

UNIVERSITI PUTRA MALAYSIA

SOLUTIONS OF DIOPHANTINE EQUATION $x^4 + y^4 = p^k z^3$ FOR PRIMES p, $2 \le p \le 13$

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SOLUTIONS OF DIOPHANTINE EQUATION $x^4 + y^4 = p^k z^3$ FOR PRIMES p, $2 \le p \le 13$



By

SHAHRINA BT ISMAIL

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Master

November 2011

To My Beloved Father, Mr. Ismail and My Mother, Madam Jamilah,

Family and Friends



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Master of Science

SOLUTIONS OF DIOPHANTINE EQUATION $x^4 + y^4 = p^k z^3$ FOR PRIMES $p, 2 \le p \le 13$

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SHAHRINA BT ISMAIL

November 2011

Chair: Professor Dato' Hj. Kamel Ariffin Mohd Atan, PhD

Institute: Institute for Mathematical Research

The purpose of this study is to determine the existence, types and the cardinality of the solutions for the diophantine equation $x^4 + y^4 = z^3$ and $x^4 + y^4 = p^k z^3$ for p a prime, $2 \le p \le 13$ and $k \in \mathbb{Z}^+$ in the rings of integers \mathbb{Z} and Gaussian integers $\mathbb{Z}[i]$. Another aim of this study was to develop methods of finding all solutions to these equations. In finding solutions for the diophantine equation $x^4 + y^4 = p^k z^3$ in the rings of integers and Gaussian integers, the values of (p, k) are restricted to (p, k) = $\{(1, 1), (2, 1), (2, k), (3, k), (5, k), (7, k), (11, k), (13, k)\}$. Our research begins by determining the patterns of solutions to these equations. Based on our observation on these patterns, we determine the general form of solution to the equations. Tools and methods in number theory such as divisibility, congruences, properties of prime numbers and method of proof by contradiction are applied in solving these types of diophantine equations. Our result shows that there exist infinitely many solutions to these types of diophantine equations in both rings of integers and Gaussian integers for both cases x = y and $x \neq y$. The main result obtained is formulation of a generalized method to find all the solutions for both types of diophantine equations.



Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

PENYELESAIAN KEPADA PERSAMAAN DIOFANTUS $x^4 + y^4 = p^k z^3$ DENGAN p SUATU NOMBOR PERDANA, $2 \le p \le 13$

Oleh

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November 2011

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Tujuan kajian ini adalah untuk menentukan kewujudan, jenis-jenis dan kekardinalan penyelesaian untuk persamaan diofantus $x^4 + y^4 = z^3 \text{ dan } x^4 + y^4 = p^k z^3$ dengan p suatu nombor perdana, $2 \le p \le 13$ dan $k \in \mathbb{Z}^+$ dalam gelanggang integer \mathbb{Z} dan integer Gaussan $\mathbb{Z}[i]$. Tujuan lain kajian ini adalah untuk membina kaedah mencari semua penyelesaian dalam gelanggang tersebut bagi persamaan-persamaan ini. Untuk mencari penyelesaian bagi persamaan diofantus $x^4 + y^4 = p^k z^3$ ini dalam gelanggang integer dan integer Gaussan, nilai-nilai (p,k) dihadkan kepada (p,k) = $\{(1,1), (2,1), (2,k), (3,k), (5,k), (7,k), (11,k), (13,k)\}$. Penyelidikan kami bermula dengan mendapatkan pola-pola penyelesaian kepada persamaan-persamaan ini. Berdasarkan pencerapan kami terhadap pola-pola ini, kami menentukan bentuk penyelesaian am bagi persamaan-persamaan tersebut. Prinsip-prinsip dalam teori nombor seperti keterbahagian, kongruens, ciri-ciri nombor-nombor perdana dan kaedah bukti secara percanggahan diaplikasikan untuk mencari penyelesaian kepada kedua-dua persamaan diofantus jenis ini. Hasil kajian kami menunjukkan bahawa wujud takterhingga banyaknya penyelesaian bagi kedua-dua persamaan diofantus jenis ini dalam kedua-dua gelanggang integer dan integer Gaussan bagi kedua-dua kes x = y dan $x \neq y$. Hasil kajian utama yang diperolehi ialah pembinaan suatu kaedah am mencari semua penyelesaian kepada kedua-dua jenis persamaan diofantus



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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.



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