UNIVERSITI PUTRA MALAYSIA

EXPLICIT HYBRID METHODS FOR SOLVING SPECIAL SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS

FAIEZA BINTI SAMAT

FS 2012 60

EXPLICIT HYBRID METHODS FOR SOLVING SPECIAL SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS

By

FAIEZA BINTI SAMAT

Thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

June 2012

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

EXPLICIT HYBRID METHODS FOR SOLVING SPECIAL SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS

By

FAIEZA BINTI SAMAT

June 2012

Chair : Associate Professor Fudziah Ismail, PhD

Faculty : Science

The focus of this thesis is to derive new two-step explicit hybrid methods for the numerical solution of system of special second order ordinary differential equations of the form $y'' = f\left(x, y\right)$. Explicit hybrid methods of order seven have been developed by employing strategies of selecting free parameters. Dissipation relations are imposed to obtain a method with highest possible order of dissipation. Phase-lag and stability analysis are presented. Numerical results show that the methods give better accuracy compared with the existing methods. For variable step-size codes, embedded pairs of explicit hybrid methods are introduced. The phase-lag and stability interval of the methods are given and the procedure of controlling the step-size change is described. To improve the accuracy of hybrid methods, the construction of exponentially fitted explicit hybrid methods is investigated. The derivations of the methods with two stages and four stages are described in detail. The method with two stages is derived for constant step-size code whereas the method with four stages is derived for variable step-size code. Their stability regions and the numerical results are given. Finally, the construction of a block explicit hybrid method implemented on a parallel computer is discussed. This method calculates two consecutive points using two independent formulas. The stability analysis of the formula which computes the second point is presented. The parallel implementation of the method is evaluated in terms of accuracy and speedup. From the results, it is observed that the speedup is greater than 1.5 which indicates that the parallel code is faster than the sequential one. On the whole, this study reveals that the new methods are capable and efficient for solving special second order ordinary differential equations.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

KAEDAH HIBRID TAK TERSIRAT BAGI MENYELESAIKAN PERSAMAAN PEMBEZAAN KHAS PERINGKAT KEDUA

Oleh

FAIEZA BINTI SAMAT

Jun 2012

Pengerusi : **Profesor Madya Fudziah Ismail, PhD**

Fakulti : **Sains**

Tumpuan tesis ini adalah untuk menerbitkan kaedah-kaedah hibrid tak tersirat dua langkah yang baru bagi penyelesaian berangka sistem persamaan pembezaan khas peringkat kedua berbentuk $y'' = f(x, y)$. Kaedah hibrid tak tersirat berperingkat tujuh telah dibina dengan menggunakan strategi pemilihan parameter bebas. Hubungan lesapan telah dikenakan untuk memperoleh kaedah dengan peringkat lesapan setinggi mungkin. Analisis serakan dan kestabilan telah dipersembahkan. Keputusan berangka menunjukkan bahawa kaedah-kaedah ini memberikan ketepatan yang lebih baik berbanding dengan kaedah sedia ada. Kaedah pasangan benaman hibrid tak tersirat diperkenalkan untuk kod saiz langkah berubah. Nilai serakan dan selang kestabilan diberikan manakala tatacara mengawal perubahan saiz langkah diterangkan. Bagi meningkatkan ketepatan kaedah hibrid, pembinaan kaedah penyuaian eksponen hibrid tak tersirat dikaji. Terbitan kaedah yang mempunyai dua dan empat tahap dihuraikan secara terperinci. Kaedah yang mempunyai dua tahap diterbitkan untuk kod saiz langkah malar manakala kaedah yang mempunyai empat tahap pula diterbitkan untuk kod saiz langkah berubah. Rantau kestabilan dan keputusan berangka diberikan. Akhirnya, pembinaan kaedah blok hibrid tak tersirat yang dilaksanakan dalam komputer selari dibincangkan. Kaedah ini mengira dua titik berturutan dengan menggunakan dua formula bebas. Analisis kestabilan bagi formula yang mengira titik kedua dipersembahkan. Pelaksanaan selari kaedah ini dinilai dari segi ketepatan dan kecepatan. Daripada keputusan, diperhatikan bahawa nilai kecepatan lebih besar daripada 1.5, yang menunjukkan bahawa kod selari adalah lebih pantas daripada kod jujukan. Pada keseluruhannya, kajian ini menunjukkan bahawa kaedah-kaedah baru berkebolehan dan cekap untuk menyelesaikan persamaan pembezaan khas peringkat kedua.

ACKNOWLEDGEMENTS

Bismillahirrahmanirrahim. First and foremost, I would like to thank Allah s.w.t. who has given me strengths and health to undergo this tiring yet wonderful journey.

I wish to express my heartfelt gratitude to the chairman of the supervisory committee, Associate Professor Dr Fudziah Ismail for her invaluable assistance and guidance throughout the duration of the studies. This thesis would have never been completed without her continuous help in various aspects of the research.

I am grateful to the member of the supervisory committee, Yang Berbahagia Professor Dato' Dr Mohamed Suleiman and Associate Professor Dr Norihan Md Ariffin for their continuous support. Many thanks also go to Associate Professor Dr Zanariah Abdul Majid from Universiti Putra Malaysia (UPM), Dr Ummul Khair Salma from Universiti Kebangsaan Malaysia (UKM) and Professor Dr Yazid from Universiti Malaysia Terengganu (UMT) for the fruitful private communications.

Special thanks to the staffs of Department of Mathematics, Faculty of Science, staffs of Sultan Abdul Samad Library and staffs of School of Graduate Studies (SGS), UPM for their excellent facilities and services.

I thank Ministry of Higher Education and my employer, Universiti Pendidikan Sultan Idris who has funded this research and granted me study leave. My deepest appreciation goes to my family especially my beloved husband for his support and encouragement during the hard times in my studies. Finally, I would also wish to thank all my friends, whom I met along the journey, for their understanding and support.

I certify that an Examination Committee has met on **25 June 2012** to conduct the final examination of **Faieza binti Samat** on her degree thesis entitled "**Explicit hybrid methods for solving special second order ordinary differential equations**" in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the candidate be awarded the degree of Doctor of Philosophy. Members of the Examination Committee are as follows:

Leong Wah June, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Chairman)

Eshkuvatov Zainidin, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Internal Examiner)

Zarina Bibi Ibrahim, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Internal Examiner)

Abduvali Khaldjigitov, PhD
Faculty of Mechanics and Mathematics
National University of Uzbekistan
(External Examiner)

HASANAH MOHD. GHAZALI, PhD
Professor and Deputy Dean
School of Graduate Studies
Universiti Putra Malaysia
9 August 2012

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of **Doctor of Philosophy**. The members of the Supervisory Committee were as follows:

Fudziah Ismail, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Chairman)

Dato' Mohamed Suleiman, PhD
Professor
Faculty of Science
Universiti Putra Malaysia
(Member)

Norihan Md Arifin, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Member)

BUJANG BIN KIM HUAT, PhD
Professor and Dean
School of Graduate Studies
Universiti Putra Malaysia

DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

FAIEZA BINTI SAMAT
25 June 2012

LIST OF TABLES

LIST OF FIGURES

TABLE OF CONTENTS

CHAPTER

1 INTRODUCTION

2 LITERATURE REVIEW

3 EXPLICIT HYBRID METHODS

4 EMBEDDED EXPLICIT HYBRID METHODS

5 EXPONENTIALLY FITTED EXPLICIT HYBRID METHODS

6 BLOCK EXPLICIT HYBRID METHOD