



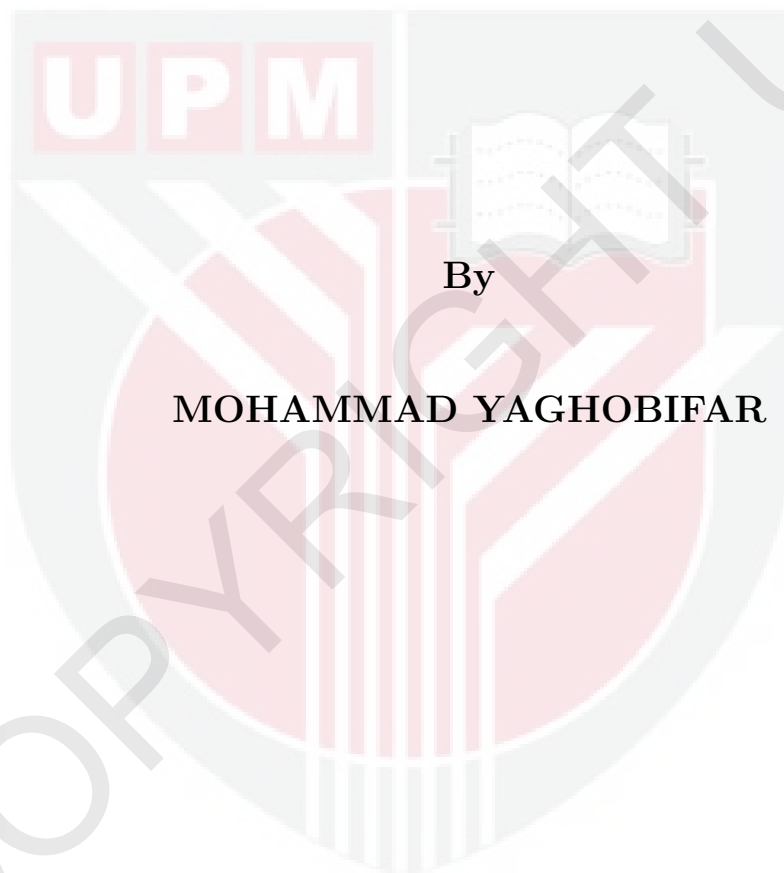
UNIVERSITI PUTRA MALAYSIA

**STRESS ANALYSIS AND THE INTERACTION BETWEEN
HOLE AND STRAIGHT CRACK IN PLANE ELASTICITY**

MOHAMMAD YAGHOBIFAR

FS 2011 80

**STRESS ANALYSIS AND THE INTERACTION
BETWEEN HOLE AND STRAIGHT CRACK IN
PLANE ELASTICITY**



By

MOHAMMAD YAGHOBIFAR

Thesis Submitted to the School of Graduate Studies, Universiti
Putra Malaysia, in Fulfilment of the Requirements for the Degree of
Doctor of Philosophy

March 2011

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Doctor of Philosophy

**STRESS ANALYSIS AND THE INTERACTION BETWEEN
HOLE AND STRAIGHT CRACK IN PLANE ELASTICITY**

By

MOHAMMAD YAGHOBIFAR

March 2011

Chair: Assc. Prof. Nik Mohd Asri Nik Long, PhD

Faculty: Science

Using complex variable methods, some criteria for homothetic transformation and rotation in an infinite plane elasticity are established. Maclaurin series is used to analyze the stresses when the pressure exerted on half plane is of the class $L^1(-\infty, \infty)$. A vibration pressure on the edge of half plane is also considered and the stresses at every point on the half plane is obtained. The problems of two dimensional hole subjected to infinite plane elasticity are considered. These mechanical problems can be written in terms of mathematical boundary value problems. The holes are mapped onto a unit circle by a rational function, then the relevant boundary value problems are solved using modified complex potential (MCP). Analytical results for the stress intensity factor and displacement functions for hypocycloid holes are presented. A circular hole in infinite plane elasticity with a single straight crack outside the circle is considered. Crack opening displacement and stress intensity factor for the mentioned crack are found. Family of hypocycloid hole is considered in an infinite plane elasticity. The stress intensity factor at the cusp points and the displacement functions are obtained. Then a straight crack is considered

outside of the hypocycloid hole. Stresses are obtained at every point outside the hole using the two Muskhelishvili complex potentials.



Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**STRESS ANALYSIS AND THE INTERACTION BETWEEN
HOLE AND STRAIGHT CRACK IN PLANE ELASTICITY**

Oleh

MOHAMMAD YAGHOBIFAR

Mac 2011

Pengerusi: Assc. Prof. Nik Mohd Asri Nik Long, PhD

Fakulti: Sains

Menggunakan kaedah pembolehubah kompleks, beberapa kriteria untuk transformasi homotetik dan putaran dalam satah anjal tak terhingga diterbitkan. Siri Maclaurin digunakan untuk menganalisis regangan apabila tekanan dikenakan ke atas satah separuh adalah dari kelas $L^1(-\infty, \infty)$. Tekanan bergetar ke atas sisi satah separuh juga dipertimbangkan dan regangan pada setiap titik atas satah separuh diperolehi. Masalah lubang dua dimensi tertakluk kepada satah anjal tak terhingga dipertimbangkan. Masalah mekanikal ini boleh ditulis dalam bentuk masalah nilai sempadan bermatematik. Lubang dipetakan ke atas bulatan unit oleh fungsi nisbah, kemudian masalah berkaitan nilai sempadan diselesaikan menggunakan potensi kompleks terubah (MCP). Keputusan beranalisis untuk faktor keamatan regangan dan fungsi sesaran untuk lubang hiposikloid dibentangkan. Satu lubang bulat di dalam satah anjal tak terhingga dengan satu rekahan lurus di luar bulatan dipertimbangkan. Sesaran bukaan dan faktor keamatan regangan untuk rekahan tersebut ditemui. Famili lubang hiposikloid dipertimbangkan dalam satah anjal tak terhingga. Faktor keamatan regangan pada titik bucu dan fungsi sesaran dipovolehi. Kemudian satu rekahan lurus dipertimbangkan di luar lubang hiposikloid. Regangan

diperolehi pada setiap titik di luar lubang menggunakan dua potensi kompleks
Muskhelishvili.



ACKNOWLEDGEMENTS

First of all, praise be to *Allah Subhanahu Wa Taala* for giving me the strength, guidance and patience to complete this thesis. May blessing and peace be upon Prophet Muhammad *Sallallahu Alaihi Wasallam*, who was sent for mercy to the world.

I am particularly grateful to Assc. Prof. Dr. Nik Mohd Asri Nik Long, chairman of the supervisory committee, for his excellent supervision, invaluable guidance, helpful discussion and continuous encouragement. I am grateful for having the opportunity to work under his supervision. His invaluable assistance and comments in the preparation and completion of this thesis are also highly appreciated. I would like to thank the members of supervisory committee, Assc. Prof. Dr. Z.K. Eshkuvatov and Assc. Prof. Dr. Z.A.Majid for their invaluable discussion, comments and help.

I also wish to express my thanks to all of my friends and colleagues during my study at Universiti Putra Malaysia. I would like to thank all staffs of Institute For Mathematical Research (INSPEM) and the Department of Mathematics for their continuous help, encouragement and support.

My deepest gratitude and love to my parents, especially my father in law *Ali Daryanian*, and all of my relatives, for their supports, encouragements, and pray for my success.

This thesis is the result of almost three and a half years of hard work where I have been accompanied by some people. I now have the pleasant opportunity to express my gratitude to all of them.

I certify that a Thesis Examination Committee has met on 28th March 2011 to conduct the final examination of **Mohammad Yaghubifar** on his thesis entitled “**Stress analysis and the interaction between hole and straight crack in plane elasticity**” in accordance with Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

Member of the Thesis Examination Committee were as follows:

Dr. Zarina Bibi Ibrahim, PhD

Department of Mathematics
Universiti Putra Malaysia
(Chairman)

Dr. Mohamed Suleiman , PhD

Professor
Department of Mathematics
Universiti Putra Malaysia
(Internal Examiner)

Dr. Fudziah Ismail, PhD

Associate Professor
Department of Mathematics
Universiti Putra Malaysia
(Internal Examiner)

Dr. P.A. Martin, PhD

Professor
Department of Mathematical and Computer Science
Colorado School of Mines Golden, USA.
(External Examiner)

BUJANG KIM HUAT, PhD

Professor and Deputy Dean
School of Graduate Studies
Universiti Putra Malaysia
Date:

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of Supervisory Committee were as follows:

Nik Mohd Asri Nik Long, PhD

Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Chairman)

Zainidin. K. Eshkuvatov, PhD

Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Member)

Zanariah. A. Majid, PhD

Faculty of Science
Universiti Putra Malaysia
(Member)

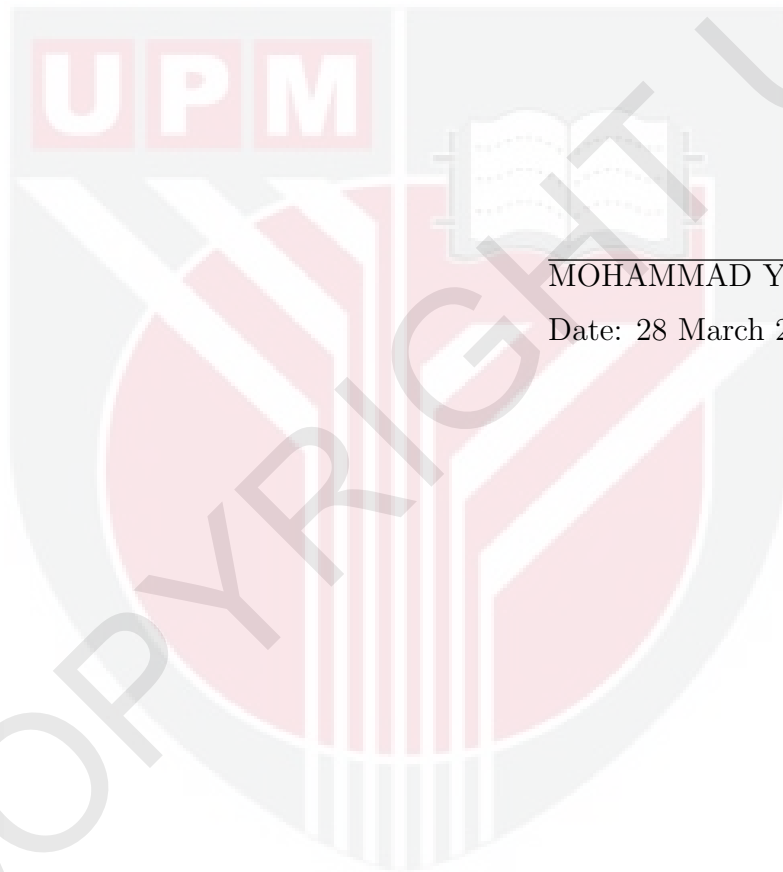
HASANAH MOHD GHAZALI, PhD

Professor and Dean
School of Graduate Studies
University Putra Malaysia

Date:

DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.



MOHAMMAD YAGHOBIFAR

Date: 28 March 2011

TABLE OF CONTENTS

	Page
ABSTRACT	ii
ABSTRAK	iv
ACKNOWLEDGEMENTS	vi
APPROVAL	vii
DECLARATION	ix
LIST OF FIGURES	xiv
CHAPTER 1	
1 INTRODUCTION AND LITERATURE REVIEW	1
1.1 Introduction	1
1.1.1 Body forces and stresses	1
1.1.2 De_finition of stress	2
1.1.3 Stress strain curve	5
1.1.4 Components of stress	6
1.1.5 Equality of shearing stresses	7
1.1.6 Two-dimensional or plane stress	8
1.1.7 Variation of stress within a body	8
1.1.8 Crack deformation modes and basic concepts	12
1.1.9 Stress intensity factor	13
1.1.10 Basic equations of complex variable function method in plane elasticity	14
1.1.11 Eigenfunction expansion form and singular stress field at the vicinity of crack tip	15
1.2 Literature review	20
1.3 Objective of the Research	23
1.4 Outline of Thesis	24
2 FUNDAMENTAL OF THE PROBLEMS	26
2.1 Conformal mapping	26
2.1.1 Simple example of conformal mapping	29
2.2 Modified Complex Potential (MCP)	33
2.2.1 Application of MCP in half-plane elasticity	34
2.2.2 Application of MCP for the case of circular boundary	38
2.3 Complex potentials in the conformal mapping	47
2.4 Integral equation with characteristic hypersingular kernel	48
2.5 Biharmonic equation	48
2.5.1 Converting biharmonic equation into complex form	50
2.6 Fourier analysis	51
2.6.1 Fourier transformation	52
2.6.2 Fourier series	53

2.7 Application of Fourier transformation for half plane elasticity	54
2.7.1 Fourier transform of Biharmonic Equation	54
3 CRITERIA FOR HOMOTHETIC TRANSFORMATION AND ROTATION FOR A HOLE IN PLANE ELASTICITY	56
3.1 Introduction	56
3.2 Governing equation	57
3.3 Homothetic transformation	57
3.4 Rotation	58
3.5 Conclusion	60
4 STRESS ANALYSIS IN HALF PLANE ELASTICITY FOR REGULAR FUNCTIONS	62
4.1 Introduction	62
4.2 Elastic Equilibrium under surface forces	64
4.3 The basis	66
4.4 Evaluation of $\phi_n(x; y)$ and $\psi_n(x; y)$	67
4.5 Evaluation of stresses	68
4.6 Example	69
4.7 Convergence of the series	69
4.8 Conclusion	71
5 ANALYSIS OF STRESS IN HALF PLANE ELASTICITY WITH PERIODIC LOAD	72
5.1 Introduction	72
5.2 Boundary value problem	73
5.3 Fundamental solutions	74
5.3.1 Case of $p(y) = a \sin by$	74
5.3.2 Case of $p(y) = a \cos by$	75
5.3.3 Case of constant pressure	75
5.4 Solution for periodic load	75
5.5 Evaluation of stresses	76
5.6 Example	76
6 ANALYTICAL SOLUTION FOR THE INTERACTION BETWEEN A STRAIGHT CRACK AND A CIRCULAR HOLE IN PLANE ELASTICITY	78
6.1 Introduction	78
6.2 Analysis	79
6.3 Stress Problems for B and C	80
6.4 Hypersingular integral equation for problem D	86
6.5 Crack opening displacement	91
6.6 Stress intensity factor	91
6.7 Examples	93
6.8 Conclusion	94

7 STRESS INTENSITY FACTOR FOR A HYPOCYCLOID HOLE AND ITS INTERACTION WITH A STRAIGHT CRACK	97
7.1 Introduction	97
7.2 Analysis	98
7.3 The coefficients	101
7.4 Stress intensity factors	102
7.5 Displacement functions and deformation of hypocycloid hole due to stress and strain	103
7.6 Examples	104
7.6.1 For $\frac{3}{4}a_{1x} = 1; \frac{3}{4}a_{1y} = \frac{3}{4}a_{1xy} = 0$	104
7.6.2 For $\frac{3}{4}a_{1x} = 0; \frac{3}{4}a_{1y} = 1; \frac{3}{4}a_{1xy} = 0$	104
7.7 Stress field around the hypocycloid hole	106
7.8 Crack opening displacement	108
7.9 Stress intensity factor	110
7.10 Conclusion	111
8 CONCLUSION AND FUTURE RESEARCH	112
8.1 Conclusion	112
8.2 Future Research	112
BIBLIOGRAPHY	114
APPENDIX	122
BIODATA OF STUDENT	124
LIST OF PUBLICATIONS	125