



**UNIVERSITI PUTRA MALAYSIA**

**MODIFICATIONS OF PARAMETER REGULA FALSI METHOD  
 $p$ -RF FOR INCLUSION OF A ZERO OF A FUNCTION WITH ONE  
REAL VARIABLE**

**NORHALIZA BINTI ABU BAKAR**

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**MODIFICATIONS OF PARAMETER REGULA FALSI  
METHOD  $p$ -RF FOR INCLUSION OF A ZERO OF A  
FUNCTION WITH ONE REAL VARIABLE**



By

**NORHALIZA BINTI ABU BAKAR**

Thesis Submitted to the School of Graduate Studies, Universiti Putra  
Malaysia, in Fulfilment of the Requirements for the Degree of Master  
of Science

January 2012

## DEDICATIONS

*I would like to dedicate this thesis to my father, Mr.Abu Bakar bin Ahmad and mother, Mrs.Norriah binti Masran and the rest of my family.*



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in  
fulfilment of the requirement for the degree of Master of Science

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**Faculty: Science**

The main objective of this thesis is to find a zero of a function using interval analysis approach. Specifically, the focus is on the well-known method called interval parameter regula falsi method ( $p$ -RF).

Three modifications had been made in order to improve the  $p$ -RF method. The new modifications namely  $p$ -RFM1,  $p$ -RFM2 and  $p$ -RFM3 methods were described widely in this thesis. This study also considers the average of central processing unit (CPU) time of the algorithms of the modified methods where they were ran on Matlab R2007a software in associated with Intlab package. The theoretical analysis of the convergence rate of the modified methods were given.

The  $p$ -RFM1 method is focusing on updating the midpoint of current interval in the inner iteration  $i$ . Another inner iteration  $l$  was introduced in  $p$ -RFM1 and the name of this modification is  $p$ -RFM2 method. The calculation of the gradient

of the function in the  $p$ -RFM1 method is approximated using the secant method. The actual gradient of the current midpoint is now replacing the approximated gradient. The modification was named as the  $p$ -RFM3 method.

All the modified methods mentioned above showed better rate of convergence than  $p$ -RF method. This is supported by lesser average CPU times tested on nine test problems. Therefore, it is concluded that the modified methods are better in term of rate of convergence and average CPU time than the original method.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

**PENGUBAHSUAIAN KAEDAH PARAMETER REGULA FALSI  
 $p$ -RF UNTUK MEMERANGKAP SUATU PENSIFAR BAGI  
SUATU FUNGSI PEMBOLEHUBAH NYATA**

Oleh

**NORHALIZA BINTI ABU BAKAR**

**Januari 2012**

**Pengerusi: Mansor Bin Monsi, PhD**

**Fakulti: Sains**

Objektif utama tesis ini ialah mencari suatu pensifar bagi suatu fungsi menggunakan pendekatan analisis selang. Secara khususnya, kajian ini difokuskan kepada suatu kaedah yang terkenal yang dipanggil Kaedah parameter Regula Falsi ( $p$ -RF).

Tiga pengubahsuaian telah dibuat untuk memperbaiki kaedah  $p$ -RF. Pengubahsuaian yang baru adalah kaedah  $p$ -RFM1,  $p$ -RFM2 dan  $p$ -RFM3 yang diterangkan dengan meluas di dalam tesis ini. Kajian ini turut mempertimbangkan purata masa pemprosesan unit (CPU) bagi algoritma kaedah yang telah diubahsuai yang mana kesemuanya dijalankan menggunakan perisian Matlab R2007a dan dibantu oleh pakej Intlab. Analisis teori bagi kadar penumpuan untuk kaedah yang telah diubahsuai diberikan.

Kaedah  $p$ -RFM1 memfokuskan ke atas mengemaskini titik tengah selang semasa pada lelaran dalaman  $i$ . Suatu lagi lelaran dalaman  $l$  telah diperkenalkan dalam

kaedah  $p$ -RFM1 dan nama pengubahsuaian ini adalah kaedah  $p$ -RFM2. Pengiraan kecerunan bagi sesuatu fungsi dalam kaedah  $p$ -RFM1 adalah dianggarkan menggunakan Kaedah Sekan. Kecerunan sebenar pada titik tengah semasa kini menggantikan kecerunan anggaran tersebut. Pengubahsuaian ini dinamakan sebagai kaedah  $p$ -RFM3.

Kesemua kaedah yang telah diubahsuai yang dinyatakan di atas menunjukkan kadar penumpuan yang lebih baik berbanding kaedah  $p$ -RF. Ini diperkukuhkan dengan singkatnya purata masa pemprosesan (CPU) yang diuji ke atas sembilan soalan ujian. Oleh itu, disimpulkan bahawa kaedah-kaedah yang telah diubahsuai adalah lebih baik dalam konteks kadar penumpuan dan purata masa CPU berbanding kaedah asal.

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I certify that a Thesis Examination Committee has met on **12 January 2012** to conduct the final examination of Norhaliza Binti Abu Bakar on her thesis entitled “**Modifications of parameter Regula Falsi Method  $p$ -RF for Inclusion of A Zero of A Function With One Real Variable**” in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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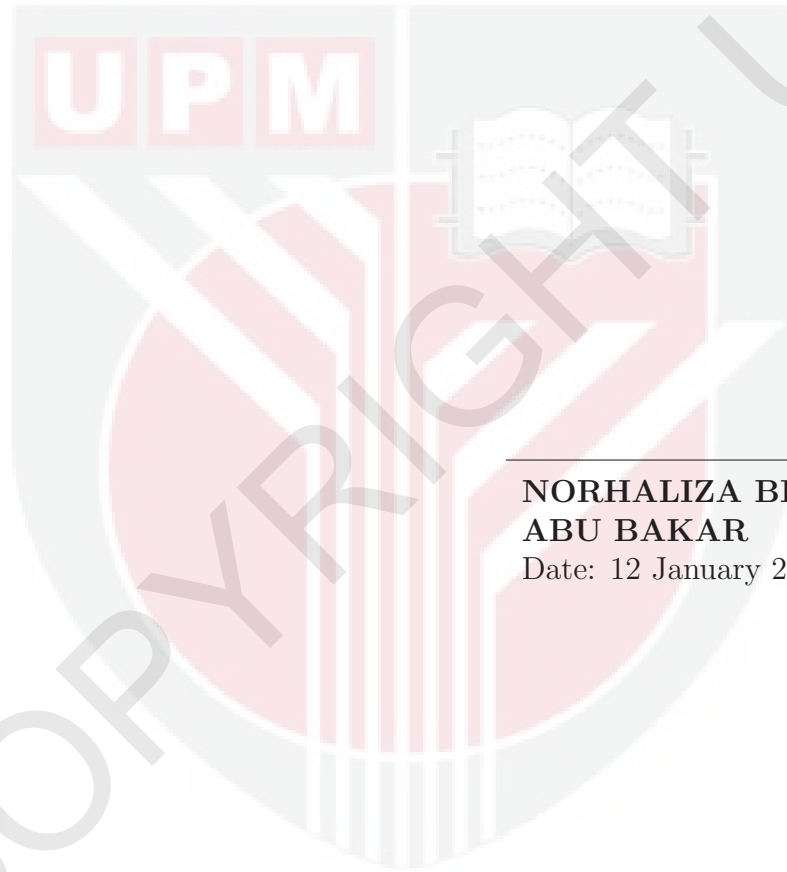
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## DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.



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**NORHALIZA BINTI  
ABU BAKAR**  
Date: 12 January 2012

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