OPTIMAL STRATEGIES OF PLAYERS IN LINEAR DIFFERENTIAL GAMES

By

MEHDI SALIMI

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

June 2011
DEDICATION

To

My Father and My Mother

For their support, encouragement and love
Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Doctor of Philosophy

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June 2011

Chair: Associate Professor Gafurjan Ibragimov, PhD

Faculty: Science

A game involves a number of players, says $N$, a set of strategies for each player, and a payoff that quantitatively describes the outcome of each play of the game in terms of the amount that each player wins or loses. A common type of game is often called the pursuit-evasion game. Pursuit-evasion game is about how to guide one or a group of pursuers to catch one or a group of moving evaders. In the general definition of a pursuit-evasion game, there will typically be $N$ players with opposing goals, each of them conflicts the other. Each player tries to fulfill his or her goals, and it is assumed that all players always do their best to fulfill their goals. These goals are formally expressed in terms of minimizing or maximizing a payoff functional.

In this thesis, we study a pursuit-evasion differential game of countably many players in Hilbert space. Motions of the players are described by the ordinary differential equations of first and second order. The control functions of players are subject to geometric and integral constraints. Resource for the control
of each pursuer is greater than that of the evader. Duration of the game is fixed. The payoff functional is the greatest lower bound of the distances between the pursuers and evader when the game is terminated. The pursuers try to minimize the payoff functional, and the evader tries to maximize it. We give a formula to calculate the value of the game and construct optimal strategies of the players. To solve the first part of the problem, the pursuit game, we use the method of fictitious pursuers.

In addition, we consider an evasion differential game of several pursuers and one evader with simple motions and integral constraints on control functions of players. We find the sufficient condition for the evader to escape from all pursuers. We present explicit strategy for the evader and show that the proposed escape is possible, no matter what control is adapted by the pursuers. We prove the admissibility of our strategy as well.

Finally, an application of pursuit-evasion game in a missile guidance system is introduced by constructing optimal strategy of pursuer missile which guarantees capturing of the evader missile.
Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

STRATEGI OPTIMUM BAGI PEMAIN DALAM PERMAINAN PEMBEZAAN LINEAR

Oleh
MEHDI SALIMI

Jun 2011

Pengerusi: Profesor Madya Gafurjan Ibragimov, PhD
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Dalam tesis ini, kami mengkaji permainan pembezaan mangsa-pemangsa dengan bilangan pemain yang boleh dikira dalam ruang Hilbert. Pergerakan pemain ini dijelaskan melalui persamaan pembezaan biasa peringkat pertama.


Akhir sekali, aplikasi permainan mangsa-pemangsa dalam sistem kawalan peluru berpandu diperkenalkan dengan membina strategi optimum bagi peluru berpandu pemangsa yang pasti dapat memintas peluru berpandu mangsa.
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I am always indebted to Assoc. Prof. Dr. Javad Laali for encouraging me to pursue my studies.
I certify that a Thesis Examination Committee has met on 14 June 2011 to conduct the final examination of Mehdi Salimi on his thesis entitled "Optimal Strategies of Players in Linear Differential Games" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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ix
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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

MEHDI SALIMI

Date: 14 June 2011
TABLE OF CONTENTS

DEDICATION          ii
ABSTRACT           iii
ABSTRAK            v
ACKNOWLEDGEMENTS   vii
APPROVAL           ix
DECLARATION        xi
LIST OF FIGURES    xiv
LIST OF ABBREVIATIONS  xv

CHAPTER

1 INTRODUCTION AND LITERATURE REVIEW     1
1.1 Introduction to Differential Games  1
1.2 Method to solve Pursuit-Evasion problems  8
1.2.1 The Lion and Man Game         9
1.2.2 A Pursuit Game with Integral Constraints  13
1.3 The Strategy of Parallel Approach (P-strategy)  15
1.3.1 Measurable Control Functions  15
1.3.2 P-strategy when Players are on One Vertical Line  17
1.3.3 P-strategy in General Case  20
1.4 Background of Study            23
1.5 Objective of Thesis           30
1.6 Outline of Thesis            31

2 OPTIMAL PURSUIT WITH COUNTABLY MANY PURSUERS AND ONE EVADER SUBJECT TO GEOMETRIC CONSTRAINTS  33
2.1 Introduction                   33
2.2 Equations of the Motion and Statement of the Problem  34
2.3 An Auxiliary Game              38
2.4 Optimal Strategies and Game Value  43
2.5 Conclusion                     49

3 PURSUIT-EVASION DIFFERENTIAL GAME WITH MANY INERTIAL PLAYERS SUBJECT TO INTEGRAL CONSTRAINTS  51
3.1 Introduction                   51
3.2 Statement of the Problem       53
3.3 An Auxiliary Game              58
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4 Main Result</td>
<td>62</td>
</tr>
<tr>
<td>3.5 Conclusion</td>
<td>69</td>
</tr>
<tr>
<td>4 EVASION FROM MANY PURSUERS IN SIMPLE MOTION DIFFERENTIAL GAME SUBJECT TO INTEGRAL CONSTRAINTS</td>
<td>71</td>
</tr>
<tr>
<td>4.1 Introduction</td>
<td>71</td>
</tr>
<tr>
<td>4.2 Statement of the Problem</td>
<td>73</td>
</tr>
<tr>
<td>4.3 Main Result</td>
<td>74</td>
</tr>
<tr>
<td>4.3.1 Reduction</td>
<td>75</td>
</tr>
<tr>
<td>4.3.2 The Case of One Pursuer</td>
<td>76</td>
</tr>
<tr>
<td>4.3.3 The Case of $k$ Pursuers</td>
<td>79</td>
</tr>
<tr>
<td>4.4 Conclusion</td>
<td>90</td>
</tr>
<tr>
<td>5 APPLICATIONS</td>
<td>91</td>
</tr>
<tr>
<td>5.1 Some Employments of Differential Games</td>
<td>91</td>
</tr>
<tr>
<td>5.2 Pursuit-Evasion Game in Missile Guidance Systems</td>
<td>93</td>
</tr>
<tr>
<td>6 GENERAL CONCLUSION AND FUTURE DIRECTIONS</td>
<td>100</td>
</tr>
<tr>
<td>6.1 General Conclusion</td>
<td>100</td>
</tr>
<tr>
<td>6.2 Future Directions</td>
<td>101</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>103</td>
</tr>
<tr>
<td>APPENDIX</td>
<td>110</td>
</tr>
<tr>
<td>BIODATA OF STUDENT</td>
<td>117</td>
</tr>
<tr>
<td>LIST OF PUBLICATIONS</td>
<td>118</td>
</tr>
</tbody>
</table>