UNIVERSITI PUTRA MALAYSIA

CONVERGENCE OF INTERVAL SYMMETRIC SINGLE-STEP METHOD FOR SIMULTANEOUS INCLUSION OF REAL POLYNOMIAL ZEROS

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FS 2012 21
CONVERGENCE OF INTERVAL SYMMETRIC SINGLE-STEP METHOD FOR SIMULTANEOUS INCLUSION OF REAL POLYNOMIAL ZEROS

By

NUR RAIDAH BINTI SALIM

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of Requirements for the Degree of Master of Science

January 2012
I would like to dedicate this thesis to my father, Mr. Salim bin Abdullah and mother, Mrs. Hamidah binti Hashim and the rest of my family.
CONVERGENCE OF INTERVAL SYMMETRIC SINGLE-STEP METHOD FOR SIMULTANEOUS INCLUSION OF REAL POLYNOMIAL ZEROS

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January 2012

Chairman: Mansor Bin Monsi, PhD

Faculty: Science

The purpose of this thesis is to find the inclusion of polynomial zeros by using interval analysis approach. We will focus on interval single-step method in order to gain the fastest speed of convergence for bounding simple polynomial zeros simultaneously.

Firstly, we will generally describe on some basic mathematical background on interval analysis approach. Then, we will briefly discuss the procedure given in the literature which has been proved by other researchers. We present some information on interval single-step IS method together with the algorithm and the analysis on the rate of convergence.
In order to improve IS method, we made several modifications using interval analysis approaches whereby it has been proved that these procedures not only including intervals for roots, but also convergent under a few assumptions. We have new modification namely ISS, IZSS and IZMSS methods which are describe precisely in this thesis. The processing time (CPU) of the algorithm of the modified methods may be done using Matlab 2007a associated with Intlab. Nevertheless, we will also present the theoretical analyses of the convergence rate of the modified procedure.

This thesis will cover the algorithms, theoretical analysis and numerical results for each modification. Based on the analysis that has been done, we finally found the rate of convergence for ISS is at least 9, for IZSS is at least 13 and for IMZSS is at least 16 while the rate of convergence of IS is at least $2(1 + \tau) > 3$.

Finally, we conclude our thesis by comparing all the factors needed in a table and we give some possible extensions for future works.
Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

KADAR PENUMPUAN KAEDAH SELANG TUNGGAL SIMMETRIK UNTUK MERANGKUMI PENSIFAR NYATA SECARA SERENTAK BAGI POLINOMIAL

Oleh

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Objektif utama tesis ini adalah untuk merangkumi pensifar nyata bagi suatu polinomial menggunakan pendekatan analisis selang. Kami akan fokus kepada suatu kaedah yang dikenali sebagai langkah selang tunggal yang digunakan untuk mendapatkan kadar penumpuannya yang lebih tinggi semasa memerangkap pensifar secara serentak. Pertama sekali, kami akan menjelaskan secara kasar tentang latar belakang matematik bagi pendekatan analisis selang. Kemudian, kami akan membincangkan tentang kaedah-kaedah terdahulu yang telah dibuktikan benar oleh penyelidik lain. Selepas itu,
kami akan membentangkan sedikit maklumat asas kaedah IS beserta dengan algoritma dan analisis ke atas kadar penumpuannya.


Tesis ini akan merangkumi aspek algoritma, teori analisis dan analisis berangka untuk setiap pengubahsuaian. Berdasarkan kepada analisis yang dilakukan, kadar penumpuan bagi ISS adalah sekurang-kurangnya 9, bagi IZSS kadar penumpuan adalah sekurang-kurangnya 13 dan IMZSS adalah sekurang-kurangnya 16. Sementara kadar penumpuan bagi kaedah IS adalah sekurang-kurangnya 3 atau $2(1+\tau)>3$.
Akhir sekali, kami simpulkan tesis ini dengan membuat perbandingan di antara kesemua faktor dan dibentangkan dalam bentuk jadual dan graf. Di samping itu, beberapa cadangan kajian lanjut turut disertakan.
ACKNOWLEDGEMENTS

Bismillahirrahmannirrahim. Alhamdulillah, I am forever grateful to Allah SWT for giving me strength and determination, health and faith, granted me with great and comfort surroundings, valuable knowledge, loving family and friends during these years. Finally, I have completed my research for my Master’s degree. I am indebted to my supervisor, Dr. Mansor bin Monsi for the guidance, advices, and motivations, constructive comments, concerns and supports without limits throughout this study.

This study would not be possible without various administrative and financial supports from UPM and KPT. I am thankful to Vice Chancellor of UPM for granting me the Graduate Research Fellowship (GRF) for two years of my study.

Special thanks go to Mr. Fakhrul Hazman Yusoff a lecturer from UITM who guide me all the way during the process of constructing the algorithm using programming language software that is MATLAB. I wish to express my sincere gratitude to the members in Department of Mathematics; especially my supervisory committee members for their concerns, Prof. Dr. Malik bin Hj. Abu Hassan and Associate Prof. Dr. Leong Wah for their hard works and assistances throughout this study. I wish to thank them for being remarkable mentors for me since 2008 until the very end of my study. A
thousand appreciations to Associate Prof. Dr. Fudziah Binti Ismail for all the financial supports.

I would like to show my gratitude to my beloved parents, Salim bin Abdullah and Hamidah binti Hashim for the endless encouragement, supports and prayers. I would also like to thank my only sister Nur Fariha Salim for always have faith in me. Finally, my sincere appreciation goes to Mohd. Shafiq bin Mohd. Noh for his continuous motivation, understanding and patience.
I certify that a Thesis Examination Committee has met on 16 January 2012 to conduct the final examination of Nur Raidah Salim on his (or her) thesis entitled “On The Convergence Speed Of The Modified Interval Single-Step Methods For Simultaneous Inclusion Of Real Zeros Of Polynomials” in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or any other institution.

NUR RAIDAH BINTI SALIM.
Date:
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