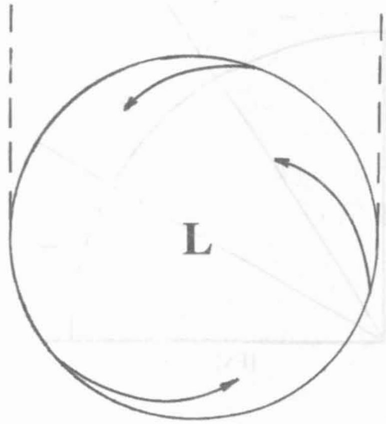

Fig 5. Circulation around a low pressure system in the northern hemisphere

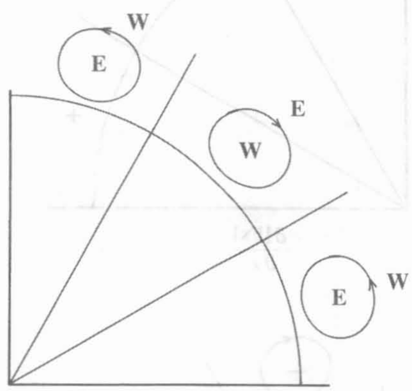

Fig 6. Observed mean meridional circulation

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