Developing a Software for Automatic Solution of Higher Order Boundary Value Problems Using the Shooting Method

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Introduction

Currently the code COLNEW is the only one for solving higher order boundary value problem (BVPs) directly which is due to Ascher et al, 1979, 1995, and based on the collocation method.

Materials and Methods

The research here develops an alternative and more efficient algorithm and a code BVPDI for solving Boundary Value Problem (BPVs) for Ordinary Differential Equations (ODEs). A generalised variable order variable stepsize Direct Integration (DI) method, a generalised Backward Differentiation method (BDF) and shooting techniques are used to solve the given BVP. When using simple shooting technique, sometimes stability difficulties arise when the differential operator of the given ODE contains rapidly growing and decaying fundamental solution modes. Then the initial value solution is very sensitive to small changes in the initial condition. In order to decrease the bound of this error, the size of domains over which the Initial Value Problem (IVPs) are integrated has to be restricted. This leads to the multiple shooting technique, which is generalisation of the simple shooting technique. Multiple shooting technique for higher order ODEs with automatic partitioning is designed and successfully implemented in the code BVPDI, to solve the underlying IVP.

Results and Discussion

The well conditioning of a higher order BVP is shown to be related to bounding quantities, one involving the boundary conditions and the other the Green's function. It is also shown that the conditioning of the multiple shooting matrix is related to the given BVP. The numerical results are then compared with the only existing direct method code COLNEW. The advantages in computational time and the accuracy of the computed solution, especially, when the range of interval is large, are pointed out. Also the advantages of BVPDI are clearer when the results are compared with the NAG subroutine D02SAF (reduction method).

Stiffness tests for the system of first order ODEs and the techniques of identifying the equations causing stiffness in a system are discussed. The analysis is extended for the higher order ODEs. Numerical. Results are discussed indicating the advantages of BVPDI code over COLNEW.

Conclusions

The success of the BVPI code applied to the general class of BVPs is the motivation to consider the same code for a special class of second order BVPs called Sturm-Liouville (SL) problem. By the application of Floquet theory and shooting algorithm, eigenvalues of SL problem with periodic boundary conditions are determined without reducing to first order system of equations. Some numerical examples are given to illustrate the success of the method. The results are then compared, when the same problem is reduced to the first order system of equations and the advantages are indicated. The code BVPDI developed here clearly demonstrates the efficiency of using DI Method and shooting techniques for solving higher order BVP for ODEs.

Benefits from the study

The new codes developed are more efficient compared to the existing codes.

Literature cited in the text

Ascher, U., Christiansen, J. and Russell, R.D. 1979. A Collocation Solver for Mixed Order Systems of Boundary Value Problem. *Math. Comput.* 33: 659-679.

Ascher, U., Matheij, R.M.M. and Russell, R.D. 1995. Numerical Solution of Boundary Value Problem for Ordinary Differential Equatious. SIAM Classics in Appd. Maths. Book Series, Philadelphia.

Project Publications in Refereed Journals

Malathi, V., Suleiman, M.B. Computing Eigenvalues of Periodic Sturm-Liouville Problems using Shooting Techniques and Direct Integration Method, *International Journal of Computer Math.* 68: 119-132.

Project Publications in Conference Proceedings

Mohamed Suleiman and Malathi, V. Computing Eigenvalues of periodic Strum-Liouville Problems. In: Proceedings of International Conference on Mathematics and its Applications in the new Millennium. 18-19 July 2000. Renaissance Palm Garden Hotel, Putra Jaya. Pp. 215-224.

Graduate Research

Malathi V. 1999. Numerical Analysis [Ph.d]. Universiti Putra Malaysia.

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