ROBUST CONTROL CHARTS
FOR CHANGE POINTS DETECTION
IN PRESENCE OF OUTLIERS

By

NG KOOI HUAT

Thesis Submitted to the School of Graduate Studies,
Universiti Putra Malaysia, in Fulfilment of the Requirements for the
Degree of Doctor of Philosophy

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DEDICATIONS

- To my family for having unconditional love for me.

- To my beloved supervisors, lecturers, teachers and friends who uplifted my life.
Abstract of Thesis Presented to the Senate of Universiti Putra Malaysia in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

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February 2012

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Control charts are used to detect whether or not a process has changed. When a control chart signals indicating that a process has changed, practitioners must initiate a search for the special cause. However, given a signal from a control chart, practitioners generally do not know what caused the process situation to change or when the process has changed. Identifying the time of the process change would simplify the seeking of the special cause. It is now evident that outliers have great impact on the parameters estimation in the setting of a control chart. The violation of assumption from normality for change point hypothesis testing method can also gravelly mislead the inferential statistics. Hence, the main focus of this research is to take remedial measures for these issues on the occasion that there is a violation of normality assumption and in the presence of contamination. We have presented
a robust individuals control chart in the context of exploratory analysis for the purpose of locating the step change position. This type of chart offers some significant advantages over the existing individuals control chart. It is about adopting the M-Scale estimator into the proposed modified procedure in the estimation of process standard deviation. The results signify that the proposed method offers substantial improvements over the existing method. On the same ground, to further enhance hypothesis testing approach in the presence of outlier for the change point statistics, the Huber Maximum-Type testing method is incorporated into the proposed modified framework. The findings indicate that the proposed approach is more efficient in detecting the correct step change position, both in normal shift and the shift in the existence of disturbances.

We also proposed a robust MM control chart for monitoring the change in process mean when there is a contamination in data collection. The newly proposed control chart is formulated through the use of S-scale estimate, which in turn yields the MM-location estimate, possessing 50% high breakdown point and 95% efficiency when the errors are under normality (Salibian-Barrera, 2004). From the results, it appears to suggest that the proposed robust MM control chart is more reliable and performs superbly in the presence of outliers.

Finally, the new robust subsample-based Modified Biweight A Scale (MBAS) chart which is resistant to outliers is proposed. A novel scale measure, namely the Modified Biweight A (MBAS) scale estimator is incorporated which provides a choice for practitioners who are interested in the detection of permanent shifts in
process variance. It is evident that the proposed chart outperforms the conventional charts when contaminated data are present. In summary, the proposed robust control-charting methodologies appear to efficiently monitor contaminated data situations and process shift, while the classical charts are not a preference for process monitoring where contamination may exist. In this thesis, all the proposed procedures were examined by real data sets and Monte Carlo simulation studies. Comparative studies among the classical and the proposed robust methods reveal that the proposed robust methods are able to rectify the issues in relation to the presence of outliers. On the contrary, the classical approaches seem to perform poorly in these circumstances.
Abstrak Tesis yang Dikemukakan kepada Senat Universiti Putra Malaysia sebagai Memenuhi Keperluan untuk Ijazah Doktor Falsafah

CARTA KAWALAN TEGUH DALAM PENGENALPASTIAN PERUBAHAN KEDUDUKAN TITIK DENGAN KEHADIRAN TITIK TERPENCIL

Oleh

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Kami juga mengemukakan carta kawalan MM yang baharu bagi memantau penukaran dalam proses purata dengan kewujudan pencemaran data. Cadangan carta kawalan yang baharu ini telah menggunakan penganggar jenis S-skala di mana penganggar titik MM yang dikemukakan mempunyai titik musnah yang tinggi sehingga mencapai 50 peratus dan 95 peratus kecekapan apabila ralatnya berada dalam situasi normal. Keputusan menunjukkan bahawa carta kawalan MM lebih diyakini dan berprestassi tinggi dalam keadaan kehadiran titik terpencil.
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I certify that a thesis Examination Committee has met on 29 February 2012 to conduct the final examination of Ng Kooi Huat on his thesis entitled “Robust Control Charts for Change Points Detection in Presence of Outliers” in accordance with Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institutions.

NG KOOI HUAT

Date: 29 February 2012
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