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SOLVING ORDINARY DIFFERENTIAL EQUATIONS USING BLOCK MULTISTEP METHOD

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By

SIAMAK MEHRKANOON

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

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Of One Essence is the Human Race,
Thusly has Creation put the Base.
One Limb impacted is sufficient,
For all Others to feel the Mace.
The Unconcern'd with Others' Plight,
Are but Brutes with Human Face.

Saadi shirazi, Persian poet(1257)

To

My Dear Father and Mother

For their encouragement

and

My wife, Noushin

For her great patience

and

My respected Teachers

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

SOLVING ORDINARY DIFFERENTIAL EQUATIONS USING BLOCK MULTISTEP METHOD

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March 2011

Chairman: Zanariah Abdul Majid, PhD

Faculty: Science

An efficient code based on block multistep method is developed for solving first and higher order initial value problems (IVPs) of ordinary differential equations (ODEs) using variable step size strategy. Unlike previous methods, the Gauss-Seidel approach is used for the implementation of the proposed method instead of Jacobi iteration. Therefore by applying the current technique, the desired order of accuracy can be achieved with less number of function evaluation, since the latest available information is used to advance the numerical solution through a given step size. The higher order IVPs are solved directly without reducing it into the first order IVPs. In addition, the stability of the proposed methods are discussed.

Furthermore, the parallel version of the proposed methods are constructed in order to solve large system of first and second order of IVPs. The parallelism across the system is considered for the implementation of the parallel block methods. Less computational time is required when the parallel versions are applied compared to sequential methods. The parallel implementation is supported by

Message Passing Interface (MPI). Both sequential and parallel algorithms were carried out on Sunfire V1280 with eight homogeneous processors located at Institute of Mathematical Research (INSPEM), University Putra Malaysia.

Subsequently, a variable block method is proposed to solve first and higher order IVPs of ODEs using variable step size and order strategy. In previous researches, in order to increase the order of the method, it supposed to use the information of more back points. In the proposed technique, attempts have been made to promote the order of the method via involving the advanced points. By utilizing the present code we are able to not only increase the order of the method but also to reach the end of the given interval faster.

In conclusion, the developed new codes deserve to be used for solving system of first and higher order IVPs of ODEs.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

MENYELESAIKAN PERSAMAAN PEMBEZAAN BIASA MENGGUNAKAN KAEDAH BLOK MULTI LANGKAH

Oleh

SIAMAK MEHRKANOON

Mac 2011

Pengerusi: Zanariah Abdul Majid, PhD

Fakulti: Sains

Suatu kod yang efisyen tentang kaedah blok multilangkah telah dibina untuk menyelesaikan masalah nilai awal (MNA) bagi persamaan pembezaan biasa (PPB) peringkat pertama dan tinggi menggunakan strategi saiz langkah berubah. Tidak seperti kaedah yang sebelum ini, pendekatan Gauss-Seidel diimplementasi bersama kaedah yang dicadangkan dan bukan dengan kaedah Lelaran Jacobi. Dengan mengaplikasikan teknik sedia ada, ketepatan peringkat yang diingini boleh dicapai dengan bilangan penilaian fungsi yang sedikit, kerana maklumat terkini yang sedia ada diguna untuk melanjutkan penyelesaian berangka menerusi saiz langkah yang diberi. MNA peringkat tertinggi telah diselesaikan secara terus tanpa menurunkan ia kepada MNA peringkat pertama. Sebagai tambahan, kestabilan kaedah yang dicadang turut dibincangkan.

Tambahan pula, versi selari kaedah yang dicadangkan adalah dibina untuk menyelesaikan sistem besar peringkat kedua dan pertama MNA. Keselarian merentasi sistem dipertimbang untuk pelaksanaan kaedah blok selari. Aplikasi versi selari mengurangkan masa pengiraan berbanding kaedah berjujukan. Pelaksanaan selari dikendalikan oleh *Message Passing Interface* (MPI). Kedua-

dua algoritma berjujukan dan selari telah dilaksanakan keatas *Sunfire* V1280 dengan lapan pemproses-pemproses homogen terletak di Institut Penyelidikan Matematik (INSPEM), Universiti Putra Malaysia.

Seterusnya, kaedah blok berubah adalah dicadangkan untuk menyelesaikan MNA bagi PPB peringkat pertama dan tinggi menggunakan strategi saiz langkah dan peringkat berubah. Dalam penyelidikan terdahulu, untuk meningkatkan peringkat sesuatu kaedah ia seharusnya menggunakan maklumat titik-titik ke belakang. Dalam teknik dicadangkan, percubaan telah dibuat untuk meningkatkan peringkat sesuatu kaedah melalui penglibatan titk-titik ke hadapan. Dengan menggunakan kod ini, kami mampu bukan sahaja meningkatkan peringkat kaedah tetapi juga untuk mencapai akhiran selang yang diberi dengan cepat.

Kesimpulannya, kod-kod baru yang dibina berhak diguna pakai untuk menyelesaikan sistem MNA bagi PPB peringkat pertama dan tinggi.

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the most merciful, the most beneficent

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I certify that a Thesis Examination Committee has met on 14 March 2011 to conduct the final examination of **Siamak Mehrkanoon** on his thesis entitled "**SOLVING ORDINARY DIFFERENTIAL EQUATIONS USING BLOCK MULTISTEP METHOD** " in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of Supervisory Committee were as follows:

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

SIAMAK MEHRKANOON

Date: 14 March 2011

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