

## An Estimating the p-adic sizes of common zeros of partial derivative polynomials

### Abstract

Let  $x = (x_1, x_2, \dots, x_n)$  be a vector in the space  $Z^n$  with  $Z$  ring of integers and  $q$  be a positive integer,  $f$  a polynomial in  $x$  with coefficients in  $Z$ . The exponential sum associated with  $f$  is defined as  $S(f; q) = \sum_{x \bmod q} e(2\pi i f(x)/q)$  where the sum is taken over a complete set of residues modulo  $q$ . The value of  $S(f; q)$  depend on the estimate of the cardinality  $V$ , the number of elements contained in the set  $V = \{X \bmod q \mid f_x \equiv 0 \pmod{q}\}$  where  $f_x$  is the partial derivatives of  $f$  with respect to  $x$ . To determine the cardinality of  $V$ , the p-adic sizes of common zeros of the partial derivative polynomials need to be obtained. In this paper we estimate the p-adic sizes of common zeros of partial derivative polynomials of  $f(x, y)$  in  $Z_p(x, y)$  of degree nine by using Newton polyhedron technique. The degree nine polynomial is of the form  $f(x, y) = ax^9 + bx^8y + cx^7y^2 + sx + ty + k$ .

**Keyword:** Exponential sums; Cardinality; p-adic sizes; Newton polyhedron.