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The Poleward Transport of Heat by the Atmosphere

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ABSTRAK

Tekanan rendah panas dan tekanan tinggi sejuk menggambarkan sumbangan kepada pertukaran tenaga potensi kepada tenaga kinetik. Di kawasan tropika, keadaan ini menggambarkan situasi pada sel Hadley. Sel ini dikenali sebagai berterusan atau 'direct'. Bagaimanapun, tekanan rendah sejuk dan tekanan tinggi panas membawa kepada sumbangan min tenaga kinetik kepada min tenaga potensi, yang menunjukkan proses lazim bagi cel Ferell (juga dikenali sebagai sel tidak berterusan atau 'indirect'). Manuskrip ini membincangkan tentang pengangkutan kaba ke kutub oleh atmosfera. Adalah diketahui bahawa eddy memainkan peranan primer dalam pengangkutan haba ke kutub. Lebih lagi, ini merupakan kaedah untuk atmosfera mengimbangkan defisit radiasi di latitud polar.

ABSTRACT

A warm low pressure and a cold high pressure imply the contribution of potential energy to kinetic energy. In the tropics, this represents the typical situation of the Hadley cell, also known as a direct cell. On the other hand, a cold low pressure and a warm high pressure indicate the contribution of the mean kinetic energy to the mean potential energy, which represents the typical process of the Ferell cell (also referred to as an indirect cell). This paper examines the poleward heat transport of the atmosphere. It is noted that eddies play the primary role in poleward heat transport. Furthermore, this is the way the atmosphere counterbalances the deficit of radiation at polar latitudes.

Keywords: poleward heat transport, eddies, zonal kinetic energy, available potential energy

INTRODUCTION

Following Holton (1972), available potential energy may be defined as the difference between the total potential energy of a closed system and the minimum total potential energy which results from an adiabatic redistribution of mass. However, only a minor fraction (0.5%) of the total potential energy of the atmosphere is available for conversion to kinetic energy (Holton 1972; Camerlengo and Nasir 1998).

The energy cycle of the atmosphere is addressed. It is verified that eddies which result from the baroclinic instability of the mean flow are to a larger extent responsible for the energy exchange in the atmosphere.

THE PROBLEM

Time Rate of Change of the Available Potential Energy

The thermodynamic equation states that:

$$\partial \theta/\partial t + V.\nabla_p\theta + \omega\,\partial\theta/\partial p = \left(\theta/c_p\,T\right)Q \qquad (1)$$

where T represents the atmospheric temperature; θ (= T $(p_{oo}/p)^k$), the potential temperature; p, the atmospheric pressure; (.), the scalar product; p_{oo}, a reference pressure; ∇_p ; the horizontal gradient in the (x, y, p) coordinate system; V (= (u, v)), the horizontal velocity; ω (= d p/ dt), the vertical velocity in the (x, y, p) coordinate system; c_p, is the specific heat at constant pressure; and Q, other forms of heating than latent heat.

The equation of continuity in the (x,y,p) coordinate system is:

$$\nabla_p . V + \partial\omega/\partial p = 0 \qquad (2)$$

Upon multiplication of equation (1) by θ and using equation (2):

$$2^{-1}\left\{\partial\left(\theta^2\right)/\partial t + \nabla_p.\left(V\theta^2\right) + \partial\left(\omega\theta^2\right)/\partial p\right\} = \left(\theta/c_p\,T\right)Q\,\theta \qquad (3)$$

is obtained.

It is convenient to define a space average of the form:

$$\overline{[\ \ ]} = \left(\int[\ \ ]dx\,dy\right)/\left(\int dx\,dy\right) \qquad (4)$$

Applying this space average in equation (3), it is obtained:

$$2^{-1}\left\{\partial\,\overline{\left(\theta^2\right)}/\partial t + \overline{\nabla_p.\left(V\theta^2\right)} + \partial\overline{\left(\omega\theta^2\right)}/\partial p\right\} = \left(\theta/c_p\,T\right)\overline{Q\theta} \qquad (5a)$$

Because $\overline{\nabla_p.\left(V\theta^2\right)} = 0$, it yields:

$$2^{-1}\left\{\partial\,\overline{\left(\theta^2\right)}/\partial t + \partial\overline{\left(\omega\theta^2\right)}/\partial p\right\} = \left(\theta/c_p\,T\right)\overline{Q\theta} \qquad (5b)$$

What does exactly $\overline{\left(\omega\theta\,\theta\right)}$ mean? The departure from the space average is symbolized by a star. Because $\overline{\omega} = 0$, it is obtained:

$$\overline{(\omega\theta\,\theta)} = \overline{\left(\theta\,\theta^*\right)^2\left(\overline{\omega} + \omega^*\right)} = \overline{\theta\,\theta\,\omega^*} + \overline{2\theta\,\theta^*\,\omega^*} + \overline{\theta^*\theta^*\omega^*} \qquad (6)$$

Therefore, equation (6) yields:

$$\overline{\left(\omega\theta\,\theta\right)} = = \overline{\theta}\,\overline{\theta}\,\overline{\omega^*} + 2\,\overline{\theta}\,\overline{\theta^*\omega^*} + \overline{\theta^*\theta^*\omega^*} \qquad (7)$$

Because $\overline{\omega^*} = 0$, it is obtained:

$$\overline{(\omega\,\theta\,\theta)} = = 2\,\bar{\theta}\;\overline{\theta^*\,\omega^*} + \overline{\theta^*\theta^*\omega^*} \tag{8}$$

Substitution of equation (8) in equation (5) yields:

$$\partial\left(\overline{\theta^2}/2\right)/\partial t + + \bar{\theta}\,\partial\left(\overline{\omega^*\theta^*}\right)/\partial\,p + \overline{\omega^*\theta^*}\partial\left(\bar{\theta}\right)/\partial\,p + \partial\left(\overline{\omega^*\theta^{2*}/2}\right)/\partial p$$

$$= \left(\theta/c_p\,T\right)\overline{Q\,\theta} \tag{9}$$

Using the continuity equation (equation 2) and applying the space average to equation (1), it yields:

$$\partial\,\bar{\theta}/\theta\,t + \nabla_p\,.\,\overline{\left(V\,\theta\right)} + \partial\,\overline{\left(\omega\,\theta\right)}/\partial\,p = \left(\theta/c_p\,T\right)\bar{Q} \tag{10}$$

Upon multiplication of equation (10) by $\bar{\theta}$, it is obtained:

$$2^{-1}\left\{\partial\overline{\theta}^2/\partial t + \overline{\nabla_p\,.\,\left(V\,\theta^2\right)}\right\} + \bar{\theta}\,\partial\,\overline{\left(\omega\theta\right)}/\partial\,p = \left(\theta/c_p\,T\right)\bar{\theta}\,\bar{Q} \tag{11}$$

However, as stated above, $\overline{\nabla_p\,.\,\left(V\,\theta^2\right)} = 0$. Therefore, equation (10) yields:

$$\partial\left(\bar{\theta}^2/2\right)/\partial t + \bar{\theta}\,\partial\,\overline{\left(\omega\theta\right)}/\partial\,p = \left(\theta/c_p\,T\right)\bar{\theta}\,\bar{Q} \tag{12}$$

Because $\bar{\omega} = 0$, it is obtained:

$$\overline{\theta\,\omega} = \bar{\theta}\,\bar{\omega} + \overline{\theta^*\,\omega^*} = \overline{\theta^*\,\omega^*} \tag{13}$$

Upon subtraction of equation (12) from equation (9), it yields:

$$\partial\left(\overline{\theta^{*2}/2}\right)/\partial t + \left(\overline{\omega^*\theta^*}\right)\partial\bar{\theta}/\partial p + \partial\left(\left(\overline{\omega^*\theta^{*2}}\right)/2\right)/\partial\,p = \left(\theta/c_p T\right)\overline{\theta^* Q^*} \tag{14}$$

The third term on the LHS of equation (14) involves the space average of the product of three quantities, each of which in itself represents a departure from a space average. Such "triple correlations" are often negligible.

The division of equation (14) by $\partial\,\bar{\theta}/\partial\,p = \Gamma$, yields:

$$\Gamma^{-1}\left\{\partial\left(\overline{\theta^{*2}/2}\right)/\partial t\right\} + \overline{\omega^*\theta^*} = \left\{\left(\theta/c_p T\right)\overline{\theta^* Q^*}\right)\right\}\Gamma^{-1} \tag{15}$$

The equation of horizontal momentum states that:

$$\partial\,V\partial t + V\cdot\nabla_p\,V + \omega\,\partial\,V/\partial\,p + k\,x\,V\,f + g\,\nabla_p\,Z = F \tag{16}$$

where F represents the external horizontal body force (of friction) per unit mass; k, the normal unit vector; and (x) the vectorial product. Scalar multiplication of equation (16) by V yields:

$$\partial\left(V^2/2\right)\partial t + \nabla_p \cdot \left(V^2/2\right) + g\,V \cdot \nabla_p z = V \cdot F \tag{17}$$

Upon integration of equation (17) over the whole mass of the atmosphere, it is obtained:

$$\int_M \left[\partial\left(V^2/2\right)\partial t\right] d\,m + g\int_M \left[V \cdot \nabla_p z\right] d\,m = \int_M V \cdot F\,dm \tag{18}$$

The second term on the LHS represents the work done by pressure forces, or the rate of generation of kinetic energy, KE, and the first term on the RHS the dissipation of KE by friction forces. Because

$$d\,m = \rho\,d\,V = \rho\,d\,x\,d\,y\,dz = \left(d\,p/g\right) dx\,dy \tag{19}$$

and upon integration by parts of equation (18), the second term on the LHS becomes:

$$\int_M \left[V \cdot \nabla_p z\right] d\,x\,d\,y\,dp = \int_M \nabla_p \cdot \left(V\,z\right) d\,x\,d\,y\,dp - \int_M z\,\nabla_p \cdot \left(V\right) d\,x\,d\,y\,dp \tag{20}$$

The continuity equation states that:

$$V_p \cdot V + \partial\,\omega/\partial\,p = 0 \tag{21}$$

It is obtained:

$$\int_M z\left(\partial\,\omega/\partial\,p\right) d\,x\,d\,y\,dp = \int_M \left[\partial\left(z\,\omega\right)/\partial\,p\right] d\,x\,d\,y\,dp - \int_M \omega\left(\partial\,z/\partial\,p\right) d\,x\,d\,y\,dp \tag{22}$$

However, the value of ω at the top and at the bottom of the atmosphere is zero. Furthermore, by the hydrostatic approximation, $\left(\partial\,z/\partial\,p\right) = -\alpha/g$. Therefore,

$$\int_M z\left(\partial\,\omega/\partial\,p\right) d\,x\,d\,y\,dp = \int_M \left(\alpha\,\omega\right) d\,m \tag{23}$$

Upon using the equation of state for ideal gases, $\alpha = R\,T/p$, it follows that:

$$\begin{aligned}\int_M \left[V \cdot \nabla_p z\right] d\,x\,d\,y\,dp &= \int_M \left(\alpha\,\omega\right) d\,m \\ &= \left(R/g\right)\int_M \left(T\,\omega\right) d\,x\,d\,y\,d\left(\text{In}\,p\right)\end{aligned} \tag{24}$$

Equation (24) defines the rate of generation of KE due to the pressure forces.

Rearrangement of equation (24) yields to:

$$\begin{aligned}\int_M \left[V \cdot \nabla_p z\right] d\,x\,d\,y\,dp &= (R/g)\int_M (T/p\,\theta)(\omega\,\theta)\,d\,x\,d\,y\,dp \\ &= \int_M (\rho\theta)^{-1}(\omega\,\theta)\,d\,x\,d\,y\,dp\end{aligned} \tag{25}$$

Because $\bar{\omega} = 0$, the space average of $\overline{(\omega\,\theta)}$ yields:

$$\overline{(\omega\,\theta)} = \bar{\omega}\,\bar{\theta} + \overline{\omega^*\ \theta^*} = \overline{\omega^*\ \theta^*} \tag{26}$$

Therefore

$$\int_M \overline{(\omega\,\theta)}\,d\,x\,d\,y\,dp = \int_M \left(\overline{\omega^*\ \theta^*}\right) d\,x\,d\,y\,dp \tag{27}$$

Thus, the space average of equation (25) yields:

$$\int_M \overline{\left[V \cdot \nabla_p\, z\right]}\,d\,x\,d\,y\,dp = \int_M (\rho\theta)^{-1}\left(\overline{\omega^*\ \theta^*}\right) d\,x\,d\,y\,(dp/g) \tag{28}$$

The equation of the available potential energy, APE is:

$$\bar{A} = \left(c_p\,k\right)\left(2\,g\,p_{oo}{}^{k}\right)^{-1}\int_0^{P_o}\overline{\theta^2}\,p^{k-1}\,(-\Gamma)^{-1}\left(\overline{\theta^*}/\bar{\theta}\right)^2 d\,p \tag{29}$$

may be rewritten as:

$$\bar{A} = \left(c_p\,k/2\right)\int\left(\bar{p}/p_{oo}\right)^k\left[\overline{\theta^{*2}}/(-\bar{\chi}\,\partial\,\theta/\partial\chi)\right] d\,y\,d\,x\,(d\,z/g) \tag{30}$$

where $\chi = \bar{p}$.

Because $\theta = T\ (p_{oo}/p)^k$, equation (30) yields:

$$\begin{aligned} \bar{A} &= \left(c_p\,k/2\right)\int (T/\theta)/\left[\chi\left(-\,\partial\bar{\theta}/\partial p\right)\right] d\,x\,d\,y\,(d\,p/g) \\ &= 0.5\int (R\ T/p\,\theta)/\left[-\,\partial\,\bar{\theta}/\partial p\right] d\,x\,d\,y\,(d\,p/g) \\ \bar{A} &= 0.5\int \theta^{*2}(\rho\theta)^{-1}/\left[-\,\partial\,\bar{\theta}/\partial p\right] d\,x\,d\,y\,(d\,p/g) \end{aligned} \tag{31}$$

Upon multiplication of equation (15) by $(\rho\theta)^{-1}$ it is obtained:

$$\begin{aligned} &\left\{(\rho\theta)^{-1}/\left[\left(-\partial\,\bar{\theta}/\partial p\right)\right]\right\}\partial\left(\overline{\theta^{*2}}\right)/\partial t \\ &= -\overline{\omega^*\,\theta^*}\,(\rho\theta)^{-1} + \left\{(\rho\theta)^{-1}/\left[\left(-\partial\,\bar{\theta}/\partial p\right)\right]\right\}\left(\theta/c_p\,T\right)\overline{Q^*\,\theta^*} \end{aligned} \tag{32}$$

where the terms of equations (31) and (32) are somewhat similar. Therefore, the LHS of equation (32) is $\partial A/\partial t$. It follows naturally that:

$$\int \partial A/\partial t\,dm = -\int \overline{\omega^*\ \theta^*}\,(\rho\theta)^{-1} d\,m + \int\left(R/g\,c_p\right)\overline{Q^*\ \theta^*}/\left[\left(-\partial\,\bar{\theta}/\partial p\right)\right] d\,m \tag{33}$$

In other words:

$$\partial/\partial\,t\left(\int \bar{A}\ dm\right) = -\int \overline{\omega^*\ \theta^*}\,(\rho\theta)^{-1} d\,m + \int (k/g)\,\overline{Q^*\ \theta^*}\,d\,x\,d\,y\,d(\mathrm{In}\,p) \tag{34}$$

where equation (34) represents the *time rate of change of the APE.*

Zonal and Eddy Available Potential Energy

The thermodynamic equation may also be written as:

$$c_v \, dT/dt + p \, d\alpha/dt = Q \qquad (35)$$

Another form of the continuity equation is:

$$d\alpha/dt = \alpha \nabla_3 . V3 \qquad (36)$$

where ∇_3, represents the three-dimensional gradient operator; and V_3 (=(u,v,w)) the velocity vector in three dimensions; and w, the vertical velocity component in the (x,y,z) coordinate system. Combination of equations (35) and (36) yields:

$$c_v \, dT/dt + p \alpha \nabla_H . V + p \alpha \partial w/\partial z = Q$$

or

$$\rho c_v \, dT/dt + p \nabla_H . V + p \partial w/\partial z = \rho Q \qquad (37)$$

Upon making usage of the continuity equation:

$$\rho \, d(\,)/dt = \rho \partial(\,)/\partial t + \nabla_3 . [\rho(\,)_3] \qquad (38)$$

and upon integration of equation (37) it yields:

$$\partial/\partial t\left(\int_M \rho c_v T \, dm\right) = -\int_V p \nabla_H \cdot V \, dV - \int_V \rho(\partial w/\partial z) dV + \int_V \rho Q \, dV \qquad (39)$$

However, upon usage of the hydrostatic approximation it yields:

$$\int p (\partial w/\partial z) dz = -\int w \rho g \, dz \qquad (40)$$

It is obtained:

$$\partial/\partial t \int_M (\rho c_v T) dm = -\int_V p \nabla_H \cdot V \, dV - \int_V w \rho g \, dV + \int_M \rho Q \, dm \qquad (41)$$

The term of the LHS represents the time rate of change of the total internal energy, while the first term on the RHS represents the rate at which the internal energy is created or destroyed by expansion or compression, respectively. The second term of the RHS may be rewritten in the form:

$$\int_V w \rho g \, dV = \int_V \rho g \, (dz/dt) dV = \partial/\partial t \left(\int g z \, dm\right) \qquad (42)$$

Equation (42) represents the time rate of change of the gravitational potential energy. From equations (41) and (42) it is obtained:

$$\partial/\partial t \int_M \left(\rho c_v T \, dm + \int_M g z \, dm\right) = \int_V p \nabla_H \cdot V \, dV + \int_M \rho Q \, dm \qquad (43)$$

Upon usage of the hydrostatic approximation, the second term of the LHS of equation (43) yields:

$$\int_M g\, z\, d\, m = \int_x \int_y \int_0^{p_o} (z\, d\, p)\, d\, y\;\; d\, x \tag{44}$$

Upon usage of the ideal gas law and upon integration by parts, it yields:

$$\int_M g\, z\, d\, m = \int_x \int_y \int_z (p\, dz)\, dy\, dx = \int_x \int_y \int_z (R\, T\, \rho)\; d\, V = \int_v (R\, T\, \rho) d\, V \tag{45}$$

Combining equations (43) and (45) it follows naturally that:

$$\partial/\partial\, t \left(\int_M c_p\, T \right) d\, m = -\int_V p\, \nabla_H \cdot V\, d\, V + \int_M \rho\, Q\, d\, m \tag{46}$$

It is convenient to split the APE into its zonal, Az, and its eddy component, A_E, in such a way that:

$$A = A_z + A_E \tag{47}$$

Therefore,

$$\int (\;)\,(T^*)^2\, d\, m = \int (\;)[T"]^2\; dm + \int (\;)\,(T')^2\, d\, m \tag{48}$$

where

$$T = \bar{T} + T^* = \bar{T} + [T]" + T' \tag{49}$$

Thus,

$$\overline{(T - \psi)^2} = \overline{([T]" + T')^2} = \overline{[T]"^2} + \overline{T'^2} \tag{50}$$

where $\psi = \bar{T}$; ('), represents the departure from x;("), the departure from y, (*), the departure from the x y average; ($\bar{\ }$) the x y average; and ($\tilde{\ }$) the y-average. Upon rewriting equation (14), it yields:

$$\partial\left(\overline{\theta^{*2}/2}\right)/\partial t + \left(\overline{\omega^*\theta^*}\right)\partial\bar{\theta}/\partial\, p + \partial\left(\left(\overline{\omega^*\theta^{*2}}\right)/2\right)/\partial\, p = \left(\theta/c_p\, T\right)\; \overline{\theta^*\, Q^*} \tag{51}$$

where

$$\theta = \theta^* + \bar{\theta} = [\tilde{\theta}] + [\theta]" + \theta'$$

$$\omega = \bar{\omega} + \omega^* = \omega^* = [\omega]" + \omega' \tag{52}$$

Therefore,

$$\overline{\omega^*\theta^*} = \overline{([\theta]" + \theta')([\omega]" + \omega')} = \widetilde{[\theta]"[\omega]"} + \widetilde{[\theta'\, \omega']} \tag{53}$$

$$\overline{\omega^*\theta^*\theta^*} = \overline{([\theta]" + \theta')^2([\omega]" + \omega')} = \overline{[\theta]"[\theta]"[\omega]"} + 2\,\overline{[\theta'\,\omega']"[\theta]"} + \overline{[\theta'\,\theta'\,\omega']} \quad (54)$$

$$\overline{\theta^*\theta^*} = \overline{[\theta]"[\theta]"} + \overline{[\theta'\,\theta']} \quad (55)$$

Upon inclusion of equations (53), (54) and (55) in equation (51), it yields:

$$2^{-1}\left\{\partial\overline{[\theta]"^2}/\partial t + \partial\overline{[\theta]'^2}/\partial t\right\} + \overline{[\theta]"[\omega]"}\partial\overline{[\theta]}/\partial p + \overline{[\theta'\,\omega']}\,\partial\overline{[\theta]}/\partial p$$

$$+ \partial\overline{\{[\theta]"[\theta'\,\omega']"\}}/\partial p + \partial\ \overline{\{[\theta]"[\theta]"[\omega]"/2\}}\ /\partial p + \partial\ \overline{\{[\theta'\,\theta'\,\omega']/2\}}\ /\partial p$$

$$= (\theta/c_p\,T)\ \{\overline{[\theta]"[\theta]"} + \overline{[\theta'\,\theta']}\} \quad (56)$$

Upon multiplication of the thermodynamic equation, in spherical coordinates:

$$\partial[\theta]/\partial t + R^{-1}\,\partial\,\{R\,[\theta\,v]\}/\partial y + \partial\,[\theta\,\omega]/\partial p = (\theta/c_p\,T)\ [Q] \quad (57)$$

by $[\theta]$, and averaging in the y-direction, it follows that:

$$2^{-1}\,\partial\overline{[\theta]}^2/\partial t + \overline{[\theta]}R^{-1}\,\partial\,\{R\overline{[\theta\,v]}\}/\partial y + \partial(R\overline{[\theta\,\omega]})/\partial p$$
$$= (\theta/c_p T)\ \overline{[\theta][Q]} \quad (58)$$

In other words:

$$2^{-1}\,\partial\overline{[\theta]}^2/\partial t + \overline{[\theta]}R^{-1}\ \{\partial(\overline{R[\theta][v]})/\partial y + \partial(\overline{R[\theta][\omega]})/\partial p\} +$$

$$\overline{[\theta]R^{-1}\ \{\partial(R[\theta'\,v'])/\partial y} + \partial\ \overline{(R[\theta'\,\omega'])/\partial p\}} = (\theta/c_p T)\ \overline{[\theta][Q]} \quad (59)$$

The zonal average of the continuity equation yields:

$$R^{-1}\,\{\partial(R[v])/\partial y + \partial(R\,[\omega])/\partial p\} = 0 \quad (60)$$

Upon differentiation of both the second and the third terms of the LHS of equation (59), it yields:

$$([\theta]^2\,R^{-1})\{\partial\,(R\,[v])/\partial y + \partial\,(R\,[\omega])/\partial p\} + [\theta]\,[v]\,\partial[\theta]\}/\partial y + [\theta]\,[\omega]\partial\,[\theta]/\partial p. \quad (61)$$

Due to the continuity equation, the sum of the first two terms equates to zero. Therefore, the y average of equation (61) remains as:

$$\overline{[\theta][v]\partial[\theta]\}}/\partial y + \overline{[\theta][\omega]\partial[\theta]}/\partial p$$

$$= \overline{[v]\partial\left([\theta]^2/2\right)}/\partial y + \overline{[\omega]\partial\left([\theta]^2/2\right)}/\partial p\} \tag{62}$$

This expression is called the *advective form.* Applying the continuity equation

$$[v]\partial q/\partial y + [\omega]\partial q/\partial p = R^{-1}\{\partial(R\, q\, [v])/\partial y + \partial(R\, q\, [\omega])/\partial p\} \tag{63}$$

in equation (62), it follows that:

$$R^{-1}\left\{\partial\left(\overline{R[\theta]^2[v]/2}\right)/\partial y + \partial\left(\overline{R[\theta]^2[\omega]/2}\right)/\partial p\right\} \tag{64}$$

where the first term is, obviously, zero. The remaining term is split in the following form:

$$\overline{[\theta][\theta][\omega]} = \overline{\left(\tilde{[\theta]} + [\theta]''\right)\left(\tilde{[\theta]} + [\theta]''\right)[\omega]''}$$

$$= 2\,\tilde{[\theta]}\overline{[\theta]''[\omega]''} + \overline{\left([\theta]''\right)^2[\omega]''} \tag{65}$$

Therefore,

$$\partial\left(\overline{R[\theta]^2[\omega]/2}\right)/\partial p\} = \partial\left\{\tilde{[\theta]}\overline{[\theta]''[\omega]''} + \overline{\left([\theta]''\right)^2[\omega]''/2}\right\}/\partial p\} \tag{66}$$

Upon inclusion of equation (66) in equation (59) it yields:

$$(1/2)\,\partial\tilde{[\theta]}^2/\partial t + \tilde{[\theta]}\left\{\partial\left(\overline{[\theta]''[\omega]''}\right)/\partial p\right\} + \left(\overline{[\theta]''[\omega]''}\right)\partial\tilde{[\theta]}/\partial p$$

$$+ \partial\left(\overline{\left([\theta]''\right)^2[\omega]''/2}\right\}/\partial\partial p + \overline{[\theta]R^{-1}\left\{\partial\left(R[\theta' v']\right)/\partial y + \partial\left(R[\theta' \omega']\right)/\partial p\right\}} \tag{67}$$

$$= \left(\theta/c_p T\right)\overline{[\theta][Q]}$$

Upon usage of the continuity equation, the equation:

$$d\, q/dt = \partial q/\partial t + u\,\partial q/\partial x + v\,\partial q/\partial y + \omega\,\partial q/\partial p \tag{68}$$

may be placed in the form:

$$d\, q/d\, t = R^{-1}\{\partial(R\, q)/\partial t + \partial(q\, R\, u)/\partial x + \partial(q\, v\, R)/\partial y + \partial(q\, \omega\, R)/\partial p \tag{69}$$

This is known as the *convergence form.* Equation (69) may also be written in the following form:

$$d\,q/d\,t = R^{-1}\,\partial\,(R\,q)/\partial\,t + (R^{-1}\,q)\,\{\partial\,(R\,u)/\partial\,x + \partial\,(v\,R)/\partial\,y + \partial\,(\omega\,R)/\partial\,p\}$$
$$+\,u\,\partial\,q/\partial\,x + v\,\partial\,q/\partial\,y + \omega\,\partial\,q/\partial\,p \tag{70}$$

Upon average of the thermodynamic equation, it yields:

$$\partial\overline{[\theta]}/\partial t + R^{-1}\left\{\partial\overline{[R\,v\,\theta]}/\partial\,y\right\} + \partial\,\overline{[\theta\,\omega]}/\partial\,p = [Q] \tag{71}$$

Due to the fact that an averaging in the y-direction is taken, the second term in the LHS is zero. Upon multiplication of equation (71) by $[\theta]$, it yields:

$$\partial\left(\overline{[\theta]^2/2}\right)/\partial\,t + \overline{[\theta]}\,\partial\left(\overline{[\theta]''[\omega]''}\right)[\theta]/\partial\,p + \overline{[\theta]}\,\partial\left([\theta'\,\omega']\right)/\partial\,p$$
$$= \left(\theta/c_p\,T\right)\overline{[\theta][Q]} \tag{72}$$

Subtraction of equation (72) from equation (67) leads to

$$(1/2)\;\partial\overline{[\theta]''^{2}}/\partial t + \left(\overline{[\theta]''[\omega]''}\right)\partial\,\overline{[\theta]}/\partial\,p + \partial\;\left(\overline{[\theta]''[\theta]''[\omega]''/2}\right\}/\partial\,p$$
$$+\,\overline{[\theta]''}R^{-1}\quad\left\{\partial\left(\overline{R[\theta'\,v']''}\right)/\partial\,y + \partial\left(\overline{R[\theta'\,\omega']''}\right)/\partial\,p\right\} = \left(\theta/c_p\,T\right)\;\overline{[\theta]''[Q]''} \tag{73}$$

Bearing in mind that $\left\{\partial\,\left(\overline{[\theta]''\;[\theta'\,v']''}\right)/\partial\,y = 0\right.$, equation (73) may be rewritten as:

$$(1/2)\;\partial\overline{[\theta]''^{2}}/\partial t + \left(\overline{[\theta]''[\omega]''}\right)\partial\,\overline{[\theta]}/\partial\,p + \partial\left(\overline{[\theta]''[\theta'\,\omega']''}\right\}/\partial\,p$$
$$-\,\overline{[\theta'\,v']''}\;\partial\,\overline{[\theta]''}/\partial\,y - \overline{[\theta'\,\omega']}\partial\,\overline{[\theta]''}/\partial\,p) = \left(\theta/c_p\,T\right)[\theta]''[Q]'' \tag{74}$$

Subtraction of equations (74) from equation (56) yields:

$$(1/2)\;\partial\overline{[\theta]}^{2}/\partial t + \overline{[\theta'\,\omega']}\partial\,\overline{[\theta]}/\partial\,p$$
$$+\left(\overline{[\theta'\,v']''\partial\,[\theta]''}\;/\partial\,y\;+\overline{[\theta'\,\omega']\;\partial\,[\theta]''}/\partial\,p =$$
$$= \left(\theta/c_p\,T\right)\overline{[\theta'\,Q']} \tag{75}$$

Zonal Kinetic Energy:

The u and v momentum equations, in spherical coordinates, are:

$$\partial u/\partial t + \partial(u\,u)/\partial x + R^{-1}\,\partial(R\,u\,v)/\partial y + \partial(u\,\omega)/\partial p - (u\,v\tan\varphi)/r$$
$$- f\,v + g\,\partial z/\partial x + F_x = 0 \qquad (76)$$

$$\partial v/\partial t + \partial(u\,v)/\partial x + R^{-1}\,\partial(R\,v\,v)/\partial y + \partial(v\,\omega)/\partial p - (u\,u\tan\varphi)/r$$
$$+ f\,u + g\,\partial z/\partial y + F_y = 0 \qquad (77)$$

Upon taking the zonal averaging of equations (76) and (77), it yields:

$$\partial[u]/\partial t + R^{-1}\,\partial(R\,[u\,v])/\partial y + \partial[u\,\omega]/\partial p - ([u\,v]\tan\varphi)/r - f\,[v] + [F_x] = 0 \qquad (78)$$

$$\partial[v]/\partial t + R^{-1}\,\partial(R\,[v\,v])/\partial y + \partial[v\,\omega]/\partial p - ([u\,u]\tan\varphi)/r$$
$$+ f\,[u] + g\,\partial[z]/\partial y + [F_y] = 0 \qquad (79)$$

where the fifth and sixth terms in the LHS are the largest in equation (79). The variables u and v, in equations (76) and (77), are of the same order of magnitude. Therefore, they are not very useful. However, the ratio $[v]/[u] \ll 1$, behaves in the same manner as the ratio of divergence/vorticity ($\ll 1$). Therefore, it may be stated that {v] is a measure of the divergence, while [u] is a measure of the vorticity.

Upon multiplication of equations (78) and (79) by [u] and [v], respectively, it yields:

$$\partial([u]^2/2)/\partial t + ([u]R^{-1})\,\partial(R\,[u\,v])/\partial y + [u]\,\partial[u\,\omega]/\partial p$$
$$- ([u][u\,v]\tan\varphi)/r - f\,[v][u] + [F_x]\,[u] = 0 \qquad (80)$$

$$\partial([v]^2/2)/\partial t + ([v]R^{-1})\,\partial(R\,[v\,v])/\partial y + [v]\,\partial[v\,\omega]/\partial p$$
$$+ ([v][u\,u]\tan\varphi)/r + f\,[v][u] + (g\,[v])\partial[z]/\partial y + [F_y]\,[v] = 0 \qquad (81)$$

The second and fourth terms of the LHS of equation (80) may be recombined in the following manner:

$$([u]R^{-1})\,\partial(R^2\,[u\,v])/\partial y \equiv ([u]R^{-1})\,\partial(R\,[u\,v])/\partial y - ([u][u\,v]\tan\varphi)/r \qquad (82)$$

It is worth mentioning that [u]/R is a measure of the angular velocity. Equation (80) may then be rewritten as:

$$\partial([u]^2/2)/\partial t + ([u]R^{-1})\{R^{-1}\,\partial(R^2\,[u\,v])/\partial y + \partial(R\,[u\,\omega])/\partial p\}$$
$$- f\,[v][u] + [F_x]\,[u] = 0 \qquad (83)$$

In a similar manner, equation (81) yields:

$$\partial([v]^2/2)/\partial t + [v]\{R^{-1}\partial((R\,[v\,v])/\partial y + \partial[v\,\omega]/\partial p\}$$
$$+ ([v][u\,u]\tan\varphi)/r + (g\,[v])\partial[z]/\partial y + f\,[v][u] + [Fy]\,[v] = 0 \qquad (84)$$

The *zonal kinetic energy*, $[K_x]$, may be defined in the following manner:

$$[K_x] = \int_M [u]^2/2 \, dm \tag{85}$$

Likewise,

$$[K_y] = \int_M [v]^2/2 \, dm \tag{86}$$

Upon integration over the entire mass of the system, it yields:

$$\partial[K_x]/\partial t + \int_M \left\{\left([u]R^{-1}\right)\left\{R^{-1}\partial\left(R^2[uv]\right)/\partial y + \partial\left(R[u\omega]\right)/\partial p\right\} dm - \int_M f[v][u]\, dm + \int_M [F_x][u]\, dm = 0 \tag{87}$$

$$\partial[K_y]/\partial t + \int_M [v]\left\{R^{-1}\partial\left(R[vv]\right)/\partial y + \partial[u\omega]/\partial p\right\} dm + \int_M \left(\left([v][uu]\tan\varphi\right)/r\right\} dm + \int_M \left(g[v]\right)\partial[z]/\partial y\, dm + \int_M f[v][u]\, dm + \int_M [F_y][v]\, dm = 0 \tag{88}$$

The first integrand in equation (87) represents the rate of generation of zonal kinetic energy. It may be integrated by parts in the following manner:

$$\int_M \left\{\left([u]R^{-1}\right)\left\{R^{-1}\partial R^2[uv]\right)/\partial y + \partial R[u\omega]\right)/\partial p\right\} dm =$$

$$\int_M \left\{\left(R^{-1}\right)\partial\left(R[u][uv]\right)/\partial y + \partial\left([u][u\omega]\right)/\partial p\right\} dm -$$

$$\int_M \left\{\left([uv]R\right)\partial\left([u]/R\right)/\partial y + R[u\omega]\right)\partial\left([u]/R\right)/\partial p\right\} dm \tag{89}$$

Upon integration from Pole to Pole, the first term in the RHS of equation (89) equates to zero. Therefore, the rate of generation of zonal kinetic energy is equal to the rate of destruction (this is denoted by the minus sign) of eddy kinetic energy. Upon introduction of equation (89) in equation (87), it yields:

$$\partial[K_x]/\partial t - \int_M \left\{\left([uv]R\right)\partial\left([u]/R\right)/\partial y + R[u\omega]\right)\partial\left([u]/R\right)/\partial p\right\} dm - \int_M f[v][u]\, dm + \int_M [F_x][u]\, dm = 0 \tag{90}$$

Proceeding in the same fashion as above with equation (88), it yields:

$$\partial\left[K_y\right]/\partial t-\int_M\left\{[v\,v]\,\partial\,[v]/\partial y+[v\,\omega]\,\partial\,[v]/\partial p\right\}d\,m$$
$$+\int_M\left\{\left([v][u\,u]\tan\varphi\right)/r\right\}d\,m+\int_M\left(g\,[v]\right)\partial\,[z]/\,\partial y\,d\,m+\int_M f[v][u]\,d\,m+$$
$$\int_M\left[F_y\right][v]\,d\,m=0 \tag{91}$$

Equations (90) and (91) represent the total rate of change of zonal kinetic energy. It is worth mentioning that the zonal kinetic energy receives contributions of energy from both the Coriolis and the friction terms and the eddies.

Total Kinetic Energy

If

$$u=[u]+u' \tag{92}$$

it is straightforward that:

$$[u^2]=[u]^2+[(u')^2] \tag{93}$$

Upon multiplication of equations (76) and (77) by u and v, respectively, followed by their respective zonal average, it yields:

$$\partial\,[u^2/2]/\,\partial\,t+(2\,R\,)^{-1}\,\partial(R\,[u\,u\,v])/\,\partial\,y+2^{-1}\,\partial\,[u\,u\,\omega]/\,\partial\,p$$
$$-\,([u\,u\,v]\tan\varphi)/r-f\,[u\,v]+g\,[u\partial\,z/\,\partial x]+[u\,F_x]=0 \tag{94}$$

$$\partial\,[v^2/2]/\partial\,t+(2\,R)^{-1}\,\partial\,(R\,[v\,v\,v])/\partial\,y+2^{-1}\,\partial\,[v\,v\,\omega]/\partial\,p$$
$$-\,([u\,u\,v\,]\tan\varphi)/r+f\,[u\,v]+g\,[v\,\partial\,z/\partial\,y]+[v\,F_y\,]=0 \tag{95}$$

Upon usage of the definitions of equations (85) and (86), the last two equations upon integration over the whole mass of the system may be rewritten as:

$$\partial\left[K_x\right]/\partial\,t-\int_M\left\{\left([u\,u\,v]\tan\varphi\right)/r\right\}d\,m-\int_M f\left[v\,u\right]d\,m+$$
$$\int_M g\left[u\partial\,z/\partial x\right]d\,m+\int_M\left[u\,F_x\right]d\,m=0 \tag{96}$$

$$\partial\left[K_y\right]/\partial\,t+\int_M\left\{\left([u\,u\,v]\tan\varphi\right)/r\right\}d\,m+\int_M f\left[v\,u\right]d\,m+$$
$$\int_M g\left[v\partial\,z/\partial y\right]d\,m+\int_M\left[v\,F_y\right]d\,m=0 \tag{97}$$

The set of equations (96) and (97) represent the total rate of change of total kinetic energy.

Eddy Kinetic Energy

A formulation for the total rate of change of the eddy kinetic energy is sought. For this purpose, the addition of equations (96) and (97) is subtracted from the addition of equations (90) and (91). Keeping in mind that [A B] = [A][B] + [A'B'], after some trivial algebraic manipulations, it yields:

$$\partial K'/\partial t + \int_M \Big\{\big([u\,v]\,R\big)\partial\big([u]/R\big)/\partial y + \big(R[u\,\omega]\big)\partial\big([u]/R\big)/\partial p + \big([v\,v]\partial[v]/\partial y\big) + \big([v\,\omega]\partial[v]/\partial p\big) - \big([v][u\,u]\tan\varphi\big)/r\Big\}\,dm + g\int_M \big\{\partial[v'\partial z'/\partial y] + [u'\partial z'/\partial x]\big\}\,dm + \int_M \big([u'F_x'] + [v'F_y']\big)\,dm = 0 \tag{98}$$

where

$$K' = \int_M \big([u'^2]/2\big)\,dm \tag{99}$$

is the eddy kinetic energy. The first integrand represents the rate of conversion of eddy kinetic energy to zonal kinetic energy. The second integrand represents the release of potential energy (baroclinic instability mechanism) while the third integrand represents the dissipation of eddy kinetic energy by friction.

The first integrand of equation (98) may be rewritten as:

$$\int_M \Big\{\big([u\,v]\,R\big)\partial\big([u]/R\big)/\partial y + \big(R[u\,\omega]\big)\partial\big([u]/R\big)/\partial p + \big([v\,v]\partial[v]/\partial y\big) + \big([v\,\omega]\partial[v]/\partial p\big) - \big([v][u\,u]\tan\varphi\big)/r\Big\}\,dm =$$

$$\int_M \Big\{\big([u][v]\,R\big)\partial\big([u]/R\big)/\partial y + \big(R[u][\omega]\big)\partial\big([u]/R\big)/\partial p + \big([v][v]\partial[v]/\partial y\big) + \big([v][\omega]\partial[v]/\partial p\big) - \big([v][u][u]\tan\varphi\big)/r\Big\}\,dm +$$

$$\int_M \Big\{\big([u'v']R\big)\partial\big([u]/R\big)/\partial y + \big(R[u'\omega']\big)\partial\big([u]/R\big)/\partial p + \big([v'v']\partial[v]/\partial y\big) + \big([v'\omega']\partial[v]/\partial p\big) - \big([v][u'u']\tan\varphi\big)/r\Big\}\,dm \tag{100}$$

The first term may be decomposed as follows:

$$\{([u\,v]R)\partial([u]/R)/\partial y = ([u][v])\partial[u]/\partial y + (R[u]^2[v])\partial R^{-1}/\partial y =$$
$$= ([u][v])\partial[u]/\partial y - \{(R[u]^2[v])/R^2\}\partial(r\cos\varphi)/r\partial\varphi$$
$$= ([u][v])\partial[u]/\partial y + ([u]^2[v]\tan\varphi)/r \tag{101}$$

Therefore, the RHS of equation (100) may have the form:

$$\int_M \{([u][v])\partial[u]/\partial y + ([u][\omega])\partial[u]/\partial p + ([v][v])\partial[v]/\partial y$$
$$+ ([v][\omega]\partial[v]/\partial p)\} \, dm + \int_M (\text{eddy components}) \, dm$$
$$= (1/2)\int_M \{[u]\partial([u]^2 + [v]^2)\partial y + [\omega]\partial([u]^2 + [v]^2)\partial p\} \, dm$$
$$+ \int_M (\text{eddy components}) \, dm \qquad (102)$$

If the divergence form is chosen, the RHS of equation (100) yields:

$$\int_M 2^{-1}([u]^2 + [v]^2)\{(R^{-1}\partial(R[v])/\partial y$$
$$+ \partial[\omega]/\partial p\} dm + \int_M (\text{eddy components}) \, dm \qquad (103)$$

Therefore, from equations (98) and (103) it yields:

$$\partial K'/\partial t + \int_M \{[u'v']R)\partial([u]/R)/\partial y + (R[u'\omega'])\partial([u]/R)/\partial p$$
$$+ ([v'v']\partial[v]/\partial y) + ([v'\omega']\partial[v]/\partial p) - ([v][u'u']\tan\varphi)/r\} \, dm$$
$$+ g\int_M \{\partial[v'\partial z'/\partial y] + [u'\partial z'/\partial x]\} \, dm + \int_M ([u'F_x'] + [v'F_y']) \, dm \qquad (104)$$

where the first and second terms in the RHS represent the rate of conversion of $[K_x]$ and eddies, while the third, fourth and fifth terms in the RHS represent the rate of conversion of $[K_y]$ and eddies. Furthermore, the fifth term in the RHS also represents the contribution of kinetic energy in the southward flow motion.

Following the same procedure, equations (90) and (91) may be rewritten as:

$$\partial[K_x]/\partial t = \int_M \{([u'v']R)\partial([u]/R)/\partial y + (R[u'\omega'])\partial([u]/R)/\partial p$$
$$+ f[u][v] + ([v][u][u]\tan\varphi)/r - [u][F_x]\} \, dm \qquad (105)$$

$$\partial[K_y]/\partial t = \int_M \{[u'v']\partial[v]/\partial y + [u'\omega']\partial[v]/\partial p$$
$$- f[u][v] - ([v][u][u]\tan\varphi)/r - [v][F_y]$$
$$- g[v]\partial[z]/\partial y + ([v][u'u']\tan\varphi)/r\} \, dm \qquad (106)$$

DISCUSSION AND RESULTS

Equations (56), (75), (103) and (104) may be symbolized schematically as:

$$\partial K_z/\partial t = \{A_z.K_z\} + \{K_E\ K_z\} - D_z \tag{107}$$

$$\partial K_E/\partial t = \{A_E.K_E\} - \{K_E\ K_z\} - D_E \tag{108}$$

$$\partial A_z/\partial t = -\{A_z.K_z\} - \{A_z\ A_E\} + G_z \tag{109}$$

$$\partial A_E/\partial t = -\{A_E.K_E\} + \{A_z\ A_E\} + G_E \tag{110}$$

where $\{A_z.K_z\}$, is represented by the sixth term in the RHS of equation (106); $\{K_E\ K_z\}$, by the first four terms in equations (105) and (106); D_z, by fifth term on the RHS of equations (105) and (106), respectively; $\{A_E.K_E\}$, by the sixth and seventh terms on the RHS of equation (104); and D_E, by the eighth and ninth terms on the RHS of equation (104).

The conversion of Az to Kz is represented by:

$$\{A_z \cdot K_z\} = -(1/R) \int_M p^{-1} [T][\omega] d\,m = \int_M [V] \nabla [g\,z] d\,m \tag{111}$$

is often called the sinking of cold air and rising of warm air at the same elevation.

The process which generates APE may be resolved into a heating at warmer latitudes and cooling at colder latitudes, which generates zonal available potential energy (ZAPE) and a heating of warmer regions and cooling at colder regions at the same latitude, which generate eddy available potential energy (EAPE). The conversion process may be resolved into a sinking in colder latitudes and rising of warmer air at the same latitude, changing A_E to K_E.

There is no process converting A_z to K_E or A_E to K_z. There are processes transferring A_z to A_E without affecting the kinetic energy and K_z to K_E without affecting the APE. The latter process consists of horizontal or vertical transport of absolute angular momentum by eddies to latitude circles of lower angular velocity. The former process consists of horizontal or vertical transport of sensible heat by eddies toward latitude circles of lower temperature. It is assumed that both K_z and K_E are dissipated by friction, with a conversion of A_z to K_z or A_E to K_E. However, both processes do not have to proceed in the same direction due to the fact that one form of kinetic energy may serve as a source for the other.

There is observational agreement that heating at lower latitudes and cooling at high latitudes generate A_z. It is not certain whether A_E is created or destroyed by heating (Necco 1980).

It has been noted that A_E is converted to K_E, since A_E is its only source. The Hadley cell converts A_z to K_z, but the Ferrell cell, working in opposition, may have a greater effect, for it exists in an area of greater horizontal temperature gradient.

The conversion of K_E to K_z indicates the large-scale eddy motion is an "unmixing" process. The only way to look at the circulation is by considering the mechanical effect of the eddies as a large-scale turbulent friction to postulate a negative coefficient of turbulent viscosity. In such a case it is obtained:

$$G_z = \{[Q]"[T]"\gamma_d\}/\{(\bar{\gamma}_d - \bar{\gamma})\ T\} \tag{112}$$

$$G_E = \{Q'T'\gamma_d\}/\{(\bar{\gamma}_d - \bar{\gamma})\ T\} \tag{113}$$

A general formulation of the energy cycle is depicted in *Fig 1.*

The effect of both a cold high pressure and a warm low pressure system is studied. For this purpose, it is convenient to consider the atmosphere as divided into two boxes (*Fig. 2*). The height of the column of cold air is smaller than the warm column. Therefore, the circulation is as indicated in *Fig 2.* Assume also that the divergence, D, is completely balanced; i.e. $|D_1| = |D_2| = |D_3| = |D_4| = D$. Thus, the pressure term is computed in the following manner:

$$\int p\ \nabla \cdot V\ dm = \int p\ D\ d\ m \tag{114}$$

From *Fig 2,* it follows that $D_1 < 0$ and $D_3 > 0$. Therefore, equation (114) computes as follows:

$$\begin{aligned} p_2 D_2 + p_4 D_4 + p_3 D_3 + p_1 D_1 &= p_4 (-D) + p_2 D + p_1 (-D) + p_3 D \\ &= D\ \{(p_2 - p_1) + (p_3 - p_4)\} \end{aligned} \tag{115}$$

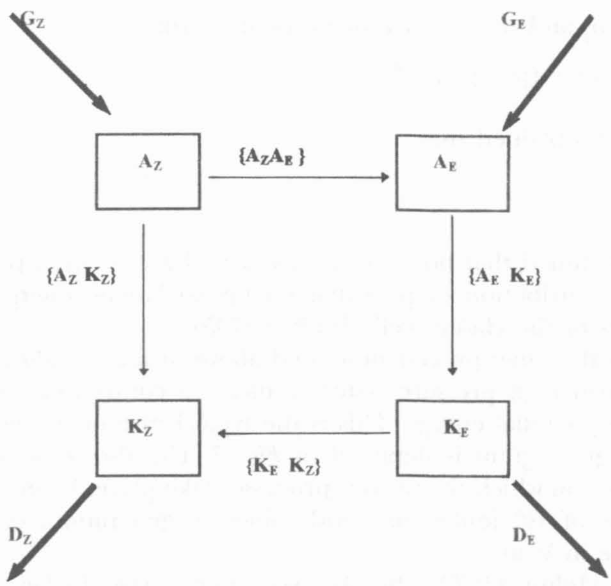

Fig 1. Energy cycle following equations (107) - (110)

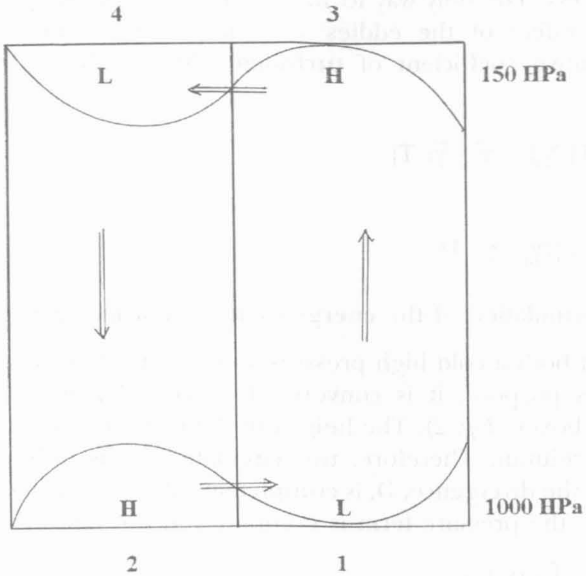

Fig 2. Resulting circulation of an idealized atmosphere divided into two boxes. The symbol ⇒ indicates the circulation of the air, while the solid line represent isobars.

Because $p_2 > p_1$ and $p_3 > p_4$, it follows naturally that:

$$D \{(p_2 - p_1) + (p_3 - p_4)\} > 0 \tag{116}$$

Therefore, it is obtained that

$$d K / d t > 0 \tag{117}$$

It may be concluded that both a warm low and a cold high pressure system indicate the contribution of potential energy to kinetic energy. This is the typical process of the Hadley cell (Hadley 1735).

Following the same process described above, it may be shown that a cold low and a warm high pressure system indicate a contribution of the kinetic energy to the potential energy. This is the typical case of the Ferrell cell.

The energy diagram is depicted in *Fig. 3*. The directions of the arrows indicate the way in which the various processes take place. Values of energy are given in units of 10^5 Joules. m^{-2}, and values of generation, conversion and dissipation are in Watts. m^{-2}.

Following Holton (1972), the observed energy cycle, in *Fig. 3* suggests the following scenario:

1) The zonal mean radiative heating generates mean ZAPE through a net heating of the tropics and cooling of the polar regions.
2) Baroclinic eddies transport warm air northward, cold air southward and transform the mean APE to EAPE.
3) EAPE is transformed into eddy kinetic energy, EKE, by the vertical motion in the eddies themselves.
4) The zonal kinetic energy, ZKE, is maintained primarily by the conversion from EKE due to the correlation [u'v'].
5) The energy is dissipated by surface friction and internal friction in the eddies and mean flow plus radiative damping in the eddies. Eddies tend to have higher vorticity than the mean flow at the top of the Ekman layer. Hence, much more of the surface dissipation is in the eddies than in the mean flow.

CONCLUSIONS

Instability of parallel inviscid flows was first addressed by Lord Rayleigh (1880). The principal conclusion indicates that if the velocity profile does not have an inflection point, the inviscid flow should be stable.

Eddies may be generated by the horizontal shear of the mean flows. If such is the case, eddies extract energy from the mean flow kinetic energy. However, eddies may extract additional energy from the mean available potential energy field through baroclinic processes. More generally, energy may be supplied to

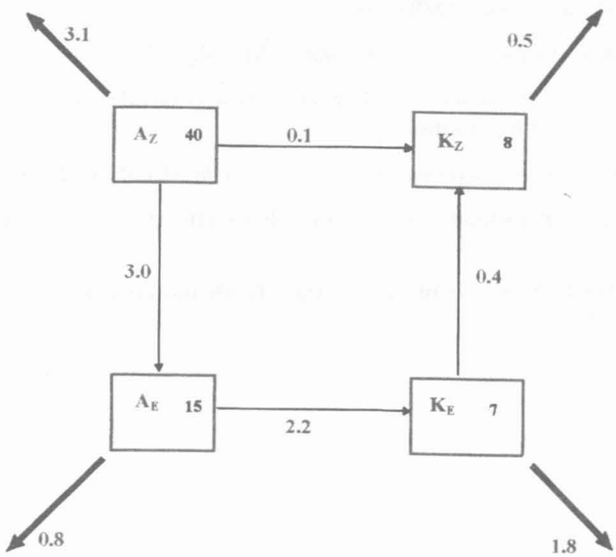

Fig 3. Atmospheric energy cycle, following Oort (1964)

the eddies by both the horizontal shear of the mean flow and the mean available potential energy field.

It may be concluded that the observed atmospheric energy cycle is consistent with the notion that eddies which result from the baroclinic instability of the mean flow are to a larger extent responsible for the energy exchange in the atmosphere. Eddies play the principal role in poleward heat transport to balance the radiation deficit in the polar regions.

In addition to the transient baroclinic eddies, forced stationary orographic waves contribute to the poleward heat transport.

On the other hand, the direct conversion of mean APE to mean KE by symmetric overturning is small and negative in mid-latitudes, but positive in the tropics, where it plays an important role in the maintenance of the mean Hadley circulation (Lorenz 1967).

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