



**UNIVERSITI PUTRA MALAYSIA**

**SKEW ARMENDARIZ RINGS AND THEIR RELATIONS**

**HAMIDEH POURTAHERIAN**

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By

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Thesis Submitted to the School of Graduate Studies, Universiti Putra  
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# DEDICATION

To

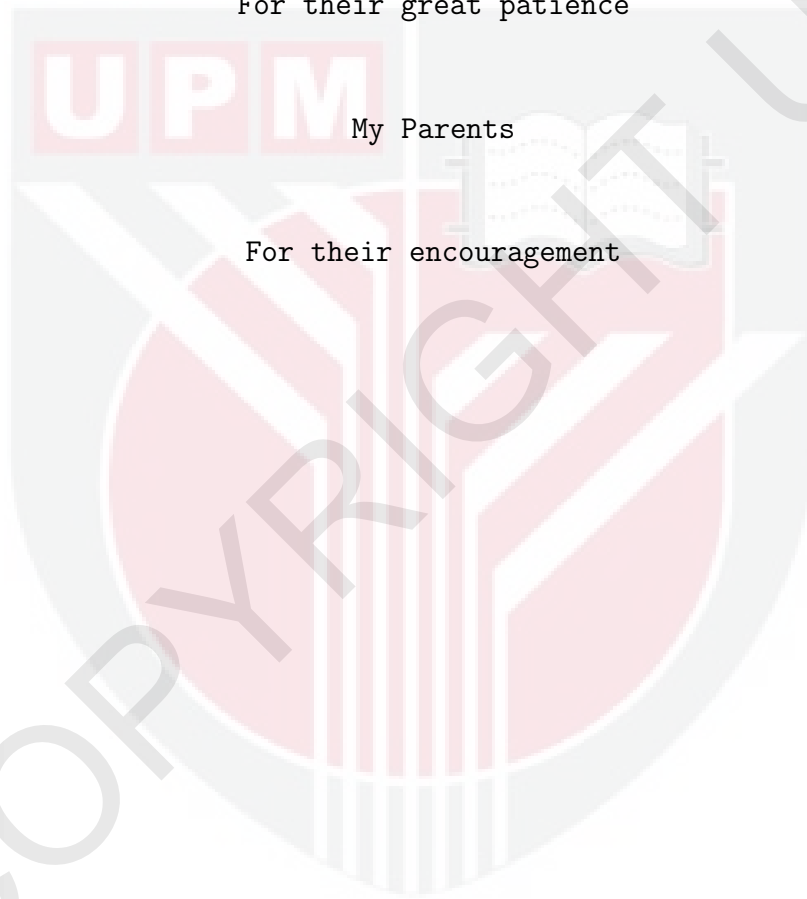
My husband, my son

Samad Doostdar, Artin

For their great patience

My Parents

For their encouragement



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in  
fulfilment of the requirement for the degree of Doctor of Philosophy

## **SKEW ARMENDARIZ RINGS AND THEIR RELATIONS**

By

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**Faculty: Faculty of Science**

The theory of rings is of paramount importance in the realm of algebra, as it deals primarily with the structures required to develop other algebraic theories and their applications. The aim of this project is to investigate a class of rings called Armendariz rings, which generalizes fields and integral domains.

Armendariz rings play an important role in algebra research and related topics. The concept of the Armendariz ring is useful in understanding the relation between annihilators of ring and its polynomial ring and skew polynomial ring. It must be noted that, throughout this research, all rings are associative with identity unless otherwise specified.

The relationships between Armendariz rings and certain other classes of rings shall be dealt with throughout the course of this discussion. Such rings include: Baer and  $p.p.$ -ring, abelian and semi-commutative rings, reversible and symmetric rings, as well as others. In addition, the generalizations of Armendariz rings and classical quotient rings will be considered. Moreover, using this properties of a ring to its polynomial and skew polynomial rings will be investigated. These factors impel the author of this discussion to consider the fact that the ring

has the property of  $*$ , as do its polynomial and skew polynomial ring. This research also defines quasi  $\alpha$ -Armendariz and quasi  $\alpha$ -skew Armendariz rings and their properties, as well as the quasi-Armendariz properties of Laurent type and power series. This is done by considering the quasi-Armendariz condition on polynomials in  $R[x, x^{-1}; \alpha]$  and  $R[[x, x^{-1}; \alpha]]$  instead of  $R[x; \alpha]$ . This research also considers the skew version of rings with respect to ring endomorphism  $\alpha$ , such as  $\alpha$ -reversible ring,  $\alpha$ -symmetric ring,  $\alpha$ -semicommutative ring and  $\alpha$ -Armendariz ring.



**GELANGGANG ARMENDARIZ PENCONG DAN HUBUNGAN  
MEREKA**

Oleh

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Teori gelanggang salah satu bahagian yang penting bagi algebra yang melibatkan struktur yang perlu untuk membangunkan teori algebra yang lain dan penggunaannya. Tujuan utama projek ini ialah untuk mengkaji kelas gelanggang yang dipanggil gelanggang Armendariz, yang mana umumnya medan dan domain integer.

Gelanggang Armendariz memainkan peranan yang penting didalam kajian Algebra dan tajuk berkaitan. Konsep gelanggang Armendariz, berguna di dalam memahami hubungan di antara gelanggang pemusnah habis dan gelanggang polinomialnya dan gelanggang polinomial pencong. Disepanjang kajian ini, semua gelanggang adalah kalis sekutuan dengan identiti kecuali hanya dinyatakan sebaliknya.

Tesis ini juga membincangkan hubungan di antara gelanggang Armendariz dan beberapa kelas gelanggang lain, seperti Baer, gelanggang  $p.p$ , gelanggang Abelian dan gelanggang semi-kalis tukar tertib, gelanggang boleh berbalik dan simetri dan lain-lain gelanggang. Maka gelanggang Armendariz dan gelanggang hasil bahangi klasik secara umum akan dikaji dan menggunakan sifat ini kepada gelanggang

polinomialnya dan gelanggang polinomial pencong. Pernyataan ini memotivasi kami untuk mempertimbangkan gelanggang yang mempunyai sifat  $*$ , begitu juga gelanggang polinomialnya dan gelanggang polinomial pencong. Kajian ini juga memberi takrifan bagi gelanggang Armendariz- $\alpha$  kuasi dan  $\alpha$ -pencong Armendariz kuasi, dan sifat-sifatnya. Begitu juga sifat Armendariz kuasi bagi jenis Laurent dan siri kuasa. Ianya dilakukan dengan mempertimbangkan syarat Armendariz kuasa. Ia dilakukan dengan mempertimbangkan syarat Armendariz-kuasi keatas polinomial di dalam  $R[x, x^{-1}; \alpha]$  dan  $R[[x, x^{-1}; \alpha]]$  daripada  $R[x; \alpha]$ . Kajian ini juga mempertimbangkan gelanggang versi pencong dengan merujuk kepada gelanggang endomorfisma  $\alpha$ , sebagai contoh gelanggang boleh berbalik- $\alpha$ , gelanggang simetri- $\alpha$ , gelanggang semi kalis tukar tertib dan gelanggang Armendariz- $\alpha$ .

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I would like to thank my parents, for their love which has made it possible for me to find my way.

My son Artin was born during this time. He always reminds me with his smile that it is time for me to end this student career.



I certify that a Thesis Examination Committee has met on **31 October 2012** to conduct the final examination of Hamideh Pourtaherian on her thesis entitled “**Skew Armendariz rings and their relations**” in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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## DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.



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