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Computing Dispersion Coefficient of Model Waste Stabilization Ponds

HOCK LYE KOH[1] and POH ENG LIM[2]
[1]*School of Mathematical Sciences*
University Science Malaysia
Minden, Penang, Malaysia.
[2]*School of Chemical Sciences, University Science Malaysia*
Minden, Penang, Malaysia.

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ABSTRAK

Kaedah yang sedia wujud untuk menentukan pekali sebaran bagi kolam penstabilan menggunakan fungsi taburan umur yang ditentukan pada hujung kolam. Biasanya, kepekatan selama dua atau tiga kali masa retensi perlu ditentukan. Ini menimbulkan masalah bagi kolam yang mempunyai masa tahanan yang panjang. Kertas ini akan membincangkan suatu kaedah cekap untuk menentukan pekali sebaran yang memerlukan masa kecerapan yang lebih singkat, biasanya kurang daripada 0.4 kali masa tahanan adalah mencukupi. Hasil yang tercapai setanding dengan atau lebih baik daripada yang terhasil dari kaedah yang sedia wujud.

ABSTRACT

Previous methods of determining dispersion coefficient of waste stabilization ponds used age distribution function observed at the outlet of the pond. Typically, concentration measurements for periods of up to two or three times the theoretical retention time are needed. This poses considerable problems for ponds with a long retention time. This paper presents an efficient method of computing the dispersion coefficient that requires a much shorter period of observation. Typically less than 0.4 times the theoretical retention time is adequate. Results obtained are comparable or superior to those obtained by previous methods.

INTRODUCTION

It has long been recognised that hydraulic flow characteristics play an important role in determining the treatment efficiency of waste stabilization ponds. Basically three types of flow patterns are assumed in the design of stabilization ponds (Thirumurthi, 1969; Thirumurthi, 1974). At the one extreme, it is assumed that the pond exhibits a plug-flow pattern; and at the other extreme the completely-mixed flow pattern. However, in a field pond the flow is neither of the above but rather an intermediate between these two patterns, namely the dispersed flow.

For a pond that exhibits the dispersed flow pattern, it is usual to assume that the dispersion coefficient D is constant over temporal and spatial variation. If S_e and S_i denote the effluent and influent pollutant strength (mg dm^{-3}) respectively, Wehner and Wilhem (1956) derived an equation as follows:

$$\frac{S_e}{S_i} = \frac{4ge^{\frac{1}{2}d}}{(1+g)^2 e^{g/2d} - (1-g)^2 e^{-g/2d}} \qquad (1)$$

In Equation (1), $d = \frac{D}{UL}$ is the dispersion number (dimensionless) $g = \sqrt{1+4kTd}$, with T (days) as the theoretical retention time and k the first-order biochemical removal coefficient (day^{-1}). Further U is the flow velocity (m day^{-1}), L is the travel length (m) of a typical particle, and D the dispersion coefficient (m^2 day^{-1}).

The dispersion number d can be evaluated by conducting tracer studies. The usual method is to inject a tracer slug into the inlet of the pond and then monitor the tracer concentration at the outlet. The resulting concentration at the outlet versus time curve is called the age distribution function which can be used to compute d by a method suggested by Levenspiel and Smith (1957) as described below.

First the mean detention time $\bar{t}$ and the standard deviation σ^2 are computed by using the formulae

$$\bar{t} = \frac{\Sigma t_i c_i}{\Sigma c_i} \tag{2}$$

and

$$\sigma_t^2 = \frac{\Sigma t_i^2 c_i}{\Sigma c_i} - \bar{t}^2 \tag{3}$$

where t_i, c_i are respectively the time and concentration at the end of regular time intervals. The summation is theoretically over an infinite time horizon. However, in practice, it takes up to three theoretical retention times. Values of c_i used in Equations (2) and (3) may be taken at every one-tenth of a retention time from the age distribution function. Finally d is calculated from Equation (4).

$$\sigma_t^2 = \bar{t}^2 \left\{ 2d - 2d^2 (1 - e^{-1/d}) \right\} \tag{4}$$

This method was used by Thirumurthi (1969) and Polprasert *et al.* (1983) to compute d of both field and laboratory-scaled ponds. However, it has several shortcomings. First, it needs concentration readings gathered over 3 retention times. Second it does not provide a measure of how good the answer is. Finally, it does not detect hydraulic flow which deviates significantly from the assumption leading to Equation (4).

In this paper we propose a method that will overcome the above-mentioned problems. Instead of determining the concentrations only at the outlet as was done earlier, we monitor the tracer concentration at five points, one at the inlet, one at the outlet and the remaining ones in the interior. The value of d is then computed by matching the observed concentration with theoretically computed concentrations as explained in the next section. More spatial observation points in excess of five could certainly be used too.

MATERIALS AND METHODS

We assumed that the tracer concentration is uniformly distributed across the plane perpendicular to the flow direction x. Then the equation governing the distribution of concentration of a conservative tracer in the pond is (where c= c(x,t))

$$\frac{\partial c}{\partial t} = -U \frac{\partial c}{\partial x} + D \frac{\partial^2 c}{\partial x^2}, \ x \ \varepsilon \ [0.\ L],\ t > 0, \tag{5}$$

where x, t are respectively the spatial and temporal variables (Arndt *et al.* 1973; Rich 1973). This equation can be reduced to the following dimensionless form

$$\frac{\partial c}{\partial t} = -\frac{\partial c}{\partial x} + d \frac{\partial^2 c}{\partial x^2}, \ x \ \varepsilon \ [0.\ 1],\ t > 0, \tag{6}$$

where $d = \frac{D}{UL}$, x and t are now dimensionless numbers. Effectively the pond has unit travel length and unit retention time.

Equation (6) will be provided with appropriate boundary and initial conditions, and the system will then be solved numerically, or analytically.

The interval [0,1] is divided into N equal subintervals each of length 1/N. Time increment is denoted by Δt.

Boundary Condition at Outlet x = 1

At the outlet, variations in concentration gradient are normally mild. The diffusion term in Equation (6) may therefore be dropped. The boundary condition is then

$$\frac{\partial c}{\partial t} = -\frac{\partial c}{\partial x} \text{ at } x = 1. \tag{7}$$

The corresponding numerical scheme, to first order, is

$$c(1,\ t + \Delta t) = c(1 - \Delta t, t) \tag{8}$$

Boundary Condition at Inlet x = 0

For a pulse release, concentration at the inlet drops to zero quickly. It is observed that this concentration attains zero values usually after 0.4 retention time. Hence we may take

$$c(0,\ t) = 0, \text{ for } t \geq 0.4 \tag{9}$$

However, if Equation (6) has to be solved starting with t < 0.4, the boundary condition ought to be changed. Following the concept expressed in (8), we adopt the following boundary condition, written in numerical scheme,

$$c(0,\ t + \Delta t) = c(0,\ t) + 2\frac{\Delta t}{\Delta x}\left[d\frac{\partial c}{\partial x} - c \right]_{x=0} \tag{10}$$

Numerical Scheme

We used the standard central difference to approximate the diffusion term, the upwind differencing scheme to approximate the advective term, and a one-step explicit method to advance in time (Richtmeyer and Morton 1957).

This scheme has a truncation error $E = 0\ (\Delta t, \Delta x)$, and is equivalent to the equation

$$\frac{\partial c}{\partial t} = -\frac{\partial c}{\partial x} + d\frac{\partial^2 c}{\partial x^2} + \alpha\frac{\partial^2 c}{\partial x^2} + \text{HOT} \qquad (11)$$

where HOT consists of higher order term. The coefficient α is known as numerical dispersion and its value is given by Roache (1972) as follows

$$\alpha = \frac{1}{2}\Delta x(1-a), \text{ where } a = \frac{\Delta t}{\Delta x} \qquad (12)$$

Further, this scheme is stable if

$$a \le 1 \text{ and } b \le \frac{1}{2} \qquad (13)$$

$$\text{where } b = d\frac{\Delta t}{(\Delta x)^2}$$

Verification of Numerical Model

Computational results using this scheme are presented in Koh and Lim (1983), where $\Delta x = 0.025$ with partition points $\{x_i\}_{i=0}^{40}$, and Δt was chosen so that $b < \frac{1}{2}$, typically 0.0005. The effects of numerical dispersion were illustrated and seen to be insignificant. Further analytic results corresponding to plug flow (d=0) were faithfully reproduced numerically by choosing $\Delta t = 0.025$ so that $\alpha = 0$.

Further numerical testing indicates that this scheme is reasonably accurate.

Identification of d

We denote by $c(x, t)$ the observed concentration (interpolated if necessary) and by $\hat{c}(x, t, d)$ the computed solution of Equation (6). The parameter d in $\hat{c}$ indicates the dependence of $\hat{c}$ on d. Values of d are adjusted so as to match the observed and computed concentration c and $\hat{c}$. This is done by minimizing the following functional with respect to d

$$\phi(d) = \sum_{j=1}^{M}\sum_{i=0}^{40}\left\{c(x_i, t_j) - \hat{c}(x_i, t_j, d)\right]^2 \qquad (4)$$

where $\left\{t_j\right\}_{j=1}^{M}$ is observation time.

This least square method has been used to determine the thermal diffusivity in a solid slab by Iordanov and Steward (1984). Values of $c(x_i, t_i)$ are obtained from observed concentration and interpolated by means of cubic spline. In the functional $\phi(d)$ in Equation (14), M typically varies from 3 to 5 with t_j lying in the time interval from t = 0.3 to t = 0.8. The minimizing value of d is obtained by dichotomy with parabolic interpolation (Tan and Koh 1977).

METHODS

A perspex rectangular tank with dimensions of 58.8 × 23.8 × 20.5 cm (*Figure 1*) was set up. To enhance plug-flow characteristics, baffles were inserted laterally such that the downflow and upflow chambers were 9.8 cm wide. The bottom edges of the underflow baffles were 5 cm above the reactor bottom. The liquid surface height was 5.5 cm above the overflow baffle. This gives an effective flow length of 113 cm. Three sampling ports were placed on one side of the reactor at effective distances of 27, 67 and 100 cm from the inlet end.

The tank was first filled with tap water to the desired level and allowed to stand a sufficient time for the current and eddies to die out. A pulse input of one litre of sodium chloride solution (70.71g of the salt dissolved in one litre of solution) was added across the width at the inlet end of the tank as carefully as possible. Immediately after the sodium chloride solution had been added, tap water was fed into the inlet end of the tank via a PVC tube of i.d. 0.5 cm by using a peristaltic pump (Watson-Marlow Ltd.). The point of addition was approximately 1 cm from the end wall of the tank at the midpoint of the width.

A small sample collected from the inlet and samples drained from taps 1, 2, and 3 as well as the outlet were analysed for the concentration of sodium chloride by titration with silver nitrate solution. Collection of samples was done very carefully so that disturbances to the bulk of the tank were kept to a minimum. Concentration was determined regularly at all the points for three theoretical times. It is estimated that the loss of sodium chloride through draining at taps 1, 2 and 3 constituted only 2% of the total amount of tracer-used. Flow rates corresponding to retention time from 0.64 day to 11.24 days were used in this study.

RESULTS AND DISCUSSION

Figure 2 depicts the spatial distribution of observed relative concentration at various times for flow with retention time of 5.76 days interpolated by cubic spline function. The observed relative concentration is the observed concentration divided by the concentration of tracer if the pulse input had been uniformly mixed throughout the entire pond initially. The distributions for other retention times are similar.

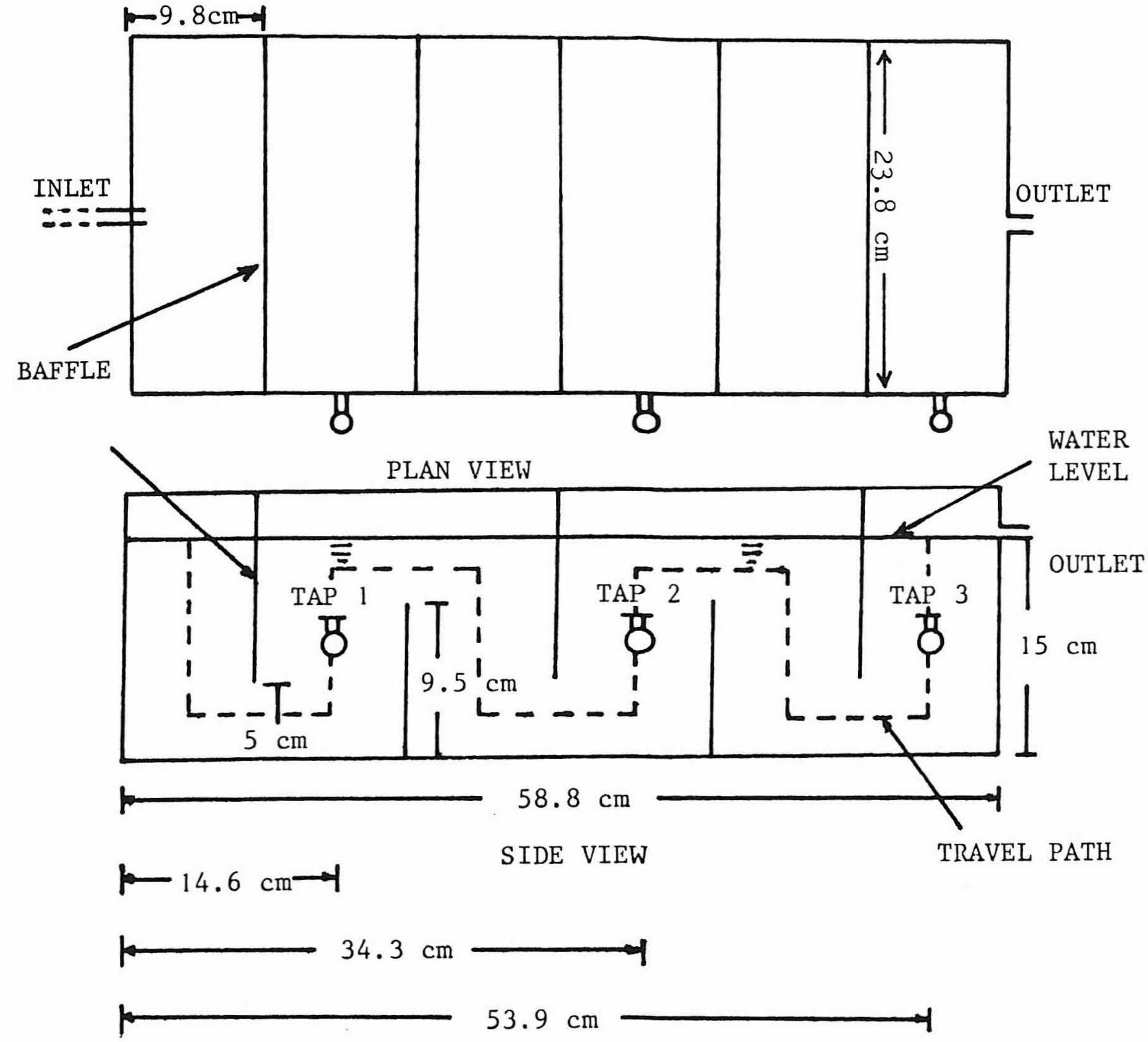

Fig. 1: Experimental set-up

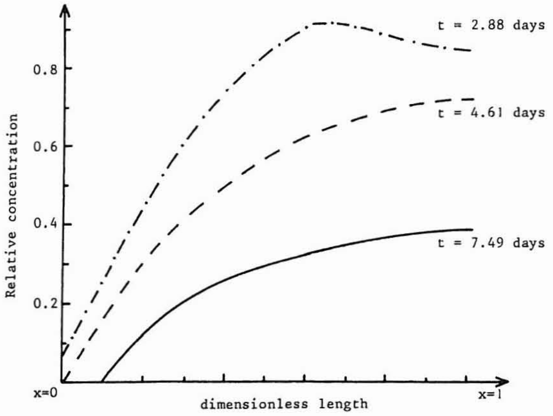

Fig. 2: Spatial distribution of observed relative concentration at various times for case No. 3: 5.76 days retention time (interpolated by cubic spline)

The dispersion coefficients $D(m^2/day)$ computed by the current numerical method and the Levenspiel and Smith method (1957) for various retention times are shown in Table 1. D_* refers to dispersion coefficients computed by the current method whereas D_2 denotes those calculated from Equation (4) in which σ_t and t are computed from Equations (2) and (3) respectively with summation taken over two theoretical retention times. D_3 denotes the corresponding values obtained with summation taken over three theoretical retention times instead. E_*, E_2 and E_3 are the root mean square error between the observed concentrations and those calculated by Equation (5) or (6) with D replaced by D_*, D_2 and D_3 respectively, that is $\sqrt{\phi(d)/(40\times M)}$, where ϕ (d) is given

TABLE 1

Dispersion coefficients D_*, D_2, D_3 with corresponding root mean square errors E_*, E_2, E_3

Case No	T (days)	$D_* \times 10^{-2}$ (m^2/day)	E_* $\times 10^{-2}$	$D_2 \times 10^{-2}$ (m^2/day)	E_2 $\times 10^{-2}$	$D_3 \times 10^{-2}$ (m^2/day)	E_3 $\times 10^{-2}$
1	11.24	3.8	4	1.4	8	2.2	6
2	8.75	5.1	5	1.9	8	2.9	7
3	5.76	8.9	3	3.3	6	5.2	5
4	2.28	13.4	2	12.2	2	20.4	4
5	1.00	27.7	6	26.6	8	*	*
6	0.64	17.7	5	43.6	6	*	*

T = Theoretical retention time in days
* = Equation (4) has no real root, indicating (4) not applicable.

by equation (14).

Figure 3 depicts the variation of dispersion coefficient D_* and D_2 with respect to retention time T for the two different computational methods. The two curves show reasonable agreement between values obtained by these two methods.

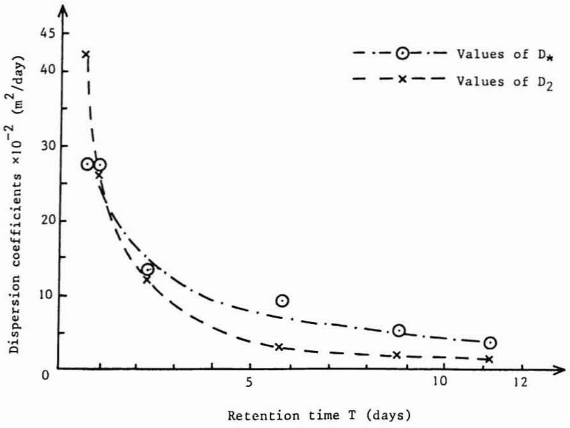

Fig. 3: Plots of dispersion coefficients D_* and D_2 vs retention time T.

Unlike the Levenspiel and Smith method which necessitates concentration data gathered over a relatively long period of time, the current method uses only concentration profiles from 0.3 retention time to 0.8 retention time. This considerable saving in time is made possible by additional readings gathered at the interior points and the inlet, which enables the spatial and temporal distribution of tracer concentrations within the tank to be known. In the case of the Levenspiel and Smith method, the effect of advection and dispersion would be detected only after a longer time lapse since the tracer concentrations are monitored only at the outlet.

By comparing the root mean square errror E_*, E_2 and E_3, it can be seen that the dispersion coefficient computed by this method is consistently superior to the Levenspiel and Smith method for various retention times.

Further, the computed value of D_* which is the minimizing value in the functional $\phi(d)$ in Equation (14) $(d = \frac{D}{UL})$, is not sensitive with respect to variations in observation time t_j. However, it was observed that this insensitivity was no longer valid when there is significant deviation in the assumption assumed in this study. Experiments with tracer dispersing and diffusing from the floor of the pond rather than following the travel path (as shown in *Fig. 1*) indicate that D_* values may vary up to one order of magnitude, depending on the observation time t_j used in minimizing $\phi(d)$. Thus, the current numerical method is able to detect hydraulic flow pattern which deviates significantly from the assumption inherent in models (4) and (5). Finally, it is observed that the tracer in the form of saline solution introduced may well affect the actual flow dynamics of the vessel to some extent. This density effect is not considered in this paper. However, it should be noted that density effect is inherent in the flow dynamics of virtually all field ponds in operation.

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