

UNIVERSITI PUTRA MALAYSIA

GENERALIZED BARZILAI AND BORWEIN METHOD FOR LARGE-SCALE UNCONSTRAINED OPTIMIZATION

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FS 2011 107

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Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Master of Science

December 2011

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Master of Science

GENERALIZED BARZILAI AND BORWEIN METHOD FOR LARGE-SCALE UNCONSTRAINED OPTIMIZATION

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December 2011

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The focus of the thesis is on finding the unconstrained minimizer of a function by using the fixed steps gradient method. Specifically, we will focus on the Barzilai and Borwein (BB) method. In this thesis, we propose a generalized Barzilai and Borwein (GBB) method that can overcome some disadvantages of the standard BB methods.

The generalized Barzilai and Borwein method presented a special choice of steplength for the gradient method, which is a convex combination of two standard BB methods. Generally, the standard BB method does not guarantee a descent in the objective function at each iteration. We choose different scalar of combination between 0 and 1 to ensure that are descending in function value. This property is shown to be able to reduce the number of iteration in obtaining an approximate minimizer.

The relationship between any gradient method and the shifted power method is considered. This relationship allows us to establish the convergence of the generalized Barzilai and Borwein method when applied to the problem of minimizing any strictly convex quadratic function. To highlight the performance of the generalized Barzilai and Borwein method, we applied them in solving the convex quadratic problem for the cases n = 2, 3, and 4. The results shown the number of iteration is decreasing for almost all cases.

Finally, we concluded the achievements in our research and some future extensions are given at the end of thesis.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

KAEDAH TERITLAK BARZILAI DAN BORWEIN UNTUK SKALA-BERSAR PENGOPTIMUMAN TAK BERKEKANGAN

By

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Fokus tesis ini ialah untuk mencari peminimum tak berkekangan suatu fungsi dengan menggunakan kaedah kecerunan langkah tetap. Khususnya, kami akan memberi tumpuan kepada kaedah Barzilai dan Borwein (BB). Dalam tesis ini, kami mencadangkan satu kaedah teritlak Barzilai dan Borwein yang boleh mengatasi beberapa kelemahan bagi kaedah BB.

Kaedah teritlak Barzilai dan Borwein telah menyediakan pilihan panjang langkah khasnya untuk kaedah kecerunan, di mana ia adalah gabungan cembung dua kaedah BB. Pada unumnya, kaedah BB tidak menjamin penurunan dalam fungsi objektif pada setiap lelaran. Kami memilih skalar yang berlainan iaitu gabungan antara 0 dan 1 untuk memastikan penurunan dalam nilai fungsi. Gabungan ini akan dapat mengurangkan bilangan lelaran untuk menghampiri peminimuman. Hubungan di antara kaedah kecerunan dan kaedah kuasa beralih dipertimbangkan. Hubungan ini membolehkan kami untuk membangkitkan penumpuan kaedah teritlak Barzilai dan Borwein apabila digunakan ke atas masalah peminimuman sebarang fungsi kuadratik yang ketat cembung. Untuk menyerlahkan prestasi kaedah teritlak Barzilai dan Borwein, kami menggunakan kaedah ini dalam menyelesaikan masalah kuadratik cembung untuk kes-kes n = 2, 3, dan 4. Keputusan menunjukkan bahawa bilangan lelaran berkurangan hampir bagi semua kes.

Akhirnya, kami merumuskan pencapaian penyelidikan ini dan beberapa cadangan lanjutan dinyatakan di akhir thesis ini.

ACKNOWLEDGEMENT

First of all, I would like to express my infinite gratitude and sincere appreciation to my chairman, Prof Madya Dr. Leong Wah June for his patience, guidance, and suggestions that enabled me to finish this research. I am also grateful to my co-supervisor, Professor Dr. Malik B. Hj. Abu Hassan and Dr. Mansor B. Monsi.

Beside that, special thanks to the Head of Department and general staffs of the Institute for Mathematical Research, Universiti Putra Malaysia, for their assistance in various capacities. Also, thanks to my financial support by Universiti Putra Malaysia under the Graduate Research Fellowship.

I give my gratitude to my parents for their endless support from beginning. Besides, I appreciate the encouragement of my brothers and sister to further my study. And thanks to all my friends for their support and encouragement. Finally, thanks to you for reading my piece of work. I certify that a Thesis Examination Committee has met on 23rd December 2011 to conduct the final examination of Koo Boon Yuan on his thesis entitled "Generalized Barzilai And Borwein Method For Large-Scale Unconstrained Optimization" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the degree of Master of Science.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.



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