UNIVERSITI PUTRA MALAYSIA

NUMERICAL SOLUTIONS OF LINEAR AND NONLINEAR HIGHER-ORDER BOUNDARY VALUE PROBLEMS BY DIFFERENTIAL RANSFORMATION METHOD AND ADOMIAN DECOMPOSITION METHOD

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By

CHE HAZIQAH BINTI CHE HUSSIN

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

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Abstract of thesis presented to the Senate of Universiti Putra Malaysia
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Chair: Prof. Adem Kilicman, PhD

Faculty: Faculty of Science

In this research, we proposed the generalization of differential transformation method for solving higher-order boundary value problems of higher-order linear and non-linear differential equations. In particular, we extended the existing method to solve nth-order boundary value problems of mth-order linear and mth-order non-linear differential equations. To verify the proposed theorems, we provided proof for each theorem.

In addition, one of the main objectives of this research is to measure the accuracy level of the proposed method. To achieve this objective, we provided some numerical examples for each proposed theorem. We solved each problem in the numerical examples by using the proposed method and modified Adomian decomposition method. Then, we compared the results with the exact solution of each problem. The method with smaller error with the exact solution will be considered as an accurate method.

From the numerical examples also, we want to observe which method that is simpler and easier to implement. We also consider time consumption to calculate DTM and ADM. To achieve all these objectives, we study rigorously the differential transformation method and the modified Adomian decomposition method for solving higher-order boundary value problems.

We focus on differential transformation method for solving linear and nonlinear higher-order boundary value problems. Different types of higher-orders are chosen such as the fourth, fifth, sixth and seventh-order boundary value problems. We observed that, solving each problem by using the modified Adomian decomposition method is very hard and needs more time in calculation.

In addition, the calculation of Adomian's polynomial is tedious. On the contrary, the proposed method is simpler and easier to implement. It involves smaller number of computation steps compared to the modified decomposition method. The linearization, discretization or perturbation also is not required during the calculation processes. From the numerical results, we observed that the proposed method gives closer approximation to the exact solution since the error is smaller than the modified decomposition method. This implies that, the proposed method is more accurate than the modified Adomian decomposition method.

Besides that, we also solved system of differential equations by using differential transformation method for linear and nonlinear equation. These systems of differential equations can't be solved by using modified decomposition method since the method is very hard to implement.

From the results that we obtained, it can reinforce conclusion made by many researchers that differential transformation method is more efficient and accurate than modified Adomian decomposition method. Therefore, we proved the differential transformation method very successful and powerful tool in numerical solution for the bounded domains.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Master Sains

PENYELESAIAN BERANGKA BAGI MASALAH NILAI SEMPADAN TERTIB LEBIH TINGGI LINEAR DAN BUKAN LINEAR DENGAN KAEDAH PENJELMAAN PEMBEZAAN DAN KAEDAH PENGURAIAN ADOMIAN

Oleh

CHE HAZIQAH BINTI CHE HUSSIN

Mac 2011

Pengerusi: Prof. Adem Kilicman, PhD

Fakulti: Fakulti Sains

Dalam penyelidikan ini, kami memperkenalkan bentuk am bagi kaedah penjelmaan pembezaan untuk menyelesaikan masalah nilai sempadan tertib lebih tinggi bagi persamaan-persamaan pembezaan linear dan bukan linear. Secara khususnya, kami mengembangkan kaedah yang sedia ada untuk menyelesaikan masalah nilai sempadan tertib ke-n bagi persamaan-persamaan pembezaan linear tertib ke-m dan bukan linear tertib ke-m. Bagi mengesahkan kebenaran kaedah yang diperkenalkan, kami memberikan pembuktian untuk setiap teorem.

Di samping itu, salah satu matlamat kajian ini adalah untuk mengukur tahap kejituan kaedah yang diperkenalkan. Bagi mencapai matlamat ini, kami memberikan beberapa contoh berangka untuk setiap teorem yang diperkenalkan. Seterusnya, suatu perbandingan antara kaedah yang diperkenalkan dengan kaedah pengubahsuaian penguraian Adomian dilakukan. Kami menyelesaikan setiap masalah yang terdapat dalam contoh-contoh tersebut dengan menggunakan kaedah yang diperkenalkan serta

kaedah pengubahsuaian penguraian Adomian. Kemudian, kami membandingkan keputusan-keputusan yang diperolehi dengan penyelesaian sebenar bagi setiap masalah-masalah berangka tersebut. Kami mempertimbangkan kaedah yang memberikan penyelesaian dengan beza yang lebih kecil dengan penyelesaian sebenar sebagai kaedah yang lebih jitu.

Menerusi penyelesaian contoh-contoh berangka tersebut juga, kami mahu menilai kaedah mana yang lebih ringkas serta mudah untuk digunakan. Bagi memastikan semua matlamat kajian ini tercapai, kami menjalankan penyelidikan yang mendalam terhadap kaedah penjelmaan pembezaan dan kaedah pengubahsuaian penguraian Adomian bagi menyelesaikan masalah-masalah nilai sempadan tertib lebih tinggi.

Secara khususnya, kami memberikan tumpuan terhadap kaedah penjelmaan pembezaan bagi menyelesaikan masalah-masalah nilai sempadan tertib lebih tinggi bagi persamaan-persamaan pembezaan linear dan bukan linear. Beberapa masalah nilai sempadan dengan pelbagai jenis tertib telah dipilih seperti tertib ke-empat, ke-lima, ke-enam dan ke-tujuh. Kami mendapati bahawa, penyelesaian menggunakan kaedah pengubahsuaian penguraian Adomian adalah sangat rumit serta memerlukan masa pengiraan yang lebih panjang.

Di samping itu, pengiraan polinomial Adomian adalah meleret. Berbanding dengan kaedah pengubahsuaian penguraian Adomian, kaedah yang diperkenalkan adalah lebih ringkas serta mudah untuk dilaksanakan. Ia melibatkan bilangan langkah-langkah pengiraan yang lebih kecil. Tiada sebarang penglinearan, pengdiskritan mahupun pengusikan diperlukan sepanjang proses penyelesaian. Berdasarkan keputusan-

keputusan yang diperolehi, kami mendapati bahawa kaedah yang diperkenalkan memberikan nilai-nilai penghampiran yang lebih dekat dengan penyelesaian sebenar memandangkan perbezaan penyelesaian yang dihasilkan dengan penyelesaian sebenar adalah lebih kecil berbanding penyelesaian yang diberikan oleh kaedah pengubahsuaian penguraian Adomian. Ini menandakan bahawa kaedah yang diperkenalkan adalah lebih jitu berbanding kaedah pengubahsuaian penguraian Adomian.

Selain itu, kami juga menyelesaikan sistem persamaan pembezaan dengan menggunakan kaedah penjelmaan pembezaan untuk persamaan linear dan bukan linear. Sistem persamaan pembezaan ini tidak boleh diselesaikan dengan menggunakan kaedah pengubahsuaian penguraian kerana kaedah ini sukar untuk dilaksanakan.

Berdasarkan keputusan yang diperolehi, ia dapat mengukuhkan lagi kesimpulan yang dibuat oleh para penyelidik bahawa kaedah penjelmaan pembezaan adalah cekap dan tepat berbanding kaedah pengubahsuaian penguraian Adomian. Maka, kami telah membuktikan bahawa kaedah penjelmaan pembezaan amnya dan kaedah yang diperkenalkan khasnya adalah merupakan suatu kaedah yang baik serta berkesan dalam penyelesaian berangka bagi domain-domain bersempadan.

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May Allah S.W.T bless all of you. InsyaALLAH. Wassalam.

I certify that an Examination Committee has met on **7th March 2011** to conduct the final examination of Che Haziqah binti Che Hussin on her thesis entitled "**Numerical solutions of linear and nonlinear higher-order boundary value problems by differential transformation method and Adomian decomposition method"** in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

Member of the Examination Committee were as follows:

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of **Master of Science**. The members of Supervisory Committee were as follows:

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

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CHE HAZIQAH BINTI CHE HUSSIN

Date:

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